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THERMAL BEHAVIOUR OF OVERHEAD CONDUCTORS

Working Group
22.12

August 2002

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OF OVERHEAD CONDUCTORS

WORKING GROUP

22.12

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THERMAL BEHAVIOUR OF OVERHEAD CONDUCTORS

Study Committee 22 - Working Group 12

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INTRODUCTION

In most countries the demand for electric power is constantly increasing, and there is a corresponding requirement to increase the power transferred by transmission and distribution lines. A solution would be to build new lines, but this may not be feasible on account of economic or environmental consideration. Hence, there may be pressure to increase the load transfer capacity of both new and old lines.

The maximum load capacity of a long line is usually dictated by consideration of system stability, permissible voltage regulation or the cost of energy losses. The capacity of a shorter line may be determined by the maximum permissible operating temperature of the conductors, assuming that the joints and clamps are in good condition and are not a constraint in operation. The maximum permissible temperature is that which results in the greatest permissible sag (allowing for creep), or that which results in the maximum allowable loss of tensile strength by annealing throughout the life of the conductor.

The conductor temperature will depend on the load current, the electrical characteristics of the conductor, and the atmospheric parameters such as wind and sun. The relationship between these parameters is known as the heat equation, and the heat equation for normal operation (the steady state or quasi-steady state) is examined in detail in Section 1. In this section the heat balance equation:

Heat gain = heat loss

is applied. It is assumed in this section that the conductor is in thermal equilibrium. There is no heat stored in the conductor.

Section 1 is used to determine the thermal rating of conductors in the design and planning stages of the line. It can also be used to determine whether there is any available capacity in existing lines.

Section 2 is used to determine the temperature of the conductor while not in thermal equilibrium. This is used in the real time monitoring systems, which use weather stations and/or other thermal or tension conductor measuring devices, which measure the position of the conductor position in space. Due to the conductor changing position in space continually and that the thermal rating is a function of the position in space the thermal rating also changes continually. Section 2 applies the equation:

Heat stored in conductor = heat gain – heat loss

Section 3 is used to determine the temperature of the conductor under short-circuit conditions. This assumes that there is no cooling (adiabatic state). The equation applied is thus:

Heat stored = heat gained

SECTION 1

MATHEMATICAL MODEL FOR EVALUATION OF CONDUCTOR TEMPERATURE IN THE STEADY (OR QUASI-STEADY) STATE (NORMAL OPERATION)

1. INTRODUCTION

The meteorological parameters influencing the thermal state of the conductor include the mean wind velocity, direction and turbulence, ambient temperature and solar radiation. Assuming these, and the electrical load to be fairly constant, then the conductor temperature does not change significantly.

In this situation the heat supplied to the conductor is balanced by the heat dissipated (no heat energy is stored in the conductor), the thermal condition of the conductor is then defined as steady state. A heat balance equation can thus be written:

Heat gain = heat loss

$$P_J + P_M + P_S + P_i = P_c + P_r + P_w \tag{1}$$

where

P_J = Joule heating
P_M = magnetic heating
P_S = solar heating
P_i = corona heating
P_c = convective cooling
P_r = radiative cooling
P_w = evaporative cooling

2. HEAT GAIN

This section deals with the analysis of the terms on the left hand side of eq. (1).

2.1. CURRENT HEATING

Current heating is the heating of the conductor due to the effects of load current and includes the Joule, magnetic and skin effects.

Note that the methods described in this paper are approximate. For current densities in excess of 1.5 A/mm^2 it is recommended to refer to more detailed calculation methods.

Joule heating refers to the heating of the conductor due to the resistance of the conductor.

The magnetic effect refers to heating of the conductor due to cyclic magnetic flux which causes heating by eddy currents, hysteresis and magnetic viscosity [3]. This phenomenon occurs only with alternating current and is usually negligible with non-ferrous conductors at power frequency but could be significant with steel-cored conductors. This is because, in steel-cored conductors, a longitudinal magnetic flux is produced in the steel wires by current in the non-ferrous wires spiralling around the steel core.

The skin effect refers to the increase in conductor resistance as a function of the frequency of the alternating current.

The evaluation of the current heating phenomenon for non-ferrous conductors is best performed by evaluating the Joule effect, including the skin effect (Method 1).

The accurate evaluation of the current heating phenomenon of steel-cored conductors needs to take into account the power loss in the steel and non-uniform distribution of current density, particularly, with an odd number of layers of non-ferrous wires [20, 26, 27]. A simplified, less accurate approach is given below.

2.1.1. Joule heating for non-ferrous conductors (Method 1)

The Joule heat gain is calculated by:

$$P_J = k_j I^2 R_{dc}\left[1+\alpha\left(T_{av}-20\right)\right] \tag{2}$$

where I is the effective current, R_{dc} is the dc resistance at 20°C per unit length, α is the temperature coefficient of resistance per degree Kelvin and T_{av} is the mean temperature of the conductor. The factor k_j takes into account the increase in resistance due to skin effects.

For the average value of k_j it is suggested to use 1.0123 (refer section 2.1.2).

The ac resistance can be calculated as follows:

$$R_{ac} = k_j R_{dc} \tag{3}$$

2.1.2 Calculation of current heating effects for steel cored conductors (Method 2) [1]

The simplified theory is based on the equality of power inputs for both ac and dc currents for the same average temperature of the conductor. The dc current that will result in a certain temperature being reached is calculated and the empirical formulae are then used to convert the dc to the ac current. Similarly, should the temperature need to be calculated for a given ac current, the empirical formulae are used to evaluate the equivalent dc current and hence the rise in temperature due to it. Eq. (2) is then reduced to:

$$P_J = I_{dc}{}^2 R_{dc}\left[1+\alpha\left(T_{av}-20\right)\right] \tag{4}$$

The power input must be the same for both ac and dc for the same average temperature of the conductor. Thus:

$$I_{ac}^2 R_{ac} = I_{dc}^2 R_{dc} \tag{5}$$

The following equations are based on the results of measurements on stranded conductors [1].

For aluminium-steel conductors with 3 layers of aluminium wires e.g. 428-A1/S1A-54/7 'Zebra':

$$I_{ac} = \frac{I_{dc}}{\sqrt{1.0123 + 2.319 \cdot 10^{-5} I_{dc}}} \quad (6)$$

$$I_{dc} = I_{ac}\sqrt{1.0123 + 2.36 \cdot 10^{-5} I_{ac}} \quad (7)$$

From eq. (5) it can be seen that $R_{ac}/R_{dc} = (I_{dc}/I_{ac})^2$. Hence for the 3-layer conductor, $R_{ac}/R_{dc} = 1.0123 + 2.36 \cdot 10^{-5} I_{ac}$, from eq. (7).

Note that the value of R_{ac}/R_{dc} with $I = 0$ is 1.0123. This is the skin effect factor. The current-dependent term will vary with each 3 layer aluminium-steel construction since lay lengths differ from conductor to conductor.

For aluminium-steel conductors with 1 or 2 layers of aluminium wires and a nominal cross section A = 175 mm^2 or more:

$$I_{ac} = \frac{I_{dc}}{\sqrt{1.0045 + 0.09 \cdot 10^{-6} I_{dc}}} \quad (8)$$

For other aluminium-steel conductors, single and double layer aluminium wire with a nominal cross sectional area A < 175 mm^2 (let $I_k = I_{dc}/A$, in A/mm^2):

If $I_k \leq 0.742$ then: $I_{ac} = I_{dc}$

If $0.742 < I_k \leq 2.486$ then:

$I_{ac} = I_{dc} /[1+ 0.02(25.62-133.9I_k+ 288.6I_k^2-334.5I_k^3+226.5I_k^4-89.73I_k^5+19.31I_k^6-1.744I_k^7)]^{½}$

If $2.486 < I_k \leq 3.908$ then:

$I_{ac} = I_{dc} /[1+0.02(2.978-22.02I_k+24.87I_k^2-11.64I_k^3+2.973I_k^4-0.4135I_k^5+0.02445I_k^6)]^{½}$

If $I_k > 3.908$ then: $I_{ac} = I_{dc}/(1.1)^{½}$

2.1.3 Conductor dc resistance [2]

The dc resistance of the conductor may be calculated by using the formulae in appendix IV. The resistivity and temperature coefficient values for typical conductor materials are given in table III.

2.1.4 Radial temperature distribution [3]

It is important to take into account the radial temperature distribution within a conductor mainly for two reasons:

a) the resistance depends on the average conductor temperature T_{av}
b) the sag depends on the core temperature T_c

A complex method to calculate the radial temperature distribution in stranded conductors is given in [12]. However, since relatively little heat is generated in the steel core of cored conductors and assuming that the internal heat generation is uniform, a simplified equation to calculate the radial temperature difference can be written as:

$$T_c - T_s = \frac{P_T}{2\pi\lambda}\left[\frac{1}{2} - \frac{D_2^2}{D^2 - D_2^2}\left(\ln\frac{D}{D_2}\right)\right] \tag{9}$$

where

P_T	= the total heat gain per unit length
D	= the external conductor diameter, i.e. the outer diameter
D_2	= the internal diameter which is the diameter of the steel core
T_s	= surface temperature
T_c	= core temperature
λ	= effective radial thermal conductivity

It is found from measurements that the mean value of the radial thermal conductivity λ is about 2 W/mK. Due to the fact that the difference between the core and surface temperature is between 0.5 °C and 7 °C it is generally sufficient to assume $T_{av} = T_s$.

