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LARGE CROSS-SECTIONS AND COMPOSITE SCREENS DESIGN

**Working Group
B1.03**

June 2005



LARGE CROSS-SECTIONS AND COMPOSITE SCREENS DESIGN

CIGRE WORKING GROUP B1.03

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GENERAL

1 BACKGROUND.

Some areas in cable engineering which appear to be more and more frequently referred to, still remain uncovered by International Standards or CIGRE Recommendations.

There is an increasing demand for large conductor cross section XLPE cables for bulk power transmission. But there is no agreed method to define the value of the a.c. resistance to take into account when calculating the current-carrying capacity of such cables, whereas it is a critical parameter.

Metallic screens for extruded cables are more and more designed in some countries as a combination of aluminium or copper wires and metallic sheath or tape acting as water barrier. No method is available yet to calculate the short-circuit current performance of these composite screens and to calculate the eddy current losses associated with these cable constructions.

CIGRE Working Group B1-03 was set up in September 2001 to address these matters.

2 TERMS OF REFERENCE.

The Working Group was required to elaborate some recommendations for calculation and/or measurement of :

- a.c. resistance of large conductor cross-section cables,
- short-circuit performance of composite screens,
- losses assessment of composite screens,

for HV/EHV extruded power cables.

3 BROCHURE CONTENT

The technical brochure is split into 3 parts, corresponding to the fields stated in the terms of reference :

The first part deals with a.c. resistance of large conductor cross-section cables.

The second one gives directions for calculating composite screens short-circuit performances.

And the third part gives recommendations on the way to take into account losses in composite screens when performing cable current-carrying capacity calculations.

PART 1 - LARGE CROSS SECTIONS CONDUCTORS.

4 INTRODUCTION.

It is well known that the current rating of cables for a.c. transmission strongly depends on the a.c. resistance of the conductor.

The current-carrying capacity of large cross section stranded conductors can be seriously reduced by skin and proximity effects, i.e. electromagnetic effects which produce an uneven distribution of current over the conductor cross section and hence effectively increase its resistance.

To reduce the magnitude of these effects, segmental conductors were developed.

The IEC 60 287-1-1 standard [1] provides formulae to determine the a.c. resistance from the d.c. resistance.

These formulae were obtained by solving Maxwell's equation in solid round conductors, and using approximations of Bessel's functions.

It was experimentally established that they are also suitable for stranded conductors, with some minor amendments.

They were, next, extended to segmental conductors, introducing empirical coefficients from measurements performed on 1600 mm² - 4 segments oil-filled cables.

According to measurements performed on many conductors, this extension to segmental conductors is more questionable, especially for XLPE cables and when insulated wires are used.

It is worth noting that :

- * the effective value of the a.c. resistance of cores with bare copper strands may be higher than the IEC recommended value, which may lead to undersize the cable, and, so, may be hazardous.

- * the cable cost increase due to the use of insulated wires is not balanced with the permissible increase in cable current-carrying capacity.

In this brochure, a general formula is proposed for calculating the a.c. resistance of segmental large cross-sections.

But, due to the complexity of an accurate computation, measurement of the a.c. resistance has to be recommended.

Measurement methods are reviewed, stressing upon new solutions.

5 GENERAL CONSIDERATIONS.

5.1 Skin effect and proximity effect.

Skin effect is the name given to the tendency for current to flow predominantly in the periphery of a conductor due to the internal magnetic field in the conductor.

Proximity effect is the tendency for current to flow along one side of a conductor due to interaction of the magnetic fields of the current in the conductor considered and the currents in adjacent conductors.

The a.c. resistance of a solid circular conductor is given by :

$$R = R' (1 + y_s + y_p) \quad (1)$$

where :

- R' is the d.c. resistance,
- y_s is the skin effect factor and y_p is the proximity effect factor

Skin effect in solid round circular conductors and proximity effects between solid round circular conductors were deeply investigated, specially by A.H.M. Arnold [2 - 3 - 4 - 5], and formulae were worked out [6] for y_s and y_p , through tedious calculations to approximate the Bessel's functions involved in the solution of Maxwell's equations.

5.2 Skin effect in solid conductors.

The skin effect factor y_s for an isolated solid circular conductor is given by :

$$y_s(x) = \frac{x}{2} \left\{ \frac{ber(x)bei'(x) - bei(x)ber'(x)}{[ber'(x)]^2 + [bei'(x)]^2} \right\} - 1 \quad (2)$$

where:

$$x = \sqrt{\frac{\omega\mu}{\pi.R'}} \quad \omega = 2\pi.f$$

R' being the d.c. resistance.

μ being the magnetic permeability and f the frequency.

$ber(x)$, $bei(x)$ and their derivatives $ber'(x)$ and $bei'(x)$ are tabulated Bessel functions (e.g. see reference [7]).

The derivation of this formula is given in § 5.5.

The following approximate formulae [6] have been obtained for y_s :

$$0 < x \leq 2.8 \quad y_s(x) = \frac{x^4}{192 + 0.8x^4} \quad (2a)$$

$$2.8 < x \leq 3.8 \quad y_s(x) = -0.136 - 0.0177x + 0.0563x^2 \quad (2b)$$

$$x > 3.8 \quad y_s(x) = 0.354x - 0.733 \quad (2c)$$

The maximum error is less than 0.6 %, and is negligible.

The above formulae (2a) and (2c), for small and large values of x respectively are taken from Arnold's work, while formula (2b) has been derived to fit the values of y at intermediate values of x, and at the same time not to allow an appreciable discontinuity in the calculated values of y at x=2.8 and x=3.8.

5.3 Proximity effect between solid conductors.

Derivation of formulae for proximity effects, even in the simple case of solid conductors, is anything but straightforward.

Arnold worked out formulae for various cases, using complicated tabulated functions.

For instance, for 2 solid conductors carrying single-phase current :

$$y_p(x) = \frac{G(x) \left(\frac{d_c}{S} \right)^2}{1 - A(x) \left(\frac{d_c}{S} \right)^2} \quad (3)$$

$$\text{with : } A(x) = \frac{0.042 + 0.012x^4}{1 + 0.0235x^4} \quad G(x) = \frac{x^4}{64 + \frac{20}{11}x^4} \quad \text{if } x \leq 2.8$$

For 3 circular conductors carrying three-phase current, Neher and McGrath [8] have obtained the following formula suitable for power frequency application :

$$y_p(x) = \frac{x^4}{192 + 0.8x^4} \cdot \left(\frac{d_c}{S} \right)^2 \cdot \left[0.312 \left(\frac{d_c}{S} \right)^2 + \frac{1.18}{\frac{x^4}{192 + 0.8x^4} + 0.27} \right] \quad (4)$$

where d_c is the core diameter and S the axial spacing between conductors.

5.4 Skin effect and proximity effect in stranded conductors.

The basic contribution in this field once again comes from Arnold.

Skin effect.

The skin effect in a single-core cable is substantially the same as the skin effect in a solid round cylindrical conductor having the same d.c. resistance per unit length.

Proximity effect.

The proximity effect may be calculated using formulae developed for solid round conductors provided that the resistance of the conductor is divided by a factor k_p , which is the ratio of the resistance of the path along the strands to the resistance of the path across the strands.

This factor depends on many parameters such as the surface condition of the strands, the lay of the strands, the impregnation of the core and the tightness of the insulation on the core.

Let Arnold explain his works [4]

When the two conductors of a single-phase system are close to one another the magnetic fields due to each current are superimposed, and the effective resistance of both conductors is increased.

A formula which is easy to evaluate numerically and which has negligibly small errors has been developed for the case of two round cylindrical conductors. In the stranded core of a cable, the problem is complicated by the unknown contact resistance between the strands.

The proximity effect is due mainly to an increase of current in the part of the conductor nearest the return conductor and to a diminution of current in the part of the conductor remote from the return conductor.

If, therefore, the current followed the strands, the proximity effect would be suppressed entirely, since the strands occupy positions alternately near to remote from the return conductor.

Experimental results show, however, that the proximity effect is quite appreciable in stranded conductors and it follows, therefore, that the eddy currents follow a path across the strands.

The resistance of the path of the current across the strands is greater than the resistance along the strands and is dependent on factors such as the surface condition of the strands, the lay of the strands, the impregnation of the core, and the tightness of the insulation on the core.

Let the ratio of the resistance of the path along the strands to the resistance of the path across the strands be k .

Then the formula for the proximity effect in solid cylindrical conductors may be used for stranded cables provided that the assumed resistance of the conductor is divided by the factor k .

Experimental results indicate that the value of the factor k may vary for different stranded conductors between 0.45 and 0.8.

5.5 Losses in a wire influenced by a magnetic field.

5.5.1 General expression of the electromagnetic field

Let us consider a wire placed in a magnetic field (the peak value being noted as H).

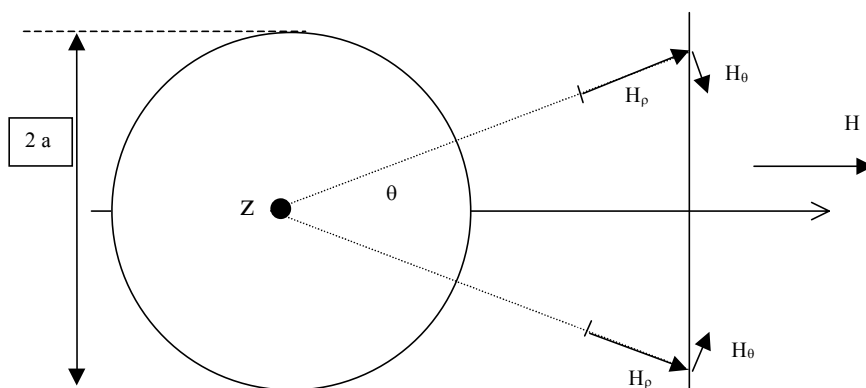


Figure 1: Magnetic field acting on a wire

From Maxwell's equations, the general expression for the longitudinal electric field E_z and the radial and transversal magnetic fields H_ρ , H_θ may be found, assuming no variation with respect to z coordinate (see appendix 1).

Inside the wire.

$$E_z = A_0 J_0(m\rho) + \sum_1^\infty A_n J_n(m\rho) \sin(n\theta + \psi_n)$$

$$H_\rho = -\frac{1}{j\omega\mu} \cdot \sum_1^\infty \frac{n}{\rho} \cdot A_n J_n(m\rho) \cos(n\theta + \psi_n)$$

$$H_\theta = \frac{m}{j\omega\mu} \left[A_0 J_0'(m\rho) + \sum_1^\infty A_n J_n'(m\rho) \sin(n\theta + \psi_n) \right]$$

where J_n is the Bessel's functions of first kind, J_n' is its derivative, and

$$m = j^{3/2} \cdot \sqrt{\omega\mu(\sigma + j\omega\epsilon)}$$

In the free space.

$$E_z = [C_{01} \ln(\rho) + C_{02}] + \sum_1^\infty [C_{n1} \cdot \rho^n + C_{n2} \cdot \rho^{-n}] \sin(n\theta)$$

$$H_\rho = -\frac{1}{j\omega\mu} \cdot \sum_1^\infty n [C_{n1} \cdot \rho^{n-1} + C_{n2} \cdot \rho^{-n-1}] \cos(n\theta)$$

$$H_\theta = \frac{1}{j\omega\mu} \left[\frac{C_{01}}{\rho} + \sum_1^\infty n [C_{n1} \cdot \rho^{n-1} - C_{n2} \cdot \rho^{-n-1}] \sin(n\theta) \right]$$

The constants A_n , C_{ni} and ψ_n are determined by expressing fields continuity at the boundary of the wire.

5.5.2 Fields and current density within the cylinder.

From calculations above :

$$E_z = \frac{I}{2\pi \cdot a} \cdot \frac{j\omega\mu}{m} \frac{J_0(m\rho)}{J_0(ma)} + 2 \cdot H \cdot \frac{j\omega\mu}{m} \cdot \frac{J_1(m\rho)}{J_0(ma)} \cdot \sin(\theta)$$

$$H_\theta = \frac{I}{2\pi \cdot a} \cdot \frac{J_0'(m\rho)}{J_0'(ma)} + 2 \cdot H \cdot \frac{J_1'(m\rho)}{J_0'(ma)} \cdot \sin(\theta)$$

Introducing k with :

$$m = j^{3/2} \cdot \sqrt{\omega\mu(\sigma + j\omega\epsilon)} \approx j^{3/2} \cdot \sqrt{\sigma\omega\mu} = j^{3/2} \cdot k$$

Following expression may be deduced :

$$E_z = -j^{\frac{3}{2}} \cdot \frac{k}{\sigma} \cdot \left[\frac{I}{2\pi \cdot a} \frac{J_0(m \cdot \rho)}{J_0'(m \cdot a)} + 2 \cdot H \cdot \frac{J_1(m \rho)}{J_0(ma)} \cdot \sin(\theta) \right]$$

5.5.3 Losses calculation

Total losses are obtained when adding losses due to the skin effect and losses due to the proximity effect.

Losses due to skin effect.

Joule losses WJ may be calculated using Poynting's theorem :

$$\frac{1}{2} \cdot \int_S (E_z \wedge H_\theta^*) dS = W_J + j \cdot \omega \cdot W_m$$

where S is the surface of the conductor .

Expressing the electric and magnetic fields :

$$2\pi \cdot a \cdot E_z \cdot H_\theta^* = \frac{1}{2} \cdot \frac{I^2}{\pi \cdot a^2} \cdot \frac{1}{\sigma} \cdot j \cdot \sqrt{\sigma \cdot \omega \cdot \mu \cdot a} \cdot \frac{J_0(ma) \cdot \left[j^{\frac{3}{2}} J_0'(ma) \right]^*}{|J_0'(ma)|^2}$$

Expressing Bessel's functions with Kelvin's functions ber and bei :

$$W_J + j \omega \cdot W_m = \frac{I^2}{2\pi \cdot a^2} \cdot \frac{1}{\sigma} \cdot [1 + F_{s0}' + j \cdot B_s] = \frac{R_{dc} \cdot I^2}{2} \cdot [1 + F_{s0}' + j \cdot B_s]$$

with :

$$F_{s0}' = \frac{ka}{2} \cdot \frac{ber(ka) \cdot bei'(ka) - bei(ka) \cdot ber'(ka)}{ber'^2(ka) + bei'^2(ka)} - 1$$

and :

$$B_s = \frac{ka}{2} \cdot \frac{ber(ka) \cdot ber'(ka) + bei(ka) \cdot bei'(ka)}{ber'^2(ka) + bei'^2(ka)}$$

Losses due to proximity effect.

Equivalent to the method using Poynting's theorem, a solution is to sum up local losses over the wire area.

$$P = \frac{1}{2} \cdot \int_0^{2\pi} \int_0^a \sigma \cdot \|E\|^2 \cdot \rho \cdot d\rho \cdot d\theta$$

$$P = \frac{1}{2} \cdot \sigma \cdot \left[\frac{2kH}{\sigma} \right]^2 \cdot \frac{1}{ber^2(ka) + bei^2(ka)} \cdot \frac{1}{k^2} \cdot \int_0^{ka} [ber_1^2(\rho') + bei_1^2(\rho')] \rho' d\rho' \cdot \int_0^{2\pi} \sin^2(\theta) d\theta$$

Using the following Bessel's functions property :

$$\int_0^x [ber_n^2(z) + bei_n^2(z)] z dz = x [ber_n(x) \cdot bei_n'(x) - bei_n(x) \cdot ber_n'(x)]$$

then :

$$P = \frac{\pi \cdot H^2}{\sigma} \cdot (ka)^2 \cdot \psi_1(ka)$$

with :

$$\psi_1(ka) = \frac{2}{ka} \cdot \frac{ber_1(ka) \cdot bei_1'(ka) - bei_1(ka) \cdot ber_1'(ka)}{ber^2(ka) + bei^2(ka)}$$

For small values of ka (up to 1) :

$$P = \frac{\pi \cdot \omega^2}{8} \cdot \sigma \cdot \mu^2 \cdot H^2 \cdot a^4$$

5.5.4 Influencing current

If the magnetic field H acting on the cylinder is generated by a current I flowing in a parallel conductor, the distance between axes S being large enough compared to the radius of the cylinder, to consider the magnetic field is nearly constant at the cylinder location :

$$H = \frac{I \cdot \sqrt{2}}{2\pi \cdot S}$$

Then :

$$P = \frac{\pi \cdot \omega^2 \cdot \sigma \cdot \mu^2 \cdot a^4}{8} \cdot \left(\frac{I \cdot \sqrt{2}}{2\pi \cdot S} \right)^2 = \frac{\omega^2 \cdot \mu^2 \cdot a^2}{16 \cdot \pi^2 \cdot R_{dc} \cdot S^2} \cdot I^2$$

And the proximity effect factor is :

$$y_p = \frac{P}{R_{dc} \cdot I^2} = \frac{\omega^2 \cdot \mu^2}{16 \cdot \pi^2 \cdot R_{dc}^2} \cdot \left(\frac{a}{S} \right)^2 = \frac{x^4}{64} \cdot \left(\frac{2a}{S} \right)^2 \quad \text{with : } x^2 = \frac{\omega \mu}{\pi \cdot R_{dc}}$$

Where x is small, this expression is consistent with Arnold's formula (3) above.

5.5.5 Multiple influences: using superposition principle

When the magnetic field acting on a wire is due to the currents flowing in several parallel conductors, the superposition principle may be used to derive the total losses in the wire.

Let us consider a conductor with M other parallel conductors, with flowing currents $I_1, I_2, \dots, I_M$.

The position of the conductor k is defined by a distance b_k from the studied conductor axis and an angle α_k as shown below.

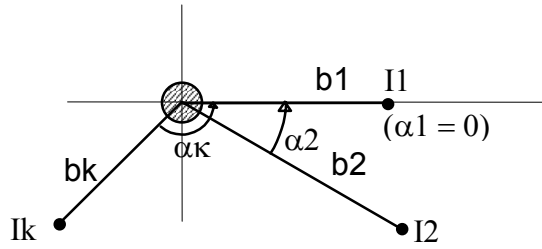


Figure 2 : Spatial references

Applying the superposition principle, the current density, taking into account the M influencing conductors is :

$$j(\rho, \theta) = j_0(\rho) + \sum_{n=1}^{\infty} j_n(\rho) \cdot \Theta_n(\theta)$$

With:

$$\Theta_n(\theta) = \sum_{k=1}^M \frac{I_k}{I} \frac{1}{a_k^n} \cos n(\theta + \alpha_k)$$

where:

$$\begin{aligned} b_1 &= b \\ b_k &= a_k \cdot b \quad (k = 1, M) \end{aligned}$$

Joule losses are given by :

$$P_t = \frac{1}{\sigma} \int_0^{2\pi} \int |j|^2 \cdot \rho \, d\rho \, d\theta$$

where σ is the conductor conductivity.

$$P_t = \frac{2\pi}{\sigma} \int_{(1)} J_0(r) \cdot J_0^*(r) \cdot r \, dr + \frac{\pi}{\sigma} \sum_{n=1}^{\infty} \Phi_M \cdot \int_{(2)} [J_n(r) \cdot J_n^*(r) \cdot r \, dr]$$

where :

$$\Phi_M = \frac{1}{\pi} \int_0^{2\pi} \Theta_n(\theta) \cdot \Theta_n^*(\theta) \cdot d\theta$$

$$\Phi_M = \frac{1}{I^2} \left[\sum_{k=1}^M \frac{I_k^2}{a_k^{2n}} + 2 \cdot \sum_{k=1}^{M-1} \frac{I_k}{a_k^n} \left\{ \sum_{j=k+1}^M \frac{I_j}{a_j^n} \cdot \cos(\varphi_k - \varphi_j) \cdot \cos n(\alpha_k - \alpha_j) \right\} \right]$$

φ_k being the phase delay of current I_k ,

Where x is small, this expression is consistent with Arnold's formula (4) above.

5.6 Segmental conductors.

For segmental conductors, the basic formula for solid conductors was extended introducing a skin effect coefficient k_s .

A k_s value of 0.435 has been used for copper bare strands, based on measurements performed on 1600 mm² - 4 segments oil-filled cables.

As early as 1968, evidence was given [9] that this has to be considered carefully.

The importance of some design features (number of sectors, direction of lay of wires in the sectors - unidirectional or contradirectional -, length of lay of wires in the sectors, degree of compaction of the sectors) was pointed out, the most sensitive parameter being the surface condition of the wires (depending on temperature and mechanical compression).

The usefulness of the segmental form of conductor construction, as regards a.c. resistance, depends upon the elimination (as far as possible) of interwire conduction.

In 1999 Jicable meeting, measurements performed on XLPE cables are displayed [10] [11] for some cores with uninsulated copper wires (4 or 5 segments) : larger values of k_s were found, typically ranging from 0.5 to 0.85. Figure 3 shows typical values measured on some large conductors.

On the other side, lower k_s values were recorded insulated copper wires.

As regards cores with aluminium wires, k_s values were introduced in IEC standard 60 287 1-1. They are derived from calculations reported in § 5.7.

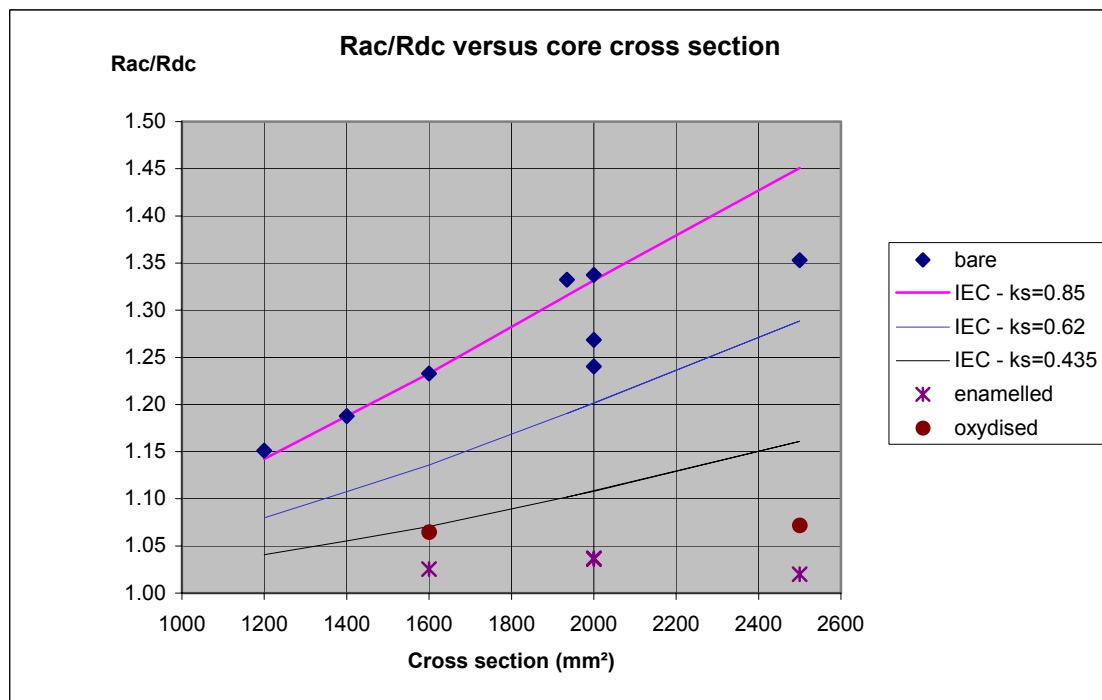


Figure 3 : a.c. resistance / d.c. resistance ratio - typical values measured on some large conductors.