Eq. (9) applies to hollow-core or steel-cored conductors. For all aluminium and all aluminium alloy conductors $D_2 = 0$, hence:

$$T_c - T_s = \frac{P_T}{4\pi\lambda} \tag{10}$$

2.2 SOLAR HEATING [4]

The solar heat gain P_S depends on the diameter of the conductor and (to a lesser extent) its inclination ζ to the horizontal, the absorptivity α_s of the surface of the conductor, the intensities I_D, the direct solar radiation on a surface normal to the beam and I_d, the diffuse sky radiation to a horizontal surface; the solar altitude H_S, the angle η of the solar beam with respect to the axis of the conductor, and the albedo (reflectance) F of the surface of the ground beneath the conductor.

2.2.1 Calculation of solar heating

The solar heat gain may be calculated if all the above variables including both the direct and diffuse solar radiation are known. However, in practice, the direct solar radiation meters prove to be expensive. On the other hand diffuse solar radiation meters need regular attention and thus it is not feasible to use them on remote sites. Global solar radiation meters are relatively inexpensive and reliable. For the above reasons the method using global solar radiation is given below. The other more complex method using all the variables given above is given in Appendix V.

The solar heating using global solar radiation can be written as:

$$P_S = \alpha_s S D \tag{11}$$

where

α_s	= absorptivity of conductor surface
S	= global solar radiation
D	= external diameter of conductor

The value of α_s varies from 0.23 for bright stranded aluminium conductor to 0.95 for weathered conductor in industrial environment. For most purposes a value of 0.5 may be used for α_s.

2.3 CORONA HEATING [4]

Corona heating is only significant with high surface voltage gradients which are present during precipitation and high wind where convective and evaporative cooling is high. Due to this fact and the fact that it is considered necessary to evaluate the maximum rating of lines based on average or high ambient steady state conditions it is not considered necessary to include formulae for the calculation of corona heating.

3. HEAT LOSS

This section deals with the analysis of the terms on the right hand side of eq. (1).

3.1 CONVECTIVE COOLING [3, 4]

The hot surface of the conductor heats the air adjacent to it, and the density of the heated air decreases, thus causing it to rise in the case of natural convection ($V = 0$), or to be carried away in the case of forced convection ($V \neq 0$). Colder air flows in to replace the heated air, thus cooling the conductor. Dimensional analysis has shown that certain non-dimensional groupings of parameters are useful in calculating convective heat transfer. These are:

i) The Nusselt number, $Nu = h_c D/\lambda_f$, where h_c is the coefficient of convective heat transfer (W/m^2K) and λ_f is the thermal conductivity of air (W/mK).

ii) The Reynolds number, $Re = \rho_r VD/\nu_f$, where V is the wind velocity (m/s), ν_f is the kinematic viscosity (m^2/s) and ρ_r is the relative air density ($\rho_r = \rho/\rho_0$, where ρ is the air density at the altitude in question and ρ_0 is the air density at sea level).

iii) The Grashof number, $Gr = D^3(T_s - T_a)g/(T_f + 273)\nu_f^2$, where T_s is the conductor surface temperature and T_a is the ambient temperature.

iv) The Prandtl number $Pr = c\mu/\lambda_f$, where c is the specific heat capacity of air at constant pressure (J/kgK) and μ is the dynamic viscosity of air (kg/ms).

The empirical equations for calculating the above variables are:

ν_f	$= 1.32\cdot10^{-5} + 9.5\cdot10^{-8}\, T_f$
λ_f	$= 2.42\cdot10^{-2} + 7.2\cdot10^{-5}\, T_f$
Pr	$= 0.715 - 2.5\cdot10^{-4}\, T_f$
g	$= 9.807\ (m/s^2)$
T_f	$= 0.5(T_s + T_a)$

ρ_r $= \exp(-1.16 \cdot 10^{-4} y)$, where y is the height above sea level (m) [7]

The convective heat loss is given by:

$$P_c = \pi \lambda_f (T_s - T_a) Nu \tag{12}$$

where the Nusselt number can be found from the equations in cl. 3.1.1 and 3.1.2 for forced and natural convection respectively.

3.1.1 Forced convective cooling

In the normal operating range of film temperature $T_f = 0.5(T_s+T_a)$ the Nusselt number can be represented by:

$$Nu = B_1 (Re)^n \tag{13}$$

where B_1 and n are constants depending on the Reynolds number and conductor surface roughness $R_f = d/[2(D-d)]$, found in table I.

Table I. Constants for calculation of forced convective heat transfer from conductors with steady crossflow of air

Surface	***Re***		B_1	n
	from	**to**		
Stranded all surfaces	10^2	$2.65 \cdot 10^3$	0.641	0.471
Stranded $R_f \leq 0.05$	$> 2.65 \cdot 10^3$	$5 \cdot 10^4$	0.178	0.633
Stranded $R_f > 0.05$	$> 2.65 \cdot 10^3$	$5 \cdot 10^4$	0.048	0.800

The wire diameter d should be the outer layer wire diameter (usually non-ferrous).

The conductor diameter D should be the overall diameter despite the fact that a stranded conductor may have a surface area of 40 - 45 % greater than a smooth conductor of the same diameter. This is because the boundary layer detaches from each wire and re-attaches at the next, thus forming stagnant zones at the interstices. The increase, with regard to forced convective cooling, between stranded and smooth conductors is a function of the roughness and the Reynolds number.

When dealing with a transmission line it may be of interest to note that the wind velocity is dependent on the height above ground, terrain and other factors. Details may be found in [8].

The wind direction plays an important role in the effectiveness of the forced convective cooling. The Nusselt number varies as the sine of the angle of attack δ (with respect to the axis of the conductor) as follows [6]:

$$Nu_\delta = Nu_{90}\left[A_1 + B_2(\sin\delta)^{m_1}\right] \tag{14}$$

where

$A_1 = 0.42$, $B_2 = 0.68$ and $m_1 = 1.08$ for $0° < \delta < 24°$
$A_1 = 0.42$, $B_2 = 0.58$ and $m_1 = 0.90$ for $24° < \delta < 90°$

When the wind blows parallel to the conductor axis the Nusselt number with a wind angle of 0° drops to around 0.42 Nu_{90}. This is due to swirling of the flow due to the stranding of the conductor.

With low wind velocity ($V < 0.5$ m/s), however, it has been found that there is no preferred wind direction and the Nusselt number is unlikely to go below (refer section 3.1.3 for calculation of cooling at low wind speeds):

$$Nu_{cor} = 0.55 Nu_{90} \tag{15}$$

Nu_{cor} is the corrected Nusselt number.

3.1.2 Natural convective cooling [3, 4]

The Nusselt number for natural convective cooling depends on the product of Grashof and Prandtl numbers:

$$Nu = A_2(Gr \cdot \Pr)^{m_1} \tag{16}$$

Values for the constants A_2 and m_2 for various ranges of the Rayleigh number $Gr{\cdot}Pr$ are given in table II below:

Table II. Constants for calculation of natural convective heat transfer from conductors in air

Gr·Pr		***A₂***	***m₂***
from	**to**		
10^2	10^4	0.850	0.188
10^4	10^6	0.480	0.250

3.1.3 Cooling at low wind speeds

At low wind speeds ($V < 0.5$ m/s) calculations can be based on mixed forced and natural convection [3]. However, a simplified method [9, 10] can be used. This method calculates three convective cooling values and the largest is then selected:

a) Since there is no preferred wind direction, an angle of attack of 45° is assumed and the forced convection heat loss P_{fa} is calculated using eq. (14) and (12).
b) The second value P_{fb} is calculated using eq. (15) and (12).
c) The natural convective heat loss P_n is calculated using eq. (16).

The largest value of cooling from methods (a) - (c) above is then used.

3.2 RADIATIVE COOLING [3, 6]

Due to the fact that the radiation loss is usually a small fraction of the total heat loss, especially with forced convection, it is often sufficiently accurate to write:

$$P_r = \pi D \varepsilon \sigma_B[(T_s + 273)^4 - (T_a + 273)^4] \tag{17}$$

where the emissivity ε is dependent on the conductor surface and varies from 0.23 for new conductors to 0.95 for industrial weathered conductors (a suggested value is 0.5), σ_B is the Stefan-Boltzmann constant, T_a is the ambient temperature and T_s is the conductor surface temperature.

3.3 EVAPORATIVE COOLING [3]

The cooling due to evaporation does not alter significantly with water vapour being present in the air or with water droplets being entrained in the flow around the conductor. It does alter significantly as soon as the conductor is wetted. The evaporative cooling effects are generally ignored, and are therefore not dealt with in this document.