5.7 Coefficients for cores using aluminium wires.

5.7.1 General

First, the calculation method is illustrated in the case of a solid conductor : it is shown that the skin effect factor derived is close to the exact value.

Then, the method is applied to segmental cores : the skin effect factor derived that way was introduced in IEC 60 287 – 1 – 1 for cores with aluminium strands.

5.7.2 Solid conductor.

The current density is assumed to be constant in the core.

The magnetic field generated by this current distribution produces electro-motive forces (emf).

Local eddy currents are produced, linked to the difference between the local emf and the mean emf, as illustrated below.

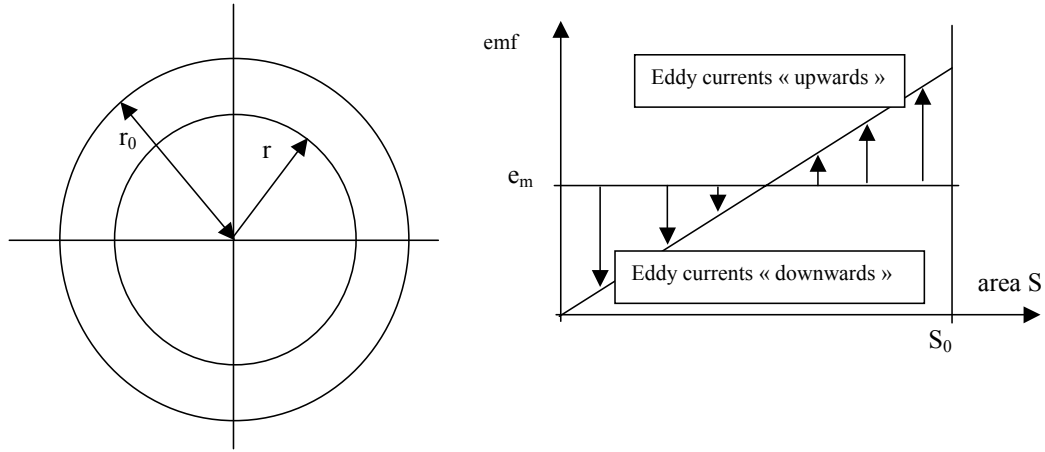


Figure 4 : Electro-motive forces in a solid conductor (radius r_0)

At distance r from the centre, the magnetic field H and the emf e are expressed as :

$$H = \frac{I}{2\pi \cdot r} \cdot \frac{\pi r^2}{\pi r_0^2}$$

$$e = -j\omega\mu \cdot \int_0^r H \cdot dr = -j\frac{\omega\mu \cdot I}{4\pi} \cdot \frac{r^2}{r_0^2} = -j\frac{\omega\mu \cdot I}{4\pi} \cdot \frac{S}{S_0}$$

Where I is the current flowing in the core, r_0 is the core radius, and S is the area of circle r as radius.

The eddy current density is given by :

$$di = \sigma \cdot (e - e_m)$$

where σ is the wire conductivity and e_m is the mean of e , that is :

$$e_m = \frac{1}{S_0} \int e \cdot dS = -j\frac{\omega\mu \cdot I}{4\pi} \cdot \frac{1}{2}$$

Losses due to eddy currents are :

$$W = \int_0^{S_0} \frac{di^2}{\sigma} dS = \sigma \cdot \left(\frac{\omega\mu I}{4\pi} \right)^2 \cdot \int_0^{S_0} \left(\frac{S}{S_0} - \frac{1}{2} \right)^2 dS = \sigma \cdot \left(\frac{\omega\mu I}{4\pi} \right)^2 \cdot \frac{S_0}{12}$$

Introducing the d.c. resistance and parameter x_s :

$$R_{dc} = \frac{1}{\sigma \cdot S_0} \quad x_s^2 = \frac{\omega\mu}{\pi \cdot R_{dc}}$$

The following result is obtained :

$$W = R_{dc} \cdot I^2 \cdot y_s \quad \text{with} \quad y_s = \frac{x_x^4}{192}$$

5.7.3 Segmental conductor.

As for the solid conductor, the current density is assumed to be constant in the core.

Segments are formed with concentric layers.

As wires are wound and insulated, the emf in every wire in a layer is the same on a lay length : it is the mean emf, calculated in a cross-section for every wires of the layer.

The emf is derived for the central wire and for the peripheral layer and a linear variation of the eddy current distribution in the segments is assumed.

So that the eddy current density is :

$$di = \sigma \cdot \left[e_{\min} + (e_{\max} - e_{\min}) \cdot \frac{S_s}{S_{s0}} - e_m \right]$$

where $e_{\min}$ is the lower value of the emf (for central wire) and $e_{\max}$ is the higher (for peripheral layer) ; e_m being the mean emf in the core.

And S_{s0} is the segment area.

In the following, N is the number of segments.

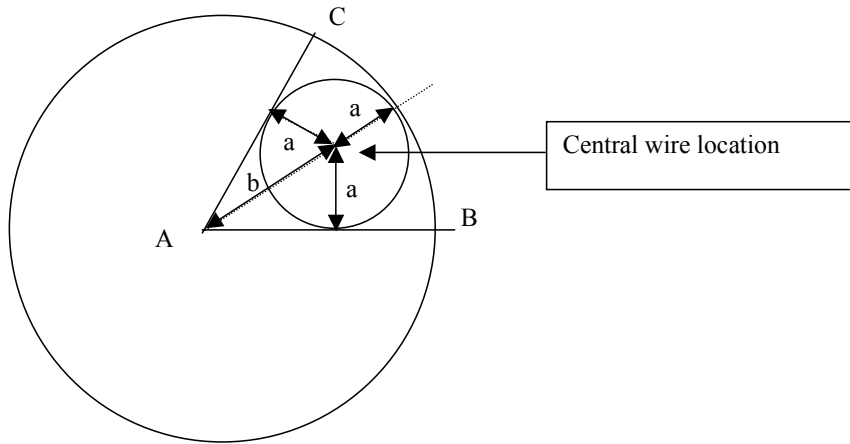


Figure 5 : location of the central wire in a segment.

The maximum value in a segment is for the peripheral layer :

$$e_{\max} = \frac{1}{l_{ABC}} \cdot \int_{ABC} e \cdot ds$$

$$l_{ABC} = \int_{ABC} ds = 2r_0 \cdot \left(1 + \frac{\pi}{N}\right)$$

$$\int_{ABC} e \cdot ds = -j \frac{\omega \mu}{4\pi} \frac{I}{r_0^2} \left[2 \cdot \int_0^{r_0} r^2 \cdot dr + \int_0^{\frac{2\pi}{N}} r_0^2 \cdot r_0 \cdot d\theta \right] = -j \frac{\omega \mu}{4\pi} \cdot I \cdot r_0 \cdot \left(\frac{2}{3} + \frac{2\pi}{N} \right)$$

$$e_{\max} = -j \frac{\omega \mu}{4\pi} \cdot I \cdot c_{\max} \quad \text{with : } c_{\max} = \frac{\left(\frac{2}{3} + \frac{2\pi}{N} \right)}{2 \cdot \left(1 + \frac{\pi}{N} \right)}$$

The minimum value is for the central wire in the segment :

$$e_{\min} = -j \omega \mu \cdot I \cdot \frac{b^2}{r_0^2} = -j \frac{\omega \mu}{4\pi} \cdot I \cdot c_{\min} \quad \text{with : } c_{\min} = \frac{1}{\left[1 + \sin\left(\frac{\pi}{N}\right) \right]^2}$$

The mean emf in the core is (as already seen) :

$$e_m = -j \frac{\omega \mu \cdot I}{4\pi} \cdot \frac{1}{2}$$

It follows that losses for every segments are :

$$W = N \cdot \int_0^{S_0} \frac{di^2}{\sigma} dS = \sigma \cdot \left(\frac{\omega \mu}{4\pi} I \right)^2 \cdot \left[\left(c_{\min} - \frac{1}{2} \right)^2 + \left(c_{\min} - \frac{1}{2} \right) \cdot (c_{\max} - c_{\min}) + \frac{(c_{\max} - c_{\min})^2}{3} \right] \cdot S_0$$

Comparing with the expression for solid conductors :

$$y_{s-segmental} = k_s^2 \cdot y_{s-solid} \quad \text{with : } k_s^2 = 12 \cdot \left[\left(c_{\min} - \frac{1}{2} \right)^2 + \left(c_{\min} - \frac{1}{2} \right) \cdot (c_{\max} - c_{\min}) + \frac{(c_{\max} - c_{\min})^2}{3} \right]$$

This leads to numerical values :

N	ks
4	0.288
5	0.195
6	0.118
7	0.070

5.7.4 Segmental conductor with peripheral strands

The emf in the central wire and in the peripheral layer in a segment may be derived from the corresponding emf in a fully segmental core with same diameter, as :

$$e'_{\min} = c.e_{\min} \quad e'_{\max} = c.e_{\max}$$

where c is the ratio of the total cross-sectional area of the segmental conductor to the total cross-sectional area of the complete conductor.

The emf in the peripheral layer may be expressed as in a stranded core.

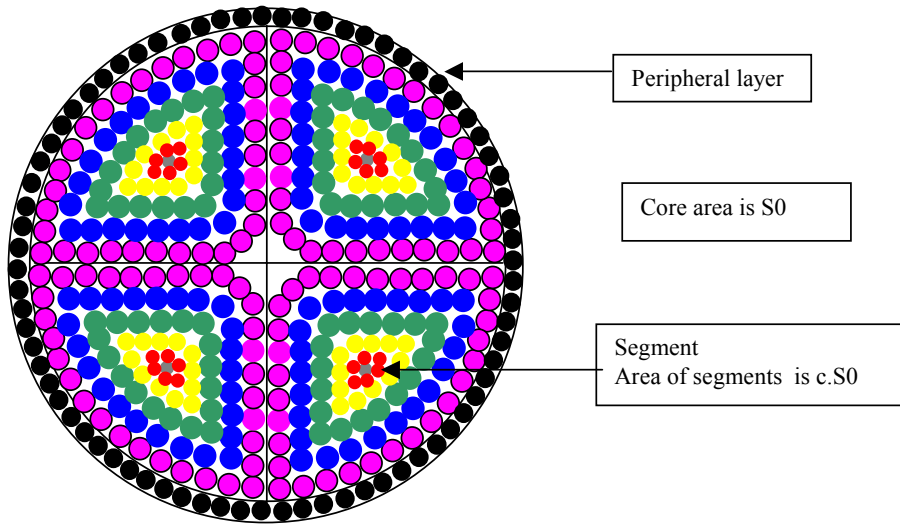


Figure 6 : Segmental core with peripheral layer.

Accordingly, the eddy current density may be expressed as :

$$di = \sigma \cdot \left[e'_{\min} + (e'_{\max} - e'_{\min}) \frac{S}{c.S_0} - \frac{1}{2} \right] \quad \text{for } S \leq c.S_0$$

$$di = \sigma \cdot \left(\frac{\omega\mu}{2\pi} I \right) \cdot \left[\frac{S}{S_0} - \frac{1}{2} \right] \quad \text{for } c.S_0 < S \leq S_0$$

Losses are :

$$W = \sigma \cdot \left(\frac{\omega\mu}{4\pi} I \right)^2 \cdot \left\{ \int_0^{c.S_0} \left[c.c_{\min} + (c_{\max} - c_{\min}) \frac{S}{S_0} - \frac{1}{2} \right]^2 dS + \int_{c.S_0}^{S_0} \left[\frac{S}{S_0} - \frac{1}{2} \right]^2 dS \right\}$$

Performing integration and comparing with the expression for solid conductor lead to :

$$k_s^2 = 12.c \cdot \left[\left(c.c_{\min} - \frac{1}{2} \right)^2 + c \cdot \left(c.c_{\min} - \frac{1}{2} \right) \cdot (c_{\max} - c_{\min}) + \frac{c^2 \cdot (c_{\max} - c_{\min})^2}{3} \right] + b \cdot (3 - 6.b + 4.b^2)$$

with $b = 1 - c$

5.8 I.E.C. present statements.

The formula for a.c. resistance of conductors, given in IEC 60 287-1-1 standard is the general formula (1) where the skin effect factor y_s and the proximity effect factor y_p , are respectively function of parameters x_s and x_p

$$x_s = x_0 \cdot \sqrt{k_s} \quad x_p = x_0 \cdot \sqrt{k_p} \quad x_0 = \sqrt{\frac{\omega \mu}{\pi R'}}$$

The formulae for y_s and y_p (for a three phase circuit) are formulae (2a) and (3) with parameters x_s and x_p respectively.

The coefficients k_s and k_p depend upon the core material and construction (see table 1).

From this table, it can be seen that main points to be dealt with are as follows :

- **Stranded conductors with aluminium or copper insulated strands**

In aluminium conductors, a tough oxide coating prevents interwire conduction, to some extent, which should turn into a value of k_p less than 1, as for cores with copper insulated strands.

- **Segmental conductors with copper bare strands**

The values of the 2 factors k_s and k_p are provisional and are not in accordance with several experimental measurements.

- **Segmental conductors with aluminium or copper insulated strands.**

For aluminium conductors, k_p is provisional; for cores with insulated copper wires, lower k_s values are recorded and lower values of k_p should be expected.

Type of conductor	Whether dried and impregnated or not	k_s	k_p
<i>Copper</i>			
Round, stranded	Yes	1	0,8
Round, stranded	No	1	1
Round, segmental ^a		0,435	0,37
Hollow, helical stranded	Yes	^b	0,8
Sector-shaped	Yes	1	0,8
Sector-shaped	No	1	1
<i>Aluminium</i>			
Round, stranded	either	1	
Round, 4 segment	either	0,28	
Round, 5 segment	either	0,19	
Round, 6 segment	either	0,12	
Segmental with peripheral strands	either	^c	
^a The values given apply to conductors having four segments (with or without central duct) and sectional areas less than 1600 mm². These values apply to conductors in which all the layers of wire have the same direction of lay. The values are provisional and the subject is under consideration. ^b The following formula should be used for k_s: $k_s = \left(\frac{d'_c - d_i}{d'_c + d_i} \right) \left(\frac{d'_c + 2d_i}{d'_c + d_i} \right)^2$ where d_i is the inside diameter of the conductor (central duct) (mm); d'_c is the outside diameter of the equivalent solid conductor having the same central duct (mm). ^c The following formula should be used to calculate k_s for cables having a conductor which consists of a central segmental portion surrounded by one or more layers of strands. $k_s = \left\{ 12c \left[(\alpha c - 0,5)^2 + (\alpha c - 0,5)(\psi - \alpha)c + 0,33(\psi - \alpha)^2 c^2 \right] + b(3 - 6b + 4b^2) \right\}^{0,5}$ where b is the ratio of the total cross-sectional area of the peripheral strands to the total cross-sectional area of the complete conductor; c is the ratio of the total cross-sectional area of the segmental conductor to the total cross-sectional area of the complete conductor, $c = (1-b)$. $\alpha = \frac{1}{(1 + \sin \pi / n)^2}$ $\psi = \frac{2\pi / n + 2 / 3}{2(1 + \pi / n)}$ n is the number of segments. This formula is applicable to aluminium conductors up to 1600 mm². If the total cross-sectional area of the peripheral strands exceeds 30% of the total cross-sectional area of the conductor then k_s may be regarded as unity. ^d Although there are no accepted experimental results dealing specifically with the coefficient k_p for aluminium conductors, it is recommended that, for stranded aluminium conductors, the values given for similar copper conductors are used.			

Table 1 : Skin and proximity effects – Experimental values for the coefficients k_s and k_p – stated in IEC 60 287

6 CALCULATION METHODS FOR SEGMENTAL CORES.

6.1 Several possible ways

In order to derive Joule losses in a segmental core, the starting point is always, of course, Maxwell's equations. The challenge is to solve this set of equations to determine the electric field in any point of the core. Several methods [12] may be used to achieve this target, namely :

- Analytical solutions
- Finite elements method
- Circuit theory
- Flux methods

Analytical solutions.

When cylindrical plain conductors are considered, the skin effect is easily calculated with analytical calculus. Only reasonable assumptions are made (such as neglecting displacement currents or end effects). Proximity effects between conductors may also be computed but accurate solutions are rather difficult to handle, since they involve a large amount of Bessel's functions. Fortunately, approximate solutions may be worked out, using polynomial approximations.

If stranded cores are dealt with, the problem is much more difficult for 2 reasons :

- The variations of the electromagnetic field in the direction of the core axis cannot be neglected.
- The relation between the current density and the electric field is no more a scalar (the conductivity of the core material) but is a tensor (whose components reflect the conductivity of both the strands and the inter-strand medium).

There are no known exact analytical solutions.

For non-segmental cores, approximate solutions are available, introducing experimental coefficients in the expressions for plain conductors.

For segmental cores, such an approach was intended but from measurements performed on many different designs, it has to be recognized that large discrepancies may be recorded.

K. Sugiyama [13] proposed a general calculation method, which may be applied to conductors with bare wires, insulated wires and to inter-layer insulated conductors, which was deeply investigated by the Working Group.

Finite elements methods.

As already mentioned, the core is an anisotropic medium.

Moreover, due to the stranding of the wires in the segments and the stranding of the segments in the core, the problem to be solved is a true 3 dimensions one.

The size of the wires and the stranding pitches are quite different, which turns into a very large mesh and would require a huge calculation power.

Circuit theory.

As a consequence of Maxwell's equations, the current density J in a strand is linked to a scalar potential V and a vector potential A .

This allows to consider each strand as an elementary conductor, characterised by a resistance and an inductance, mutual influences between the strands being expressed by mutual inductances between strands.

As the inter-strand medium is treated as vacuum, inter-strand conductivity is not considered.

Circuit theory.

The starting point are the formula linking the electric field to the scalar potential V and the vector potential A :

$$E = -\text{Grad}V - \frac{\partial A}{\partial t}$$

and the formula allowing to derive the vector potential from the current density J :

$$A = \frac{\mu}{4\pi} \cdot \int_V \frac{J}{r} \cdot dv$$

J is the current density in a volume element dv and r is the distance of the point where E is being determined from dv.

Considering sinusoidal forcing conditions and long parallel wires of circular cross-section, and assuming a current density nearly constant in every wire, this leads to :

$$\Delta V_i = - \left[R_i + \frac{j\omega\mu}{2\pi} \cdot \text{Ln} \left(\frac{r_i}{D} \right) \right] \cdot I_i - j\omega \cdot \sum_{j \neq i} A_{ij}$$

$$A_{ij} = \frac{\mu}{2\pi} \cdot I_j \cdot \text{Ln} \left(\frac{d_{ij}}{D} \right)$$

where :

ΔV_i is the longitudinal voltage drop in wire i.

d_{ij} is the axial distance between wires i and j and I_j is the current in wire j

R_i is the resistance and r_i the geometric mean radius of wire i.

D is the distance of the wires to a fictitious remote conductor carrying the return current.

Flux methods.

Basically, these methods [14,15] are similar to the circuit theory. The difference lies in the way to derive expressions for inductances. They are derived from the flux of the magnetic field between conductors.

6.2 No exact solution.

Another difficulty arises when the actual geometry of a segmental core has to be modelled: as segments are formed by compression, the distribution of the strands in the segments is not regular and the cross-section of the strands has elliptic or rhombic shapes.

All these considerations lead to the conclusion that exact computation of the eddy current losses in segmental cores cannot be achieved.

6.3 A pragmatic approach.

The final target is to derive formulae, as simple as possible, to estimate Joule losses in segmental cores, with reasonable accuracy.

Obviously, an accurate result cannot be reached when certain parameters are ignored. But, this is acceptable, as long as the result is not affected to a large extent. It has to be kept in mind that a few percent error on the skin effect factor of a segmental core has little influence on the cable current-carrying capacity.

The method applied was to use an analytical approach, with clearly identified assumptions, and to appreciate errors due to these assumptions, as far it is possible, with circuit method.

That way, a clear understanding of the problem was necessary.

Of course, the results derived from the formulae have to be in good accordance with experimental results whenever they are available.

7 DERIVATION OF PRACTICAL FORMULA FOR SKIN EFFECT

7.1 Background.

Let us consider a segmental core as illustrated below (with 4 segments).

Phenomena involved are as follows :

First, eddy currents are generated in every segment due to the skin effect.

Then, within any segment, additional eddy currents are generated, too, due to the proximity effect : they are generated by the action of the magnetic fields from other segments.

Two kinds of currents may be considered : there are always eddy currents inside every wire, and, also, circulating currents between wires, if wires are not insulated.