SECTION 2

MATHEMATICAL MODEL FOR EVALUATION OF CONDUCTOR TEMPERATURE IN THE UNSTEADY STATE

1. INTRODUCTION

The general heat equation for an homogeneous and isotropic solid can be expressed in polar co-ordinates in the form:

$$\frac{\partial T}{\partial t}=\frac{\lambda}{\gamma c}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}+\frac{\partial^2 T}{\partial z^2}\right)+\frac{q(T,\varphi,z,r,t)}{\gamma c} \tag{18}$$

where

c	= specific heat capacity
q	= power per unit volume
r	= radius
T	= temperature
t	= time
z	= axial length
γ	= mass density
λ	= thermal conductivity
φ	= azimuthal angle

If it is assumed that the conductor has cylindrical symmetry and semi-infinite length, then the terms φ and z can be neglected, so that eq. (18) reduces to:

$$\frac{\partial T}{\partial t}=\frac{\lambda}{\gamma c}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\frac{q(T,r,t)}{\gamma c} \tag{19}$$

Eq. (19) requires a numerical solution with appropriate initial and boundary conditions [11]. If we assume a radial temperature distribution, such as that given in eq. (9) for a steel-cored conductor or eq. (10) for a monometallic conductor, then we have a fixed relationship between the core temperature T_c and the surface temperature T_s for a given total power input per unit length. The mean temperature can then be calculated [12], but it is usually sufficiently accurate to assume that

$$T_{av}=\frac{T_c+T_s}{2} \tag{20}$$

Since the mass per unit length $m = \gamma A$ and $q = P/A$, where A is the cross-sectional area and P is the power per unit length, eq. (19) can be reduced to the differential equation:

$$mc\frac{dT_{av}}{dt}=P_J+P_M+P_S-P_r-P_c \tag{21}$$

In the case of a steel-cored conductor:

$$mc = m_a c_a + m_s c_s \tag{22}$$

where the subscripts a and s refer to the non-ferrous and ferrous sections, respectively. Values for the mass density and specific heat capacity of various conductor materials are given in table III. The mass density is practically constant up to 100° C, but the specific heat capacity varies linearly with temperature:

$$c(T) = c[1 + \beta(T - 20)] \tag{23}$$

The values for the temperature coefficient β are also found in table III.

The terms on the right-hand side of eq. (21) are discussed in Section 1:

P_J = Joule heat gain (cl. 2.1.1)
P_M = magnetic heat gain (cl. 2.1)
P_S = solar heat gain (cl. 2.2)
P_c = convective heat loss (cl. 3.1)
P_r = radiative heat loss (cl. 3.2)

If the magnitude of one or more of these terms, particularly the Joule heat gain and the convective heat loss, changes during the time interval dt, then the temperature of the conductor will change. If the heat gains exceed the heat losses, dT/dt will be positive, whereas, if the heat losses exceed the heat gains, dT/dt will be negative.

Eq. (21) can only be solved explicitly if the heat gains and heat losses are either constant or vary linearly with the temperature [13]. The Joule heat gain varies linearly with T_{av}, and the convective heat loss increases linearly with T_s for forced convection, but is non-linear for natural convection. The radiative heat loss is very non-linear with temperature. The magnetic heat gain is temperature dependent, because the hysteresis and eddy current losses and the current redistribution due to the transformer effect all depend on the temperature of the steel core [3, 27]. It is also dependent on the current distribution within the conductor. The solar heat gain can be assumed to remain constant if dt is relatively short, say, less than one twentieth of the thermal time constant.

2. TIME-DEPENDENT HEATING [6, 13-15]

The differential heat balance eq. (21) can be solved by numerical integration (Appendix VI), but by a certain degree of approximation it is possible to obtain a closed form solution. In particular, if the radiative heat loss can be linearized [16, 17], or the loss is small relative to the forced convective heat loss, and the solar and magnetic heat gains are assumed to be constant, the solution to eq. (21) is:

$$t = \frac{-mc\theta_m}{I^2 R_{ac} + P_S}\left[\beta(\theta - \theta_1) + (1 + \beta\theta_m)\ln\left(\frac{\theta_m - \theta}{\theta_m - \theta_1}\right)\right] \tag{24}$$

where the specific heat capacity c is calculated at the ambient temperature.

Since it can be seen from Table III that β is small,

$$t = \frac{-mc\theta_m}{I^2 R_{ac} + P_S} \ln\left(\frac{\theta_m - \theta}{\theta_m - \theta_1}\right) \tag{25}$$

where

R_{ac}	= ac resistance per unit length at ambient temperature
θ	= T_{av}-T_a = average temperature rise of conductor
θ_I	= T_{avI}-T_a = initial average temperature rise of conductor at time t_I
θ_m	= T_{avm}-T_a = asymptotic average temperature rise of conductor

Fig. 1 shows the heating characteristic for a conductor after a step increase in current.

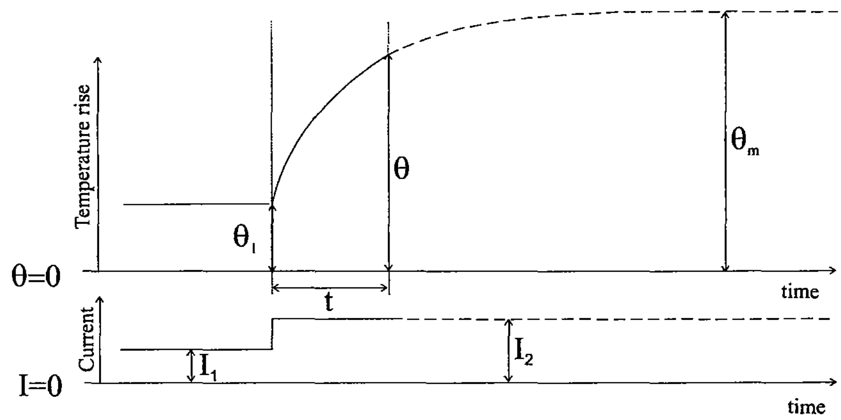

Fig. 1

The thermal time constant τ_h is the time interval for the temperature rise of the conductor to reach 63.2 % of its asymptotic (steady-state) value θ_m. The thermal time constant is given by:

$$\tau_h = \frac{mc\theta_m}{I^2 R_{ac} + P_S} \tag{26}$$

Hence, from eq. (25) and (26),

$$t \cong -\tau_h \ln\left(\frac{\theta_m - \theta}{\theta_m - \theta_1}\right) \tag{27a}$$

or

$$\theta \cong \theta_m - (\theta_m - \theta_1)\, e^{-\frac{t}{\tau_h}} \tag{27b}$$

The thermal time constant for a steel-cored conductor is found from [18]:

$$\tau_h = \frac{(m_a c_a + m_s c_s)\theta_m}{I^2 R_{ac} + P_S} + \frac{m_s c_s \ln\left(\frac{D}{D_2}\right)}{2\pi\lambda_a} \tag{28}$$

where λ_a is the radial thermal conductivity of the non-ferrous section of the conductor.

If a steady state current I_1 has produced a maximum temperature rise of the conductor θ_{m1} and the current is increased in a step to I_2, which results in a maximum temperature rise θ_{m2}, then the time interval t_{12} for the temperature rise to increase from θ_{m1} to a prescribed value θ_2 is given by:

$$t_{12} \cong -\tau_h \ln\left(\frac{\theta_{m2} - \theta_2}{\theta_{m2} - \theta_{m1}}\right) \tag{29}$$

where the current I in eq. (26) and (28) is now the current I_2. θ_{m1} and θ_{m2} are calculated according to the steady state methods given in Section 1.

Appendix VII reports the comparison between time constants obtained by means of these formulas and by means of laboratory tests. It can be seen that the agreement is good for wind speeds greater than 0.5 m/s. For wind speeds less than 0.5 m/s, there is an error of up to 20 % in the calculated examples. This discrepancy is possibly due to the neglect of mixed convection, i.e. simultaneous forced and natural convection, at low wind speeds.

3. TIME-DEPENDENT COOLING [6, 14, 18]

When the current is reduced by a step change from an initial value I_1, which resulted in a steady-state temperature rise of the conductor θ_{m1}, to a new value I_2, with constant atmospheric conditions, the solution of eq. (21) becomes:

$$t = \frac{-mc\theta_{m1}}{P_1}\left[\beta(\theta - \theta_{m1}) + \left(1 + \frac{\beta P_2 \theta_{m1}}{P_1}\right)\ln\left(\frac{P_1 \frac{\theta}{\theta_{m1}} - P_2}{P_1 - P_2}\right)\right] \tag{30}$$

where

$$P_1 = I_1^2 R_{ac}(1 + \alpha\theta_{m1}) + P_S - \alpha I_2^2 R_{ac}\theta_{m1}$$
$$P_2 = I_1^2 R_{ac} + P_S$$

Because β is small, see table III, eq. (30) approximates to:

$$t \cong \frac{-mc\theta_{m1}}{P_1} \ln\left(\frac{P_1 \dfrac{\theta}{\theta_{m1}} - P_2}{P_1 - P_2} \right) \tag{31}$$

Fig. 2 shows the cooling characteristic after a step decrease in current.