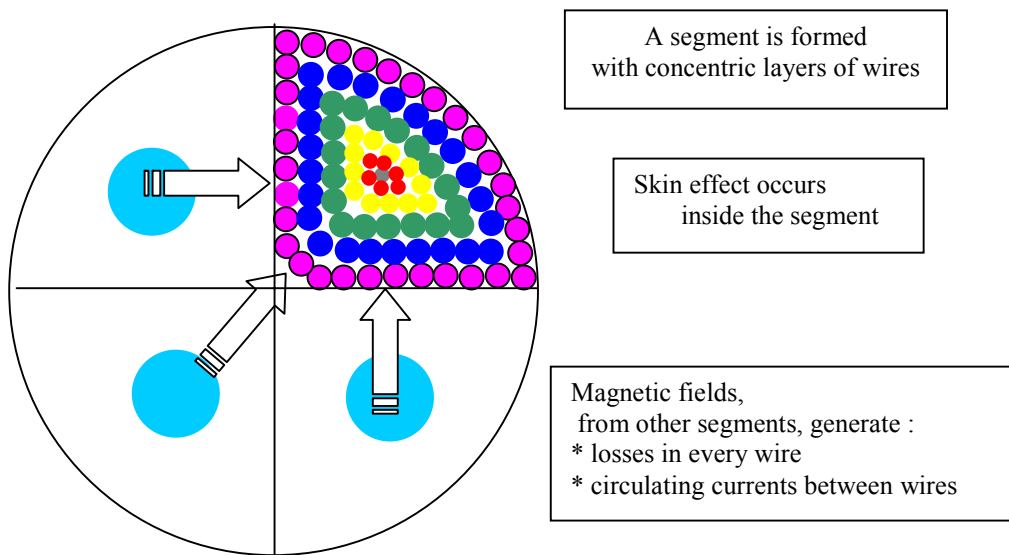


Figure 7 : skin effect in segmental cores

7.2 Skin effect in the segments.

Experimental evidence confirmed that little error is involved in using the formula for round conductors to estimate the skin effect in isolated shaped conductors.[5]

A derivation of the skin effect in a segment, taking into account the segment shape is proposed in § 5.7. It can be seen that the result is in accordance with this consideration for 4 or 5 segments but deviates for higher numbers of segments, probably due to the assumptions which are the background of the model.

7.3 Proximity effect between segments.

When dealing with proximity effect between stranded round conductors, Arnold writes [4] :

If the current followed the strands, the proximity effect would be suppressed entirely, since the strands occupy positions alternately near to remote from the return conductor.

Let us correct this consideration that proximity effect would be suppressed entirely if the strands were insulated : in such a case, an even distribution of the currents would be achieved in the segment, but the magnetic fields acting on the segment would generate eddy currents inside every strand.

Of course, considering the dimensions of the strands, this effect is not very large.

“circulating” eddy currents

The distribution of “circulating” eddy currents in a segment may be relatively simply derived, when following assumptions are made :

- A segment behaves like a concentric strand of a circular type with same cross-section.
- The magnetic field acting on a segment, produced by other segments, is constant within the considered segment, its magnitude being a mean value.
- The magnetic field generated by a segment is equal to the magnetic field due to the current flowing in the segment, assumed to be concentrated on the segment axis.

This derivation is carried out in § 7.5.4.

K. Sugiyama suggests another solution, replacing the second and third assumptions, by the following :

- The magnetic field acting on a segment is due to a constant current density within the segmental core.

§ 7.4 presents K. Sugiyama’s method.

To cope with the inter-wire conductance, an apparent conductivity is considered, as Arnold suggests.

“internal” eddy currents

Losses due to “internal” eddy currents inside every wire may be derived using a similar approach.

To assess K. Sugiyama’s solution, computations were carried out, the starting point being a simple formula which makes possible to calculate eddy current losses in a wire (radius a) located in a magnetic field H , namely :

$$Losses = \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot H_{rms}^2 \cdot a^4 \quad (5)$$

The magnetic field acting on a wire is the resultant of the magnetic fields due to the whole of other wires.

Elementary magnetic fields are derived from Ampere’s theorem, assuming every wire carries the same current.

7.4 Sugiyama’s approach.

7.4.1 Introduction

K. SUGIYAMA established equations to estimate the current distribution within segmental cores of different designs (in particular with insulated wires or inter-layer insulated segments) and worked out a relatively simple expression for the skin effect factor. In the following, the theory developed by SUGIYAMA is presented, main assumptions and derivations being highlighted.

7.4.2 Basic assumptions.

In the derivation of the skin effect factor for segmental cores, the following assumptions are introduced:

- The segment is a concentric strand of circular type (leading to an error of several percents only).
- The ratio of strand pitch and strand radius is constant.
- The conductor is sufficiently long and its end effect can be neglected.

7.4.3 Principles.

First, eddy currents are generated in every segment due to the skin effect.

The current distribution is derived from Maxwell's equations, linking the current density and the magnetic field.

Within any segment, additional eddy currents are generated, too, due to the proximity effect: they are generated by the action of the magnetic fields from other segments.

Two kinds of currents may be considered: there are always eddy currents inside every wire, and, also, circulating currents between wires, if wires are not insulated.

If the segments are supposed to be built with rectilinear wires bonded at both ends, the situation in a segment is more or less the same as for a solid conductor: the proximity effects turn into circulating currents in the wires, these currents leading to the same distribution as for a solid conductor.

If stranded uninsulated wires are used, it may be assumed that longitudinal circulating currents (i.e. parallel to the core axis) are likely to be generated as for the case of rectilinear wires. But, to flow longitudinally, they have to pass from a wire to another. To cope with the inter-wire conductance which is different from the wire conductance, an equivalent conductance of the current path is defined, as explained in APPENDIX 2.

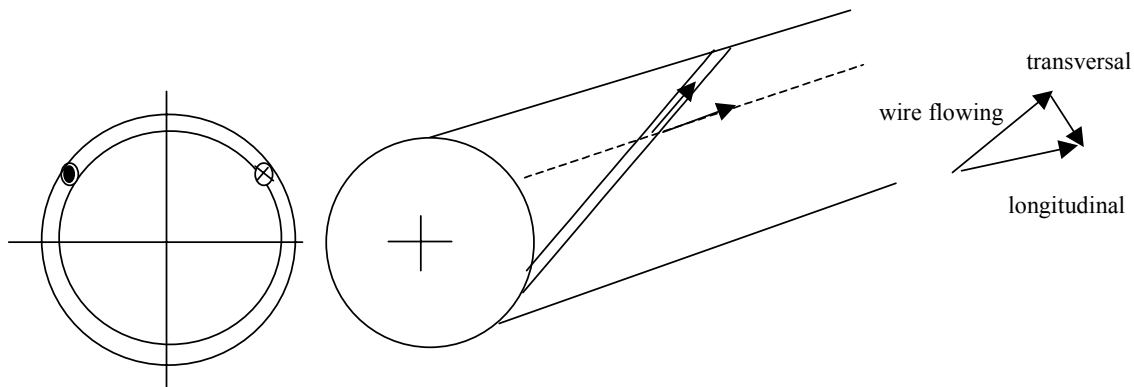


Figure 8 : Current flowing in a strand.

If the segments are stranded, using insulated wires, proximity effects are quite different:

on a lay length, any wire is influenced by external magnetic fields the same way; as a consequence, there is no circulating currents in the wires.

In this case, only local eddy currents are generated inside any wire, by external magnetic fields.

As a conclusion, the current distribution within a segment results from the skin effect in this segment and proximity effects with other segments.

Proximity effects result in:

- Eddy currents inside any wire due to magnetic fields acting on the wire.
- Circulating currents due to the magnetic field acting on the segment, when wires are not insulated. (if they are insulated, there are no circulating currents, since the flux between any 2 wires is zero).

7.4.4 Proximity effect : eddy currents.

The evaluation of the eddy currents inside any wire is going on the 3 following assumptions :

- The current density is supposed to be approximately constant within the core. It follows that the magnetic field H_c in the core is circumferential.
- Considering any wire in a segment, the eddy currents due to the “total” magnetic field H_c are the superposition of eddy currents due to the “external” magnetic field generated outside of the segment and eddy currents due to the “internal” magnetic field H_s . This magnetic field H_s within a segment due to the current distribution in this segment is circumferential in the segment (see illustration below).
- Due to compaction, wires cross-section may be considered as a square.

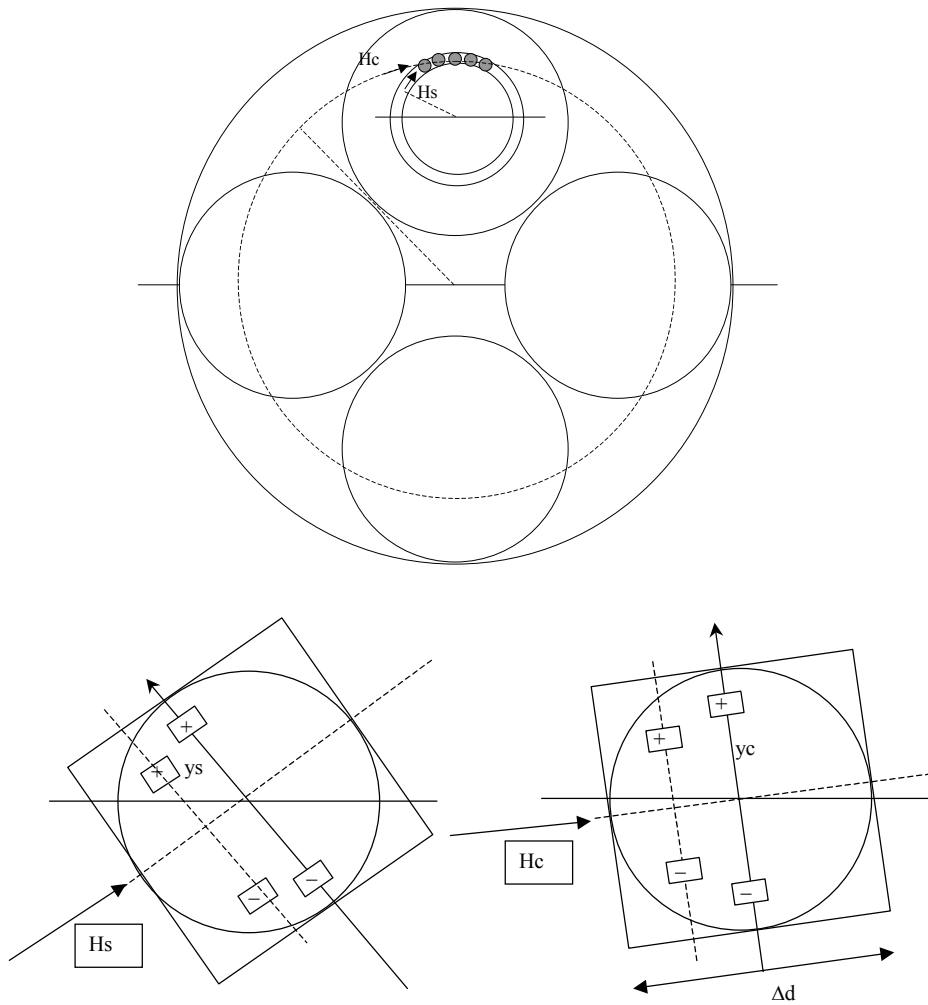


Figure 9 : Magnetic fields acting on a wire

As a consequence, following relationships may be established :

$$J_{s0}(y_s) = \bar{J}_z + j\omega \cdot \mu \cdot \sigma_0 \cdot H_s \cdot y_s \quad J_z(y_c) = \bar{J}_z + j\omega \cdot \mu \cdot \sigma_0 \cdot H_c \cdot y_c$$

the mean current density being :

$$\bar{J}_z = \frac{1}{\Delta s} \cdot \int_{\Delta s} J_z \cdot ds$$

For any wire, P(J) being the losses due to the current density J, we get :

$$P(J_z) = P(\bar{J}_z) + P(J_{H_c}) \quad P(J_{s0}) = P(\bar{J}_z) + P(J_{H_s})$$

As :

$$\int_{\Delta s} j\omega \cdot \mu \cdot \sigma_0 \cdot H_c \cdot y_c = 0 \quad \int_{\Delta s} j\omega \cdot \mu \cdot \sigma_0 \cdot H_s \cdot y_s = 0$$

Therefore :

$$P(J_z) = P(J_{s0}) + P(J_{H_c}) - P(J_{H_s})$$

As the magnetic field acting on a wire may be considered constant :

$$\int_{\Delta s} H_c^2 \cdot y_c^2 \cdot ds \approx H_c^2 \cdot \Delta d \cdot \int_{-\frac{\Delta d}{2}}^{\frac{\Delta d}{2}} y_c^2 \cdot dy_c = H_c^2 \cdot \frac{\Delta d^4}{12}$$

Losses in any wire due to Hc are :

$$P(J_{Hc}) = \frac{1}{\sigma_0 \cdot \cos \alpha} \cdot (\omega \mu \sigma_0)^2 \cdot H_c^2 \cdot \frac{\Delta d^4}{12}$$

It can be considered that the losses per unit area in the core due to the magnetic field Hc are :

$$\frac{P(J_{Hc})}{\Delta s} = \frac{1}{\sigma_0 \cdot \cos \alpha} \cdot (\omega \mu \sigma_0)^2 \cdot H_c^2 \cdot \frac{\Delta d^2}{12}$$

Total losses for every wire are obtained using :

$$\sum_{wires\ i} P_i = \int_A \frac{P_i}{\Delta s} \cdot ds$$

It is the same way with Hs.

Finally, total losses due to the skin effect and to eddy currents generated inside the wires may be expressed as :

$$P_{se} = P_{s0} + P_{e0} - P_{es}$$

$$P_{s0} = \frac{N}{\sigma_0 \cdot \cos \alpha} \cdot \int_{As} |J_{s0}|^2 \cdot ds$$

$$P_{e0} = \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0}{\cos \alpha} \cdot \frac{\Delta d^2}{12} \cdot \int_{Ac} H_c^2 \cdot ds$$

$$P_{es} = \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0}{\cos \alpha} \cdot \frac{\Delta d^2}{12} \cdot N \cdot \int_{As} H_s^2 \cdot ds$$

7.4.5 Proximity effects : circulating currents.

Considering a cylinder in a magnetic field H perpendicular to its axis, from Maxwell's equations, it follows that eddy currents generated in the cylinder may be expressed as :

$$j_c \approx j\sigma\omega\mu.\rho.H.\cos(\theta)$$

Corresponding losses are :

$$P_{cs} = \omega^2.\mu^2.\sigma.\int_{As} H^2.r^2.\cos^2\theta.ds$$

where σ is the conductivity.

The same expression is used to derive the losses in a segment, using an apparent conductivity as explained in § 7.4.3.

In the segmental core, circulating currents in the segments are derived as if the segment were plain assuming that the magnetic field acting on the segment is constant and is defined by :

$$H^2 = \frac{1}{A_c} \int_{A_c} H_c^2.ds$$

So that :

$$\int_{As} H^2.r^2.\cos^2\theta.ds = \left[\frac{1}{A_c} \int_{A_c} H_c^2.ds \right] \left[\int_{As} r^2.\cos^2\theta.ds \right] = \frac{A_s^2}{4\pi} \left[\frac{1}{A_c} \int_{A_c} H_c^2.ds \right] = \frac{A_c}{4\pi.N^2} \int_{A_c} H_c^2.ds$$

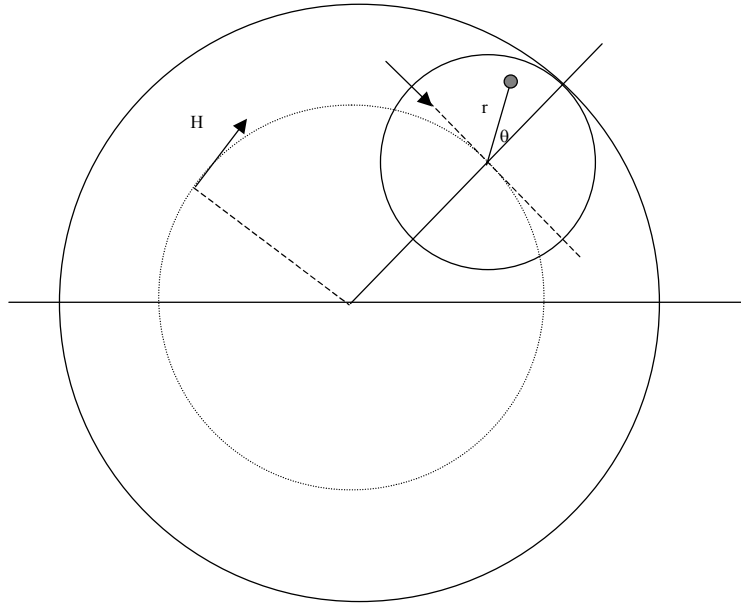


Figure 10 : Magnetic field in the core

It follows that the total losses due to circulating currents are :

$$P_c = \omega^2.\mu^2.\sigma_1.N.\int_{As} H_c^2.r^2.\cos^2\theta.ds = \omega^2.\mu^2.\sigma_1.\frac{A_c}{4\pi.N}.\int_{A_c} H_c^2.ds$$

7.4.6 Final expression.

On one hand, a.c. losses are :

$$P_{ac} = P_{s0} + P_{e0} - P_{es} + P_c$$

As a result of calculations above :

$$P_{ac} = \frac{N}{\sigma_0 \cdot \cos \alpha} \cdot \int_{As} |J_{s0}|^2 \cdot ds + \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0}{\cos \alpha} \cdot \frac{\Delta d^2}{12} \cdot \left\{ \int_{Ac} H_c^2 \cdot ds - N \cdot \int_{As} H_s^2 \cdot ds \right\} + \omega^2 \cdot \mu^2 \cdot \sigma_1 \cdot \frac{A_c}{4\pi \cdot N} \int_{Ac} H_c^2 \cdot ds$$

On the other hand, d.c. losses are :

$$P_{dc} = \frac{I_c^2}{\sigma_0 \cdot \cos \alpha \cdot A_c}$$

Therefore the a.c./d.c. resistance ratio is :

$$F_s = F_{s0} + F_{e0} - F_{es} + \Delta F$$

$$F_{s0} = \left[\frac{A_s}{I_s^2} \cdot \int_{As} |J_{s0}|^2 \cdot ds - 1 \right]$$

$$F_{e0} = \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c}{I_c^2} \cdot \frac{\Delta d^2}{12} \cdot \int_{Ac} H_c^2 \cdot ds$$

$$F_{es} = N \cdot \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c}{I_c^2} \cdot \frac{\Delta d^2}{12} \cdot \int_{As} H_s^2 \cdot ds$$

$$\Delta F = \frac{\sigma_1}{\sigma_0} \cdot \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c^2 \cdot \cos \alpha}{4\pi \cdot N \cdot I_c^2} \cdot \int_{Ac} H_c^2 \cdot ds$$

The integrals are calculated and simplification of F_{e0} and F_{es} is achieved in APPENDIX 2.

The comparison with results from other calculation methods, as illustrated in the following paragraph, lead to introduce a corrective factor for ΔF .

The final working formulae are reported in § 7.6.

7.5 Comparing calculation methods

7.5.1 General

This paragraph presents a comparison of the losses derived using Sugiyama's formula and other calculation methods.

Particularly, circuit theory was applied to core structures with a « true » geometry; the current distribution in the core is illustrated in following figures, for various configurations.

The purpose of these calculations was to check the important assumption in Sugiyama's approach, that sector-shaped segments may be considered as circular-shaped.

Of course, the actual distribution of the wires in a segment is not as fine as it may be in a simulated one, and the cross-section of the wires is no more circular. So that the calculation results using circuit theory have also to be carefully considered.

A 2000 mm² Copper core was considered, with a current flowing in the core of 1000 A.

7.5.2 Skin effect within segments and losses due to eddy currents in the wires.

Losses due to the current flow and skin effect within every segments (W/m)		
Segment number	SUGIYAMA (IEC for circular shape)	Circuit theory
4	9.2	9.3
5	9.0	9.1
6	8.8	8.9
7	8.8	8.9

Losses due to the skin effect within every segments (mW/m)	
Segment number	SUGIYAMA (IEC for circular shape)
4	570
5	370
6	260
7	190

Losses inside every wires due to magnetic fields (mW/m)			
Segment number	SUGIYAMA Cos α = 0.95	SUGIYAMA Cos α = 1	From magnetic fields due to every wires (*) Cos α = 1
4	128	116	120
5	105	95	95
6	85	77	78
7	71	64	65

(*) Losses in a wire (radius a) located in a magnetic field H are given by :

$$Losses = \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot H_{rms}^2 \cdot a^4$$

The magnetic field acting on a wire is the resultant of the magnetic fields due to the whole of other wires. Elementary magnetic fields are derived from Ampere's theorem, assuming every wire carries the same current.

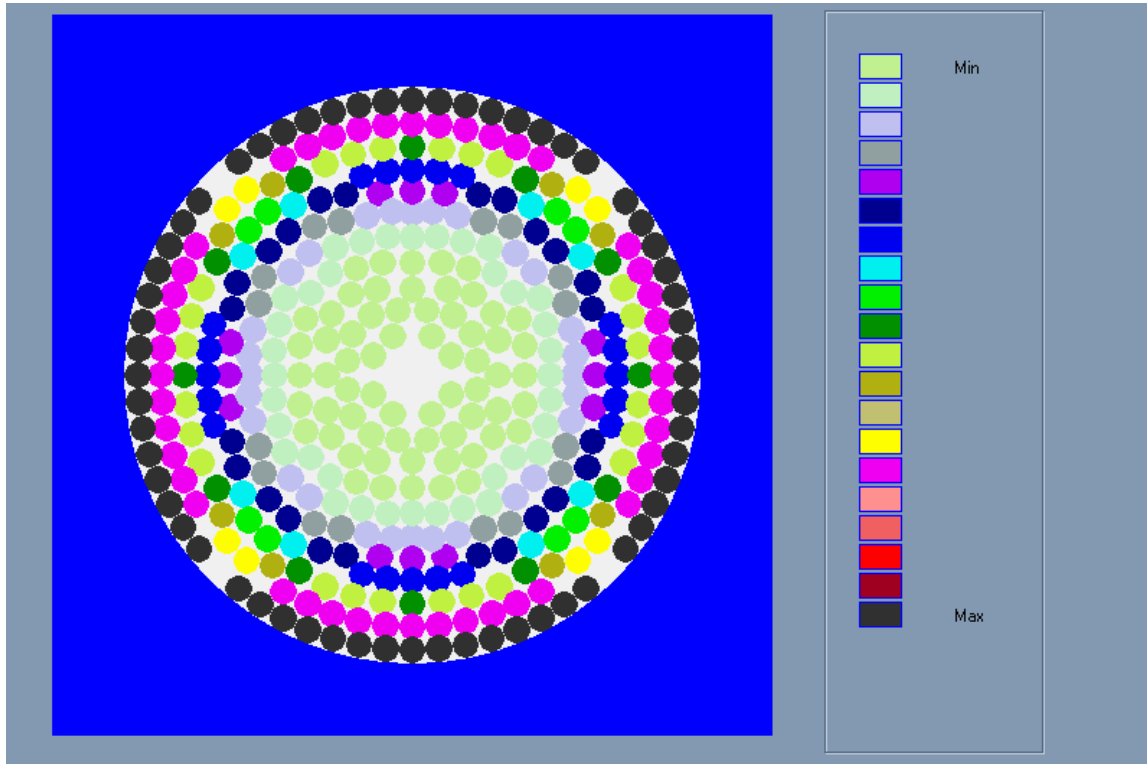


Figure 11 : Skin effect in the segments of a 4 segment core, ignoring segment insulation
(distribution as for a stranded core)