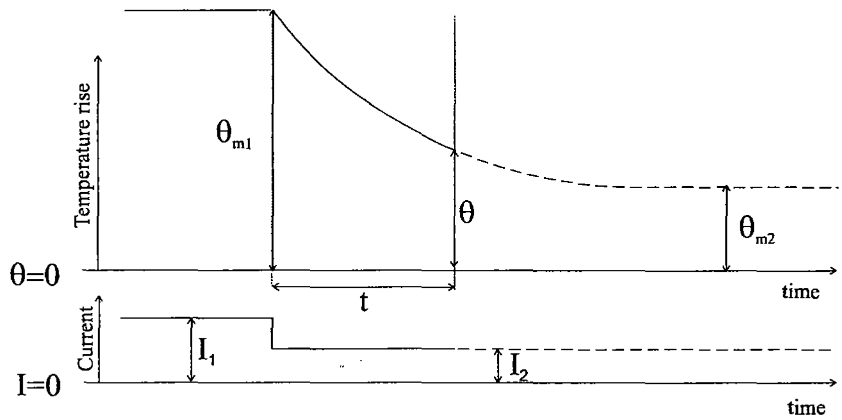

Fig. 2

The thermal time constant during cooling of the conductor is defined as the time interval for the temperature rise to fall to 36.8 % of its initial value. The thermal time constant for a monometallic conductor is given by:

$$\tau_{co} = \frac{mc\theta_{m1}}{P_1} \tag{32}$$

and the thermal time constant for a steel-cored conductor is found from [18]:

$$\tau_{co} = \frac{(m_a c_a + m_s c_s)\theta_{m1}}{P_1} + \frac{m_s c_s \ln\left(\dfrac{D}{D_2}\right)}{2\pi\lambda_a} \tag{33}$$

The time interval t_{12} for the temperature rise of the conductor to fall from θ_1 to θ_2 is given by:

$$t_{12} = -\tau_{co} \ln\left(\frac{P_1 \dfrac{\theta_2}{\theta_{m1}} - P_2}{P_1 \dfrac{\theta_1}{\theta_{m1}} - P_2} \right) \tag{34}$$

Examples of calculations of time intervals for conductor heating and cooling and of current ratings for different intervals of time are worked in Appendix II. It should be noted that the approximate method may lead to calculated time intervals which are slightly non-conservative, especially for low wind speeds.

Table III. Typical conductor material data at 20 °C [21, 22, 23, 24, 25]

Metal	γ kg/m^3	c J/kgK	ρ_{dc} nΩm	α 1/K	β 1/K
Aluminium	2703	897	28.264	$4.03 \cdot 10^{-3}$	$3.8 \cdot 10^{-4}$
Aluminium alloy(***)	2703	909	32.7	$3.6 \cdot 10^{-3}$	$4.5 \cdot 10^{-4}$
Copper	8890	383	17.241	$3.9 \cdot 10^{-3}$	$3.35 \cdot 10^{-4}$
Steel	7780	481	287.36 (*) 191.57 (**)	$4.5 \cdot 10^{-3}$	$1.0 \cdot 10^{-4}$
Aluminium-clad steel	(1)	(2)			
20 SA	6590	518	84.80	$3.6 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$
27 SA	5910	551	63.86	$3.6 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$
30 SA	5610	566	57.47	$3.8 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$
40 SA	4640	630	43.10	$4.0 \cdot 10^{-3}$	$1.4 \cdot 10^{-4}$

(*) Based on 6 % IACS conductivity
(**) Based on 9 % IACS conductivity

(1) $$\gamma = \frac{\gamma_a A_a + \gamma_s A_s}{A_a + A_s}$$

(2) $$\gamma = \frac{c_a \gamma_a A_a + c_s \gamma_s A_s}{\gamma_a A_a + \gamma_s A_s}$$

SECTION 3

MATHEMATICAL MODEL FOR EVALUATION OF CONDUCTOR TEMPERATURE IN THE ADIABATIC STATE

1. INTRODUCTION

The method for calculating the conductor temperature rise caused by a short-time current is presented in IEC Publication 60865-1 [25]. The standard states that the steel core of the aluminium conductor steel reinforced (ACSR) shall not be taken into account in the calculation. However, this assumption can lead to considerable errors in calculating the conductor temperature rise during short-circuit, especially as far as ACSR, AACSR or similar conductors with high steel content are concerned. A mathematical solution to the problem, with all the parameters assumed to vary quadratically with temperature has been given in [31]. However, in order to simplify the problem, the following assumptions have been made in IEC 60865-1 and in this brochure:

- skin-effect (magnetic influence of the conductor itself) and proximity-effect (magnetic influence of nearby parallel conductors) are disregarded, i.e. the current is assumed to be evenly distributed over the conductor cross-section area
- the resistance-temperature characteristic is assumed linear
- the specific heat capacity of the conductor is considered constant
- the heating is considered adiabatic, i.e. heat gains and losses at the surface of the conductor are not taken into account, due to the very short time span

2. CALCULATION OF THE CONDUCTOR TEMPERATURE RISE UNDER SHORT-CIRCUIT CONDITIONS

Taking the assumptions given in cl. 1 into account eq. (21) in Section 2 under short-circuit conditions reduces to:

$$mc\frac{dT}{dt} = P_J \tag{35}$$

$$\frac{dT}{dt} = I^2 \frac{R_{dc}\left[1+\alpha(T-20)\right]}{mc} \tag{36}$$

$$\frac{dT}{dt} = I^2 \frac{\dfrac{\rho_{dc}\left[1+\alpha(T-20)\right]}{A}}{A_1\gamma_1 c_1 + A_2\gamma_2 c_2} \tag{37}$$

$$\int_{T_1}^{T_2} \frac{dT}{1+\alpha(T-20)} = \int_0^t \frac{I^2 \rho_{dc}}{A(A_1\gamma_1 c_1 + A_2\gamma_2 c_2)} dt \tag{38}$$

$$\frac{1}{\alpha}\ln\frac{1+\alpha(T_2-20)}{1+\alpha(T_1-20)}=\frac{I^2\rho_{dc}}{A(A_1\gamma_1c_1+A_2\gamma_2c_2)}t \tag{39}$$

The final temperature of the conductor for a given thermal equivalent short-time withstand current I_{th} (see clause 3.2.2 of IEC 60865-1) is then:

$$T_2=\frac{[1+\alpha(T_1-20)]e^{\frac{I_{th}^2\rho_{dc}\alpha}{A(A_1\gamma_1c_1+A_2\gamma_2c_2)}t}-1}{\alpha}+20 \tag{40}$$

The following three cases are considered (subscript s means here either steel or aluminium-clad steel):

(1) Homogeneous non-ferrous conductor:

$\rho_{dc} = \rho_a$
$\alpha = \alpha_a$
$A = A_1 = A_a$
$A_2 = 0$
$\gamma_1 = \gamma_a$
$c_1 = c_a$
$\gamma_2 = 0$
$c_2 = 0$

(2) Homogeneous ferrous conductor (made of steel or aluminium-clad steel wires):

$\rho_{dc} = \rho_s$
$\alpha = \alpha_s$
$A_1 = 0$
$A = A = A_s$
$\gamma_1 = 0$
$c_1 = 0$
$\gamma_2 = \gamma_s$
$c_2 = c_s$

(3) Steel- or aluminium-clad steel-cored conductor:

$\rho_{dc} = \rho_a$
$\alpha = \alpha_a$
$A = A_1 = A_a$
$A_2 = A_s$
$\gamma_1 = \gamma_a$
$\gamma_2 = \gamma_s$
$c_1 = c_a$
$c_2 = c_s$

3. CALCULATION OF THE ALLOWED CONDUCTOR SHORT-TIME WITHSTAND CURRENT

The thermal equivalent short-time withstand current I_{th} for a given final temperature can be calculated by:

$$I_{th} = \sqrt{\frac{A(A_1\gamma_1 c_1 + A_2\gamma_2 c_2)}{\rho_{dc}\alpha t}\ln\frac{1+\alpha(T_2-20)}{1+\alpha(T_1-20)}} \qquad (41)$$

4. MATERIAL DATA FOR THE CALCULATION

The material data for the calculations is given in Section 2, table III.

Note! In case of calculating pure steel wires it is advisable to use the known value for the steel resistivity, if possible. If the resistivity is not known the relatively high value corresponding to the conductivity of 6 % IACS should be used.

5. MAXIMUM RECOMMENDED CONDUCTOR TEMPERATURES DURING A SHORT-CIRCUIT

IEC Publication 60865-1, table 6, gives the highest recommended short-time conductor temperatures. These values are also shown below in table IV. If the temperature is reached, a negligible decrease in strength can occur which does not jeopardise the safety in operation. However, the strength and lifetime of the conductor depend on the whole operational history of the conductor. Consequently, other temperatures than those in table IV may also be employed, depending on the user.

Table IV. Recommended highest temperatures for mechanically stressed conductors during a short-circuit

Type of conductor	**Maximum conductor temperature**
Copper, aluminium, aluminium alloy, steel-cored conductor	200 °C
Steel	300 °C

6. COMPARISON BETWEEN CALCULATED AND MEASURED CONDUCTOR TEMPERATURE RISES IN SHORT-CIRCUIT

Measured and calculated maximum temperature rises of copper and aluminium conductors during short circuits are compared in [28]. In the case of steel-cored conductors the calculated temperature rises are compared to laboratory measurements [29, 30] and presented in fig. 3 to 6. The conductors studied are:

- 122-A1/S1A-12/7
- 210-A2/S1A-30/7
- 106-A2/S1A-12/7
- 490-A1/S1A-54/7

The calculation method gives somewhat lower temperature values than measurements. However, the error is comparatively small. The biggest deviations are found in tests made on AACSR 106-A2/S1A-12/7 shown in figure 4 giving ca. 20 % lower calculated values. Generally the method gives ca. 10 % lower temperature values as far as high steel content conductors are concerned and only a few per cent lower values in case of low steel content conductors.

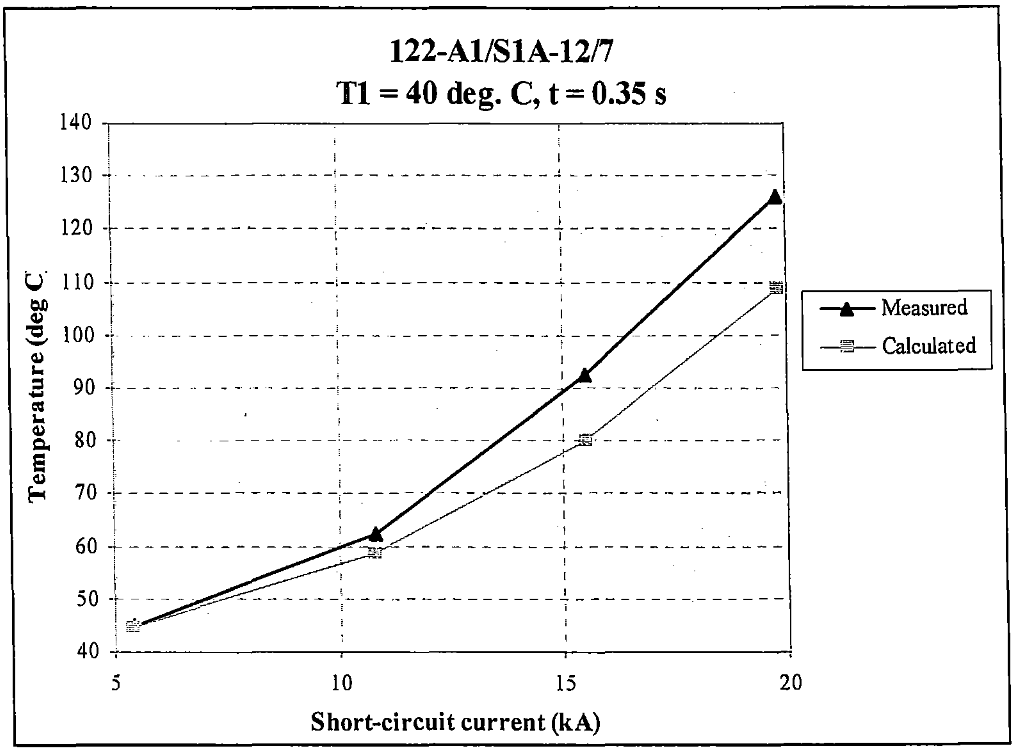

Figure 3. Temperature rise of ACSR 122-A1/S1A-12/7 during short circuit

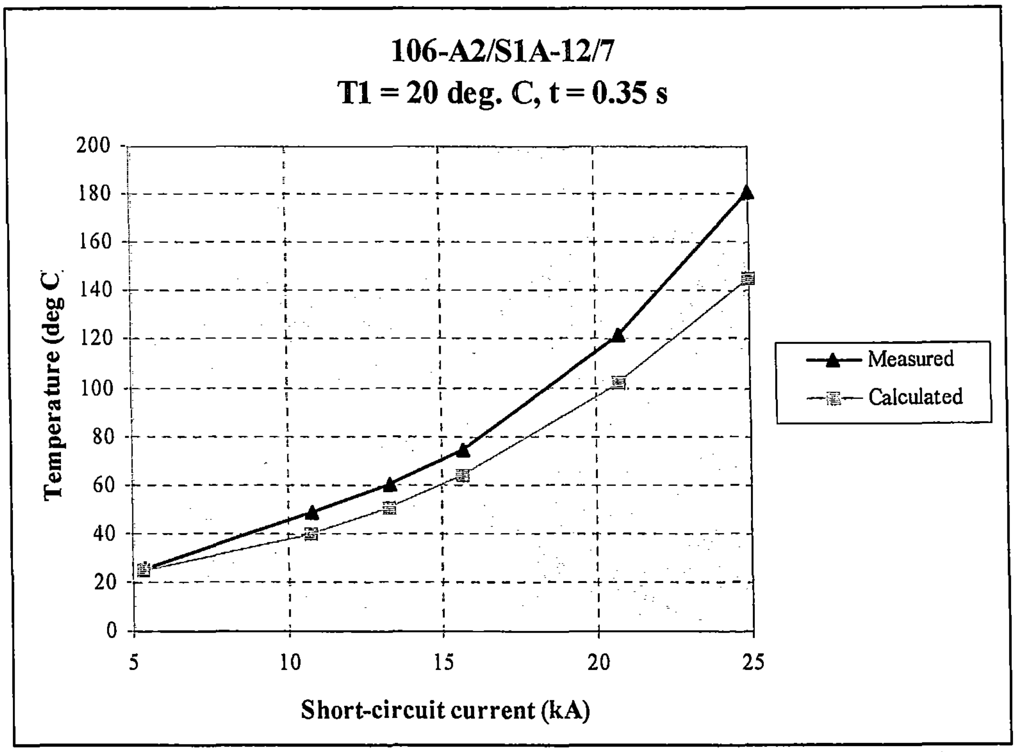

Figure 4. Temperature rise of AACSR 106-A2/S1A-12/7 during short circuit

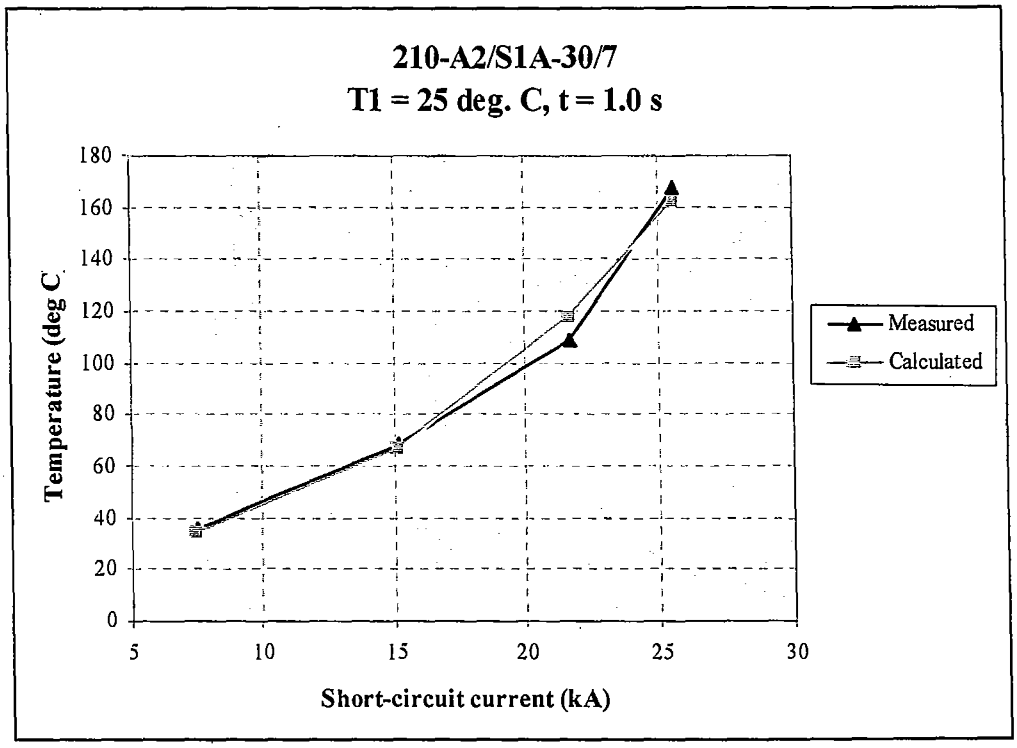

Figure 5. Temperature rise of AACSR 210-A2/S1A-30/7 during short circuit

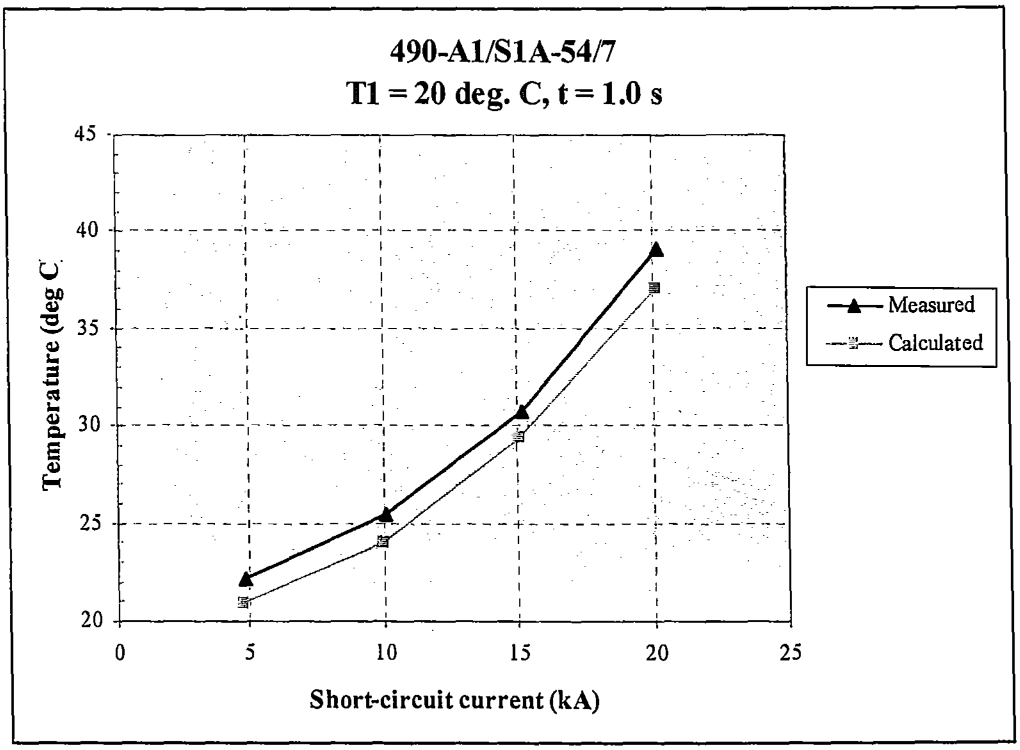

Figure 6. Temperature rise of ACSR 490-A1/S1A-54/7 during short circuit

NOMENCLATURE

A	cross sectional area, m^2
A, B	(with numerical subscripts) constants
B	$I_d(\pi/2)(1+F)$
c	specific heat capacity, J/kg K
D	diameter of circle circumscribing the conductor, m
D_2	diameter of steel core, m
d	diameter of wire in outer layer, m
d_a	diameter of non ferrous wire, m
d_s	diameter of steel wire, m
d_z	mean diameter of layer z, m
F	albedo (reflectance) of surface
g	acceleration due to gravity, m/s^2
Gr	Grashof number = $D^3(T_s-T_a)g/(T_f+273)\nu^2$
h_c	heat transfer coefficient, W/m^2K
H_S	solar altitude, deg
I	effective current, A
I_{dc}	direct current to give equivalent conductor heating to corresponding ac value, A
I_k	direct current per area, A/mm^2
I_D	intensity of direct solar radiation on surface normal to solar beam, W/m^2
I_d	intensity of diffuse solar radiation on horizontal surface, W/m^2
K_z	$\sqrt{\{1+(\pi d_z/l_z)^2}$
k_j	factor which takes into account the increase in resistance due to skin effects
l_z	lay length of layer z
m	mass per unit length, kg/m
m, n	constants
N^*	day of the year
Nu	Nusselt number = h_cD/λ_f
P	power exchange per unit length, W/m
Pr	Prandtl number = $c\mu/\lambda_f$
R	resistance per unit length, Ω/m
Re	Reynolds number = $\rho_r VD/\nu_f$
R_f	conductor roughness = $d/[2(D-d)]$
r	radius, m
q	power per unit volume, W/m^3
S	global solar radiation, W/m^2
t	time interval, s
T	temperature, °C
T_1	initial temperature, K
T_2	final temperature, K
V	velocity, m/s
y	height above sea level, m
z	axial length, m
z	layer number
Z	hour angle of sun, deg (positive before noon)

GREEK SYMBOLS

α	temperature coefficient of resistance, 1/K
α_s	solar absorptivity of surface
β	temperature coefficient of specific heat capacity, 1/K
γ	mass density, kg/m^3
ε	solar emissivity of surface
γ_C	azimuth of conductor, deg (positive from south through west)
γ_S	azimuth of sun, deg (positive from south through west)
δ	angle of attack, deg; declination, deg
ζ	inclination to horizontal, deg (positive when tilted to south)
η	angle of incidence of solar beam relative to axis of conductor, deg
λ	thermal conductivity, W/mK
μ	viscosity, kg/ms
v	kinematic viscosity, m^2/s
ρ	air density, kg/m^3; resistivity, Ωm
ρ_r	relative air density
ρ_0	air density at sea level, kg/m^3
ρ_a	resistivity of non-ferrous material, Ωm
ρ_s	resistivity of steel, Ωm
σ_B	Stefan-Boltzmann constant, $5.6697 \cdot 10^{-8}$ W/m^2K
θ	temperature rise above ambient, K
τ	thermal time constant, s
φ	latitude, deg (north positive); azimuthal angle, deg

SUBSCRIPTS

a	ambient, non-ferrous
ac	alternating current
av	mean value
c	core, convection
co	cooling
cor	corrected
dc	direct current
f	film at surface, forced
h	heating
i	ionization
J	joule
M	magnetic
m	asymptotic value
n	natural
r	radiation, relative
s	steel, surface

S	solar
T	total
w	mass transfer
z	layer number

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APPENDIX I

EXAMPLES SHOWING METHOD OF CALCULATION FOR STEADY STATE

Example 1

Conductor Type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind Speed:	2 m/s
Wind angle:	45 deg
Ambient temperature:	40°C
Height above sea level:	1600 m
I_{ac}:	600 A

Calculate the value of surface temperature T_s of the conductor.

This calculation relies on an initial guess at the value of T_s. An initial guess of T_s = 57 °C will be taken and also $T_{av} = T_s$.

The dc current corresponding to the ac current may be calculated by iteration from eq. (7):

$$I_{dc} = I_{ac}\sqrt{1.0123 + 2.36 \cdot 10^{-5} I_{ac}}$$

$I_{dc} = 607.89$ A