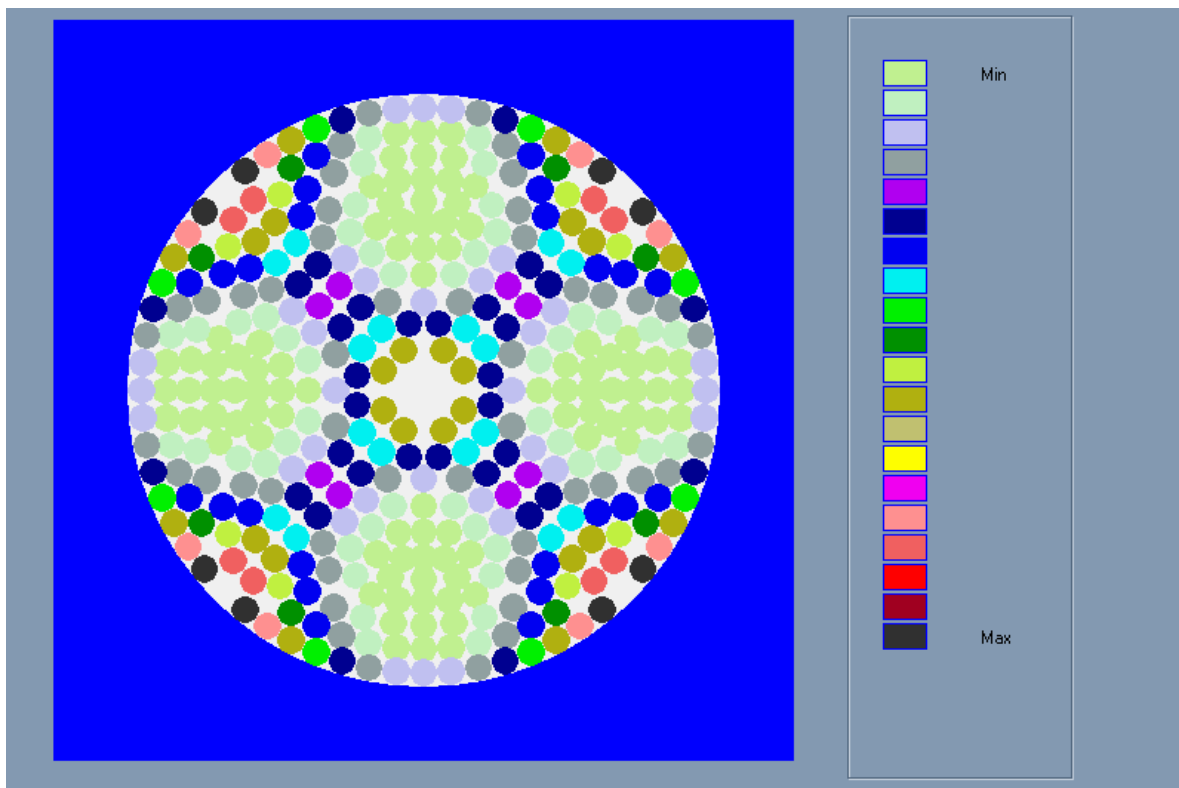


Figure 12 : Skin effect in the segments of a 4 segment core

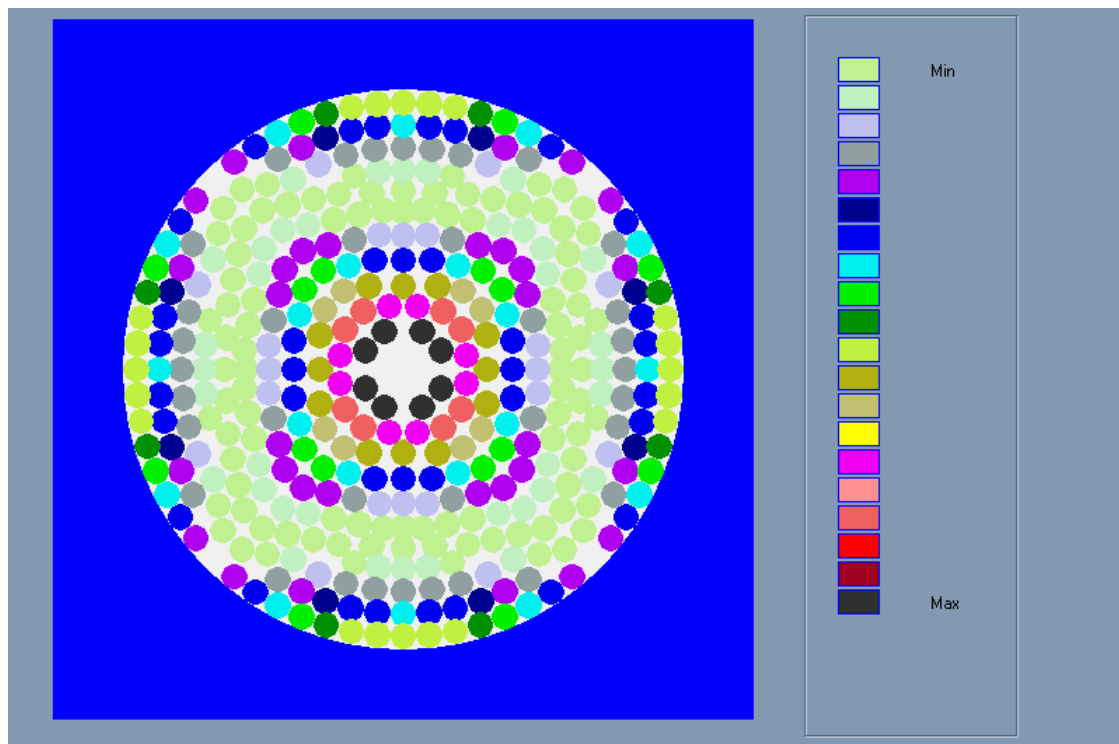


Figure 13 : Proximity effect in the segments of a 4 segment core
Current distribution in a segment is derived assuming every wire in other segments carries the same current

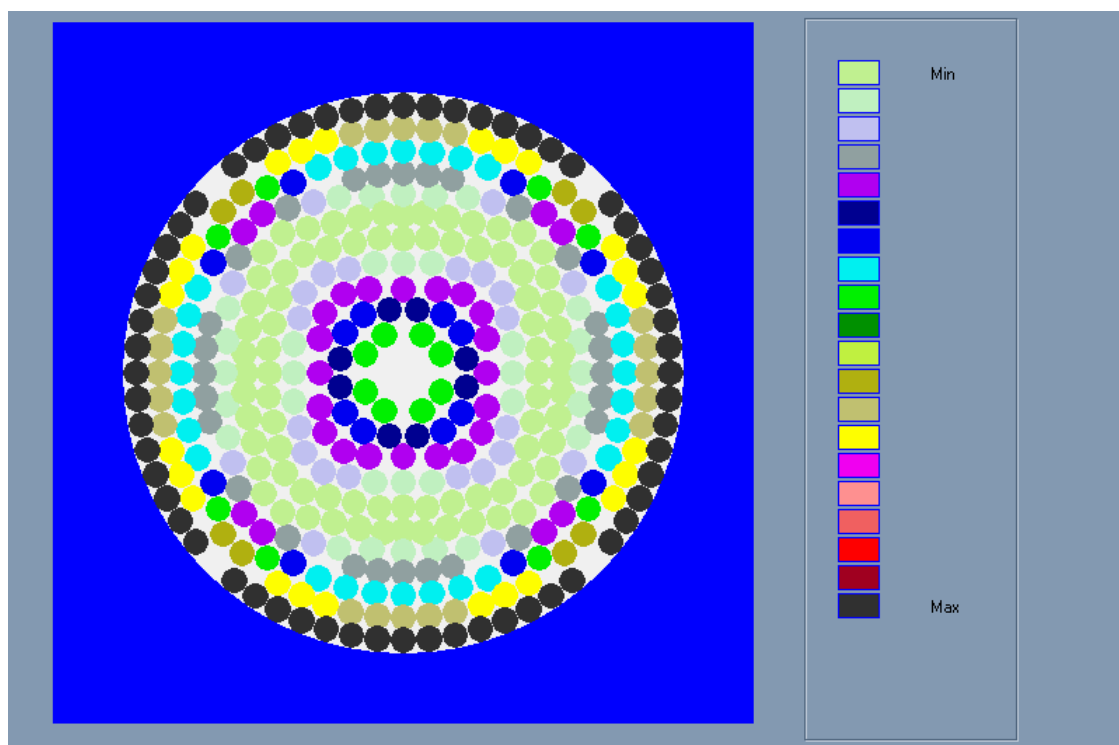


Figure 14 : Current distribution in the segments of a 4 segment core
When superposing skin effect within the segments and proximity effect assuming constant current density

7.5.3 Proximity effects between segments.

Uninsulated straight wires.

Losses due to the proximity effect between segments (W/m)				
Number of segments	SUGIYAMA	Superposition	Circuit theory	Sugiyama modified
4	14.0	10.0	6.7	7.9
5	10.7	7.1	5.8	6.8
6	8.4	5.3	5.4	5.8
7	6.8	4.1	4.7	5.0

Sugiyama

$$L_1 = R_{dc} \cdot I_c^2 \cdot \Delta F = R_{dc} \cdot I_c^2 \cdot \frac{A_c \cdot x^2 \cdot G}{16 \cdot \pi \cdot \delta_0^2} = R_{dc} \cdot I_c^2 \cdot \frac{A_c \cdot x^2 \cdot G}{16 \cdot \pi \cdot \left[\frac{2 \cdot A_c}{\pi \cdot N \cdot x^2} \right]} = R_{dc} \cdot I_c^2 \cdot \left(\frac{x^4}{64} \right) \cdot 2 \cdot N \cdot G$$

Superposition (see following paragraph)

$$L_2 \approx R_{dc} \cdot I_c^2 \cdot \left(\frac{x^4}{64} \right) \cdot \frac{N^2}{\pi^2} \cdot \Phi$$

Sugiyama modified

$$L_3 = L_1 \cdot \left(\frac{N-1}{N} \right)^2$$

The magnetic field considered by Sugiyama is the total magnetic field in the core, due to currents in every segment. If the current in the segment where the losses are derived is not taken into account, reminding that the problem is a proximity effect problem, a corrective factor has to be introduced.

It may be recognized that, for N=4 (in that case, $\Phi=5/2$) and considering G close to 1 :

$$\frac{L_2}{L_1} \approx 0.5 \approx \left(\frac{N-1}{N} \right)^2$$

7.5.4 Proximity effects between segments, using superposition.

A segment is considered as a cylinder influenced by other segments, considered as lines.