From eq. (4) and (11):

$P_J = 28.62$ W/m
$P_S = 14.02$ W/m

Corona heating is ignored as suggested in the report, thus:

$P_i = 0$

The equations for calculating convective cooling are given in section 1, cl. 3.1:

$\rho_r = 0.8306$
$\nu_f = 1.78 \cdot 10^{-5}$ m^2/s
$Re = 2670$
$\lambda_f = 0.0277$ W/mK

Conductor surface roughness is according section 1, cl. 3.1.1 (D = 28.6 mm, d = 3.18 mm):

$R_f = 0.0625$

From table I: $B_1 = 0.048$ and $n = 0.8$.

The Nusselt number for forced convective cooling from eq. (13):

$$Nu_{90} = 26.45$$

The Nusselt number for wind at an angle of 45° from eq. (14) :

$$Nu_{45} = 22.34$$

The convective cooling given by eq. (12):

$$P_c = 33.04 \text{ W/m}$$

The radiative cooling given by eq. (17):

$$P_r = 5.76 \text{ W/m}$$

Evaporative cooling is ignored as suggested in the report, thus:

$$P_w = 0$$

Now from eq. (1):

$$P_J + P_M + P_S + P_i = P_c + P_r + P_w$$

$$42.62 = 38.80$$

The result is not correct, thus the guess for T_s was incorrect. Another guess must be taken, let us say T_s = 58°C. Finally, following the same iterative procedure we can reach the conductor temperature value of ca. 58.5 °C.

In order to calculate the conductor core temperature eq. (9) is used (P_T = 42 W/m):

$$T_c = 58.5 + 1.2 \text{ °C} = 60 \text{ °C}$$

which does not differ much from the average temperature.

Example 2

Conductor type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind speed:	0.2 m/s
Wind angle:	variable
Ambient temperature:	40 °C
Height above sea level:	1600 m
I_{ac}:	600 A

Calculate the value of surface temperature T_s of the conductor.

As in example 1, this calculation relies on an initial guess at the value of T_s. A guess of $T_s = 93$ °C is taken.

The value of I_{dc} remains the same as in the previous example.

$I_{dc} = 607.89$ A
$P_J = 32.23$ W/m
$P_S = 14.02$ W/m
$v_f = 1.95 \cdot 10^{-5}$ m^2/s
$Re = 243.8$
$\lambda_f = 0.0290$ W/mK

Calculation of forced convective cooling:

From table I, $B_l = 0.641$ and $n = 0.471$.

$Nu = 8.53$
$Nu_{45} = 7.20$

For low wind speeds the minimum Nusselt number is given by eq. (15):

$Nu_{cor} = 4.69$

Since 7.20 > 4.69 the former value is chosen for calculation of forced cooling.

Calculation of natural convective cooling:

From equations in clause 3.1:

$Gr = 94387$
$Pr = 0.698$
$Gr \cdot Pr = 65882$

From table II: $A_2 = 0.48$ and $m_2 = 0.25$.

$Nu = 7.69$

Since Nu(natural) > Nu(forced) we take the highest value, i.e. $Nu = 7.69$.

$P_c = 37.13$ W/m
$P_r = 21.27$ W/m

From eq. (1) we get the following:

$$32.23 + 14.02 = 37.13 + 21.27$$
$$46.25 = 58.40$$

These two values are not equal, thus we must take another guess at the value of T_s. Let us say T_s = 82 °C. Again with iterative procedure we can finally establish the conductor temperature as 83 °C.

Example 3

Conductor type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind speed:	2 m/s
Wind angle:	45°
Ambient temperature:	40 °C
Max. allowable surface temperature:	75 °C
Height above sea level:	1600 m

Calculate the maximum current allowed under these circumstances.

As in previous examples we get:

$P_J = I_{dc}^2 \cdot 8.23 \cdot 10^{-5}$ W/m
$P_S = 14.02$ W/m

$\nu_f = 1.866 \cdot 10^{-5}$ m^2/s
$\rho_r = 0.8306$
$Re = 2547.5$
$\lambda_f = 0.0283$ W/mK
$B_1 = 0.641$ and n = 0.471
$Nu_{90} = 25.77$
$Nu_{45} = 21.8$

$P_c = 67.8$ W/m
$P_r = 12.92$ W/m

Now from eq. (1):

$$P_J + P_M + P_S + P_i = P_c + P_r + P_w$$
$$I_{dc}^2 \cdot 8.23 \cdot 10^{-5} + 14.02 = 67.8 + 12.92$$

$$I_{dc} = 900.25 \text{ A}$$

We must now take the magnetic effect into account and work out the related I_{ac} value using eq. (6):

$$I_{ac} = 885.9 \text{ A}$$

Example 4

Conductor type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind speed:	0.4 m/s
Wind angle:	variable
Ambient temperature:	40 °C
Max. allowable surface temperature:	75 °C
Height above sea level:	1600 m

Calculate the maximum current allowed under these circumstances.

As before:

$P_J = I_{dc}^2 \cdot 8.23 \cdot 10^{-5}$ W/m
$P_S = 14.02$ W/m
$\nu_f = 1.866 \cdot 10^{-5}\ m^2/s$
$\rho_r = 0.8306$
$Re = 509.6$
$\lambda_f = 0.0283$ W/mK

Calculating forced convective cooling:

$B_1 = 0.641$ and $n = 0.471$
$Nu = 12.08$
$Nu_{45} = 10.2$
$Nu_{cor} = 6.64$
$Nu_{cor} < Nu_{45}$

Calculating natural convective cooling:

$Gr = 69922.7$
$Pr = 0.701$
$Gr \cdot Pr = 48990$
$A_2 = 0.48$ and $m_2 = 0.25$
$Nu = 7.14$

Since Nu(natural) < Nu(forced) we take the highest value, i.e. $Nu = 10.2$.

$P_c = 31.78$ W/m
$P_r = 12.92$ W/m

Substituting into eq. (1):

$I_{dc}^2 \cdot 8.23 \cdot 10^{-5} + 14.02 = 31.78 + 12.92$
$I_{dc} = 610.3$ A

Now taking the magnetic effect into account by eq. (6), the relative I_{ac} value is calculated:

$I_{ac} = 602.4$ A

Example 5

Conductor Type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind Speed:	2 m/s
Wind angle:	45 deg
Ambient temperature:	40 °C
Height above sea level:	300 m
I_{ac}:	600 A

Following example 1, and simple rule of temperature change 1°C per 1000 m, we can assume as a guess value:

$T_s(1600) - T_s(300) = (1600\text{-}300)/1000 \cdot 1$ °C $= 1.3$ °C
$T_s(300) = 58.5 - 1.3$ °C $= 57$ °C
$I_{dc} = 607.89$ A
$P_j = 28.62$ W/m
$P_S = 14.02$ W/m
$\rho_r = 0.966$
$v_f = 1.78 \cdot 10^{-5}$ m^2/s
$Re = 3106$
$\lambda_f = 0.0277$ W/mK
$B_1 = 0.048$ and n = 0.8
$Nu_{90} = 29.85$
$Nu_{45} = 25.21$

$P_c = 37.28$ W/m
$P_r = 5.76$ W/m

$P_j + P_S = P_c + P_r$
$28.62 + 14.02 = 37.28 + 5.76$
$42.64 = 43.04$

The results are very similar and thus we can accept the initial assumption of $T_s = 57$ °C for conductor surface temperature.

APPENDIX II

EXAMPLES SHOWING METHOD OF CALCULATION FOR UNSTEADY STATE

Conductor type:	428-A1/S1A-54/7 'ZEBRA'
Global solar radiation:	980 W/m^2
Wind velocity:	see below
Wind angle:	45°
Ambient temperature:	40 °C
Height above sea level:	300 m
Radial thermal conductivity:	2 W/m/K
Max. allowable surface temperature:	85 °C

Example 1: Conductor heating

Initial current I_{ac1}: 600 A
Final current I_{ac2}: 1200 A

Following example 5 of Appendix I the surface temperature of the conductor is θ_{m1} = 57 °C for a wind velocity of 2 m/s. If the current is increased in a step to I_{ac2}, to obtain the time t_{12} to reach the maximum allowable temperature, with constant atmospheric conditions, it is necessary to calculate the steady state temperature with I_{ac2} (θ_{m2} = 98.4 °C), the solar heating (P_S = 14 W/m) and the Joule heating at ambient temperature ($R_{ac}I_{ac}^2$ = 134 W/m).

The thermal time constant is τ_h = 619 s from eq. (28).

The time needed to reach 85 °C (eq. (29)) is t_{12} = 701 s. Table V shows the results in the case of wind velocity ranging between 0 and 2 m/s.

Table V

Wind velocity (m/s)	2.0	1.5	1.0	0.5	0.0
Steady-state temperature θ_{m1} with I_{ac1} (°C)	57.0	59.9	63.4	70.5	83.2
Steady-state temperature θ_{m2} with I_{ac2} (°C)	98.4	107.5	120.3	145.3	166.2
Solar heating (W/m)	14.0	14.0	14.0	14.0	14.0
Joule heating (W/m)	134	138	144	154	164
Thermal time constant during heating (s)	619	713	844	1101	1317
Time to reach 85 °C (s)	701	534	403	237	28.5

Table VI shows the results obtained using the numerical method reported in Appendix VI.

Table VI

Wind velocity (m/s)	2.0	1.5	1.0	0.5	0.0
Time to reach 85 °C (s)	678	501	376	218	23.5

Example 2: Conductor cooling

Initial current I_{ac1}: 900 A
Final current I_{ac2}: 200 A
Final temperature: 70 °C

For a wind velocity of 2 m/s the interval time to cool the conductor from an initial temperature of θ_{m1} = 72.7 °C (with I_{ac1} = 900 A) to a final temperature of 70 °C with a current I_{ac2} = 200 A can be obtained by calculating the thermal time constant from eq. (33) with P_1 = 82.3 W/m and P_2 = 17 W/m: τ_{co} = 520 s. The interval of time to reach 70 °C is then from eq. (34): t_{12} = 57 s. Table VII shows the results in the case of wind velocity ranging between 0 and 2 m/s.

Table VII

Wind velocity (m/s)	2.0	1.5	1.0	0.5	0.0
Steady-state temperature θ_{m1} with I_{ac1} (°C)	72.7	78.3	85.4	99.5	117.2
P_1 (W/m)	82.3	83.6	85.2	88.5	92.6
P_2 (W/m)	17.0	17.0	17.0	17.0	17.0
Thermal time constant during cooling (s)	520	597	691	867	1071
Time to reach 70 °C (s)	57	190	380	824	1479

Table VIII shows the results obtained using the numerical method reported in Appendix VI.

Table VIII

Wind velocity (m/s)	2.0	1.5	1.0	0.5	0.0
Time to reach 70 °C (s)	55	185	374	834	1804

Example 3: Current ratings at 3, 10, 30 minutes

In order to find the ratings at different intervals of time it is necessary to iterate the operations illustrated in example 1 changing the value of current in order to obtain intervals of heating equal to the prefixed ones (in this case 3, 10 and 30 minutes).

Table IX shows the current ratings (A) at the above intervals of time in the case of wind velocity ranging between 0 and 2 m/s for an initial current I_{ac1} = 600 A.

Table IX. Current ratings (A)

		Interval of overload (min)		
		3	**10**	**30**
Wind velocity (m/s)	**2.0**	1785	1238	1077
	0.5	1321	930	797
	0.0	740	653	626

Table X shows the results obtained using the numerical method reported in Appendix VI.

Table X. Current ratings (A)

		Interval of overload (min)		
		3	**10**	**30**
Wind velocity (m/s)	**2.0**	1750	1230	1077
	0.5	1281	916	793
	0.0	716	644	624

APPENDIX III

EXAMPLES SHOWING THE METHOD OF CALCULATION FOR ADIABATIC STATE

Example 1: Calculation of conductor temperature rise

Conductor type:	428-A1/S1A-54/7 'ZEBRA'
A_a:	$428.9 \cdot 10^{-6}$ m^2
A_s:	$55.6 \cdot 10^{-6}$ m^2
Thermal equivalent short-circuit current:	43 kA
Fault time:	1 s
Initial temperature:	40 °C

$\rho_{dc} = \rho_a = 28.264$ nΩm
$\alpha = \alpha_a = 0.00403$ 1/K
$A = A_1 = A_a = 428.9 \cdot 10^{-6}$ m^2
$A_2 = A_s = 55.6 \cdot 10^{-6}$ m^2
$\gamma_1 = \gamma_a = 2703$ kg/m^3
$\gamma_2 = \gamma_s = 7780$ kg/m^3
$c_1 = c_a = 897$ J/kg K
$c_2 = c_s = 481$ J/kg K

From eq. (40) the final temperature reached after 1 s is 169.3 °C.

Example 2: Calculation of maximum permissible short-time current

Conductor type:	10 - S1A - 7
$A = A_2 = A_s$:	$67.8 \cdot 10^{-6}$ m^2
Fault time:	1 s
Initial temperature:	40 °C
Maximum allowed temperature:	300 °C

$\rho_{dc} = \rho_s = 287.36$ nΩm (corresponding to 6 % IACS)
$\alpha = \alpha_s = 4.5 \cdot 10^{-3}$ 1/K
$A_1 = 0$
$A = A_2 = A_s = 67.8 \cdot 10^{-6}$ m^2
$\gamma_1 = 0$
$c_1 = 0$
$\gamma_2 = \gamma_s = 7780$ kg/m^3
$c_2 = c_s = 481$ J/kg K

From eq. (41) the maximum allowed short-circuit current to reach the final temperature of 300 °C is 3.1 kA.

Using the value of 191.57 nΩm (corresponding to 9 % IACS) for the resistivity of steel the maximum allowed short-circuit current to reach the final temperature of 300 °C would be 3.8 kA.

APPENDIX IV

CALCULATION OF CONDUCTOR RESISTANCE

Conductance of steel core (if present):

$$\frac{1}{R_s} = \frac{\pi d_s^2}{4\rho_s}\left[1 + \sum_{1}^{n_s} \frac{6z_s}{K_{sz}}\right] \tag{42}$$

where

$$K_{sz} = \sqrt{1 + \left(\frac{\pi d_{sz}}{l_{sz}}\right)^2} \tag{43}$$

d_s = diameter of steel wire
ρ_s = resistivity of steel at 20°C
z_s = layer number of steel wires
d_{sz} = mean diameter of layer z
l_{sz} = lay length of layer z
n_s = number of layers of steel wires

The conductance of each non-ferrous layer:

$$\frac{1}{R_{az}} = \frac{\pi d_a^2 n_{az}}{4\rho_a K_{az}} \tag{44}$$

where

$$K_{az} = \sqrt{1 + \left(\frac{\pi d_{az}}{l_{az}}\right)^2} \tag{45}$$

d_a = diameter of non-ferrous wire
ρ_a = resistivity of non-ferrous material at 20°C
d_{az} = mean diameter of layer z
n_{az} = number of non-ferrous strands in layer z
l_{az} = lay length of layer z
n_a = number of layers of non-ferrous wires

The total resistance of the conductor is found from:

$$\frac{1}{R_{dc}} = \frac{1}{R_s} + \sum_{1}^{n_a} \frac{1}{R_{az}} \tag{46}$$

APPENDIX V

COMPLEX DETERMINATION OF SOLAR HEAT GAIN WITH DIRECT AND DIFFUSE RADIATION KNOWN [4]

For isotropic diffuse radiation on a horizontal conductor, the total solar heat received per unit length of the conductor is given by:

$$P_S = \alpha_s D\left[I_D\left(\sin\eta + \frac{\pi}{2} F \sin H_S \right) + B \right] \tag{47}$$

where

I_D = direct solar radiation
α_s = absorptivity of conductor surface
η = angle of the solar beam with respect to the axis of the conductor
F = reflectance or albedo of ground
H_S = solar altitude
I_d = diffuse solar radiation
Z = hour angle of the sun
N^* = day of the year

where

$I_D \approx 1280 \sin H_S/(\sin H_S + 0.314)$
$B = I_d(\pi/2)(1+F)$
$I_d = (570\text{-}0.47 I_D)(\sin H_S)^{1.2}$ [3]
$\eta = \arccos[\cos H_S \cos(\gamma_S - \gamma_C)]$
$H_S = \arcsin[\sin\varphi \sin\delta_S + \cos\varphi \cos\delta_S \cos Z]$
$\gamma_S = \arcsin[\cos\delta_S \sin Z/\cos H_S]$
$\delta_S \approx 23{,}4 \sin[360°(284+N^*)/365]$

γ_S and γ_C are the azimuths of the sun and conductor respectively, φ is the latitude, δ_S is the declination, Z is the hour angle of the sun, N^* is the day of the year. The hour angle Z increases by 15 degrees for every hour from zero at solar noon. To obtain solar time, add 4 minutes per degree of longitude east of standard time, or subtract 4 minutes per degree west of standard time; there is also a small time correction not exceeding 16 minutes, for perturbations in the earth's rotation.

The intensity of the direct solar beam I_D varies with the air mass traversed, and hence the solar altitude H_S and the turbidity of the atmosphere. The turbidity of the atmosphere is generally ignored in the determination of the steady state temperature or conductor ampacity.

The intensity of direct radiation I_D increases by 7 - 13 % from sea level to 1000 m above sea level, the higher value occurring in summer, and by 13 - 22 % from sea level to 2000 m. The albedo F is approximately 0.05 for water ($H_S > 30°$), 0.1 for forests, 0.15 for urban areas, 0.2 for soil, grass and crops, 0.3 for sand, 0.4 - 0.6 for ice and 0.6 - 0.8 for snow. The albedo tends to increase as the solar altitude increases. The value for α_s varies from 0.23 for bright stranded aluminium conductor to 0.95 for weathered conductor in an industrial environment. The recommended value is 0.5.

APPENDIX VI

NUMERICAL SOLUTION OF THE THERMAL DIFFERENTIAL EQUATION

The thermal differential equation. can be solved by taking small increments of temperature rise ($d\theta$) and calculating at each step the power input, the power loss and the heat capacity for the mean temperature rise.

The interval of time is calculated using equation (21):

$$dt_i = \frac{mc(\theta_i)\cdot d\theta}{P_J(\theta_i)+P_M(\theta_i)+P_S-P_c(\theta_i)-P_r(\theta_i)} \tag{48}$$

where θ_i is the temperature rise at step i.

Given the initial temperature and the prevailing meteorological and load conditions (constant or not), the time to reach a particular temperature is obtained from the sum of all the increments of time, $\sum_i dt_i$, up to the temperature.

In order to calculate the rating of a line for a given time interval it is necessary to repeat the above described operation changing the value of the line current until the difference between the calculated time and the time interval is less than a chosen value.

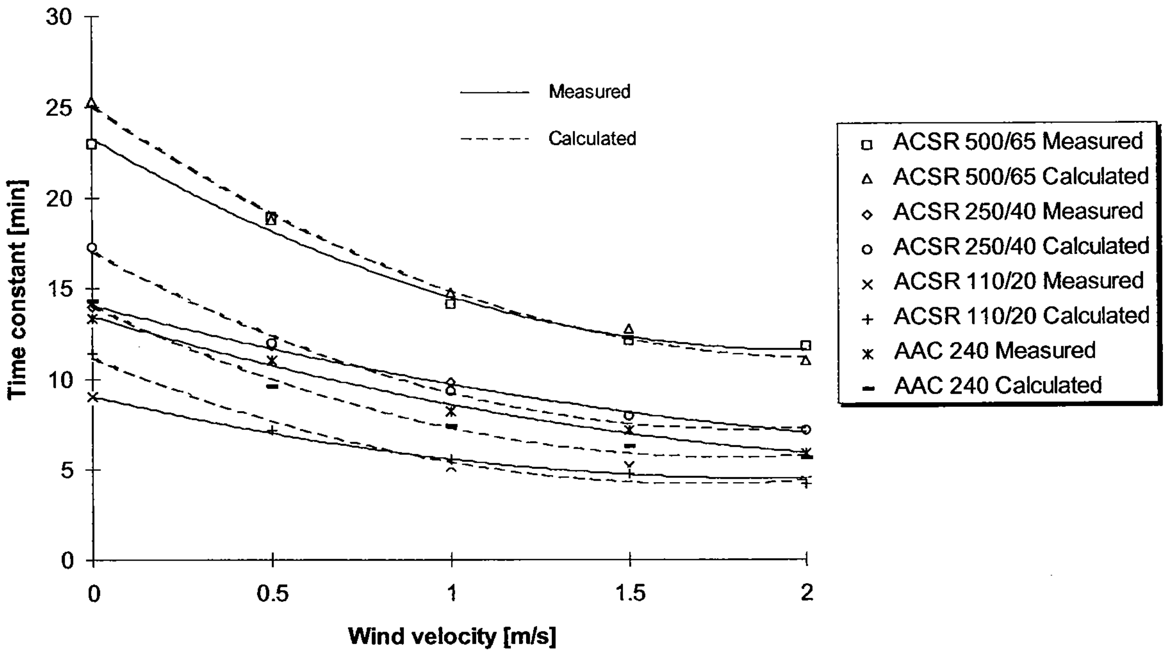
Note

Continuous on-line evaluation of conductor temperature is possible by computing the power terms of eq. (21) at a specific time interval, on the basis of the weather data measured by means of meteorological stations and on the basis of the current flowing through the conductor, and by integrating the equation by means of easily available mathematical routines. In order to achieve a good accuracy in the determination of conductor temperature it is necessary to choose a time interval (after which to solve the equation) that allows the model to follow the rapid dynamic changes in the meteorological conditions and in the current load. Any errors due to a bad estimation of the initial conductor temperature will vanish in a time in the order of the time constant.

APPENDIX VII

COMPARISON BETWEEN CALCULATED AND MEASURED TIME CONSTANTS OF CONDUCTORS

The graph below shows the comparison between the calculated and measured time constants for three ACSR conductors of different diameter and for one AAC conductor. The measurements were carried out [19, 20] in a wind tunnel with an initial temperature of 80 °C and a final temperature of 110 °C (ambient temperature 20 °C). The calculated data have been obtained with eq. (26) (for the AAC conductor) and eq. (28) (for the ACSR conductors) using the same temperatures as in the tests.



The graph shows that for wind velocity ≥ 0.5 m/s there is a good agreement between measured and calculated data while for wind velocity < 0.5 m/s the calculated times are higher than the measured ones.

The discrepancies at low wind speeds may be due to the neglect of the natural convective heat loss. Other published data [31] show good agreement between measured and calculated thermal time constants for monometallic and ACSR conductors in the range $200 < Re < 30000$.