First, the losses in a segment, due to the magnetic field from an adjacent segment, are computed.

These losses are derived using the formula for a wire (circular area - radius a) located in a magnetic field H (rms value) :

$$P_{cs} = \frac{\pi}{4} \cdot \omega^2 \cdot \mu^2 \cdot \sigma \cdot H^2 \cdot a^4$$

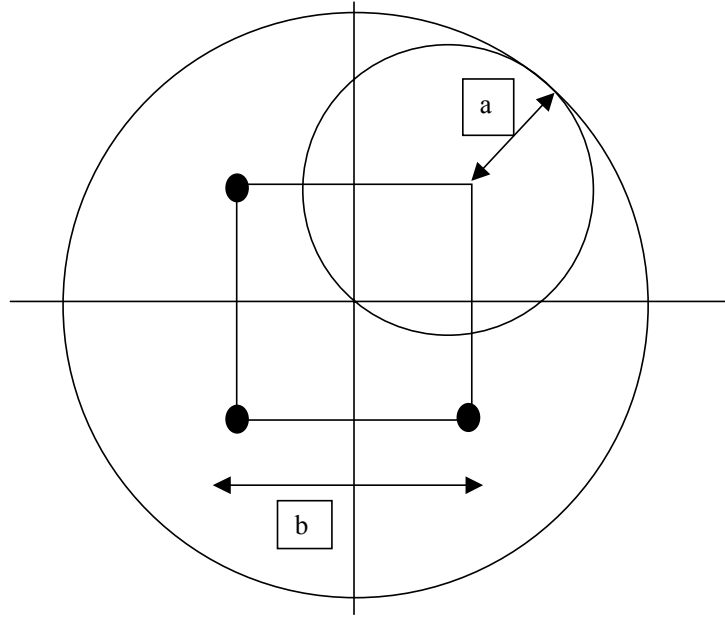


Figure 15 : Segmental core model

The magnetic field from an adjacent segment is derived as the magnetic field at the center, due to the current of the adjacent segment, localised at the center.

$$H = \frac{I_c}{2.\pi.N.b}$$

where b is the distance between 2 adjacent segment centers.

Using :

$$N.R_{dc} = \frac{1}{\sigma\pi a^2} \quad x^2 = \omega\mu\sigma.a^2$$

we get :

$$P_{cs} = N.R_{dc} \cdot \left(\frac{I_c}{N}\right)^2 \cdot \left(\frac{2a}{b}\right)^2 \cdot \frac{x^4}{64}$$

which is consistent with the formula for proximity effect from Arnold.

Now, there is the relationship between a and b :

$$b \approx 2.a.\sin\left(\frac{\pi}{N}\right) \approx 2.a.\frac{\pi}{N}$$

Therefore :

$$P_{cs} = N.R_{dc}.I_c^2 \cdot \frac{1}{\pi^2} \cdot \frac{x^4}{64}$$

To get the losses due to all other segments, superposition principle is applied.
Finally total losses are :

$$P_{cs} = R_{dc} \cdot I_c^2 \cdot \frac{x^4}{64} \cdot \frac{N^2}{\pi^2} \cdot \phi$$

where ϕ is the superposition factor.

Where $N=4$, the superposition coefficient is derived with :

$$a_1 = 1 \quad a_2 = \sqrt{2} \quad a_3 = 1$$

$$\alpha_1 = 0 \quad \alpha_2 = -\frac{\pi}{4} \quad \alpha_3 = -\frac{\pi}{2}$$

So :

$$\Phi \approx 1 + \frac{1}{2} + 1 - \left[\frac{\sqrt{2}}{2 \cdot \sqrt{2}} + 0 - \frac{\sqrt{2}}{2 \cdot \sqrt{2}} \right] = \frac{5}{2}$$

7.5.5 Looking for validation.

If a stranded core is dealt with, fictitious segments may be considered.
Losses in these fictitious segments are still mainly coming from a « skin effect » within the segment due to the magnetic field from the wires of this segment, and a « proximity effect » from other segments.

Adding losses to both effects derived from circuit theory, losses are found to be 14.7 W/m for a 6 segments core, not very far from the value for the stranded core, which is 13.9 W/m

With Sugiyama formula, losses are 17.2 W/m and with the modified formula : 14.6 W/m.

Insulated wires.

If insulated wires are considered, circulating currents due to the proximity effect between segments, should be negligible.

First, on a lay length, the mean potential vector is the same for all wires of a layer, due to regular wire arrangement.

Secondly, if circular layers are assumed, this mean potential vector value is the same for every layer.

When using circuit theory, modelling of such a configuration is achieved, considering layers as conductors.
Mutual impedances between two layers are the average of individual mutual impedances of every wire of both layers, taking into account layers in the same position in other segments.

Losses in a segmental core with insulated strands (W/m)		
Segment number	SUGIYAMA ($\sigma_1 = 0 - \cos \alpha = 1$)	Circuit theory
4	9.3	9.5
5	9.1	9.2
6	8.9	9.0
7	8.9	9.0

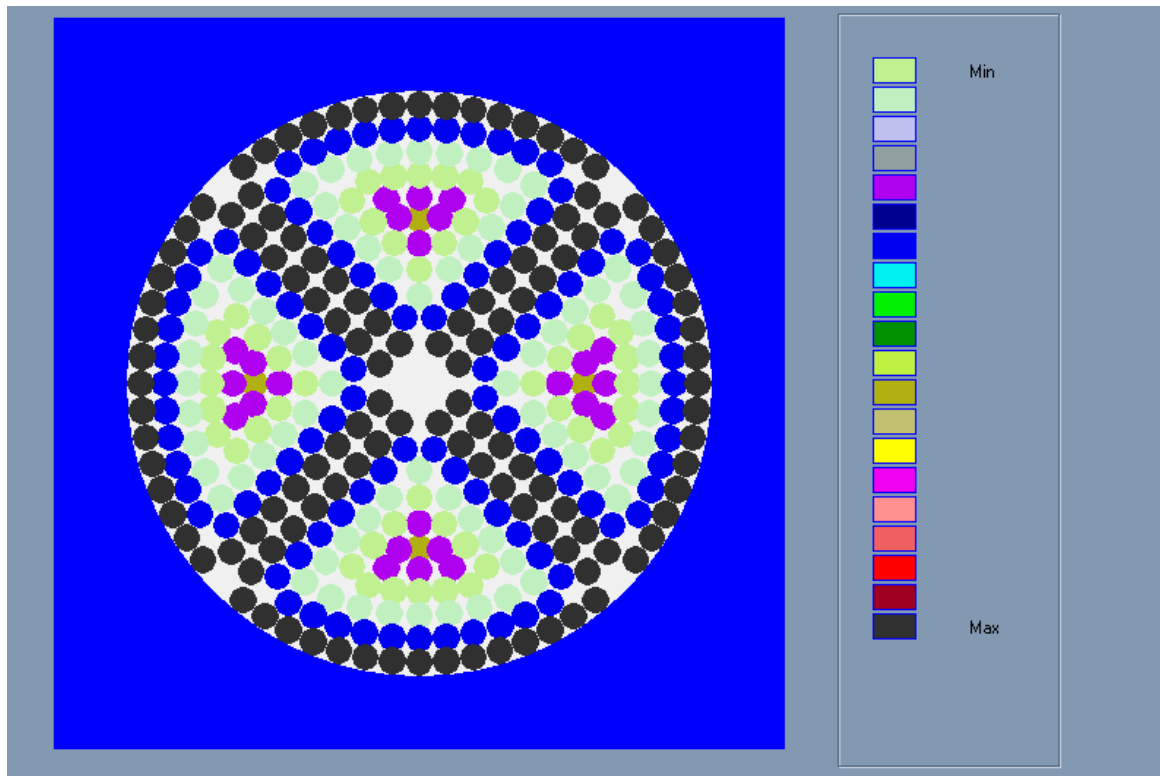


Figure 16 : Current distribution in a 4 segments core, with insulated wires

7.6 Practical formula.

A general formulation may be proposed for the skin effect factor as :

$$y_s = F_{s0} + F_e + \Delta F \quad (6)$$

F_{s0} is the skin effect factor of a solid conductor with the same cross-section as the sectors.

F_e takes into account eddy current losses inside every wire.

ΔF is a term due to eddy currents flowing from strand to strand, depending on the inter-facial conductance.

$$F_{s0} = \frac{x^4}{192 + 0.8 x^4}$$

$$F_e = \left[\frac{A_c^2}{n.N} \cdot \frac{1}{24.\pi.\delta_0^4} \right] \left[G - \frac{1}{N} \right]$$

$$\Delta F = s_r \cdot \frac{A_c \cdot x^2 \cdot G}{16.\pi.\delta_0^2} \left[\frac{N-1}{N} \right]^2$$

$$G = \frac{D_2^2 - 3.D_1^2}{D_2^2 - D_1^2} + \frac{4.D_1^4}{(D_2^2 - D_1^2)^2} \cdot \text{Ln} \left(\frac{D_2}{D_1} \right) \quad \text{where } D_1 > 0$$

$$G = 1 \quad \text{where } D_1 = 0$$

$$x^2 = \frac{\omega\mu}{\pi.N.R_{dc}}$$

$$\delta_0^2 = \frac{2}{\omega.\mu.\sigma_0}$$

Symbol	Represented quantity
A _c	nominal core cross-section
N	number of segments
n	number of wires in the segments
R _{dc}	d.c. resistance
D ₁	inner diameter
D ₂	outer diameter
σ ₀	strand conductivity
s _r	ratio of the apparent longitudinal conductivity in the core to the strand conductivity

7.7 Comparison with classical formula

The above formula may be written :

$$y_s = [F_{s0} + F_e] + \Delta F_0 \cdot s_r$$

The calculation of the losses due to the circulation currents linked to the proximity effect between segments lies on assumptions which are likely to be correct when the inter-wire conductance is low (small values of sr). Moreover, an error on these losses is less important in this case, since these losses are smaller than losses due to the skin effect in the segments.

When the inter-wire conductance is high (sr close to 1), the segmental core is expected to behave as a stranded core. In that case, the classical formula for core skin effect apply with k_s close to 1.

$$y_s = \frac{x^4}{192 + 0.8 x^4} \quad \text{with} \quad x^2 = \frac{\omega\mu}{\pi.R_{dc}}.k_s$$

On the figure 17 hereafter, the value of the k_s coefficient to apply in the classical formula to get the experimental skin effect coefficient is plotted as a function of the value of sr to be used in the working formulae to get the experimental skin effect coefficient (measurements were carried out at 3 temperatures on 10 cores, from 1600 mm² to 2500 mm², with 4 to 7 segments).

To fit the worked out formula for low sr and the classical formula for high sr , the following relationship may be used :

$$k_s \approx \sqrt{k_{s0}^2 + (1 - k_{s0}^2).sr} \quad \text{with} \quad k_{s0}^2 \approx 192. \frac{F_{s0} + F_e}{\left[\frac{\omega\mu}{\pi.R_{dc}} \right]^2}$$

This is the equation of the curve plotted on the figure 17, k_{s0} being chosen to fit the mean value of k_s for cores with enamelled strands ($sr = 0$).

Figure 18 details the theoretical k_s value to be used in classical formula as a function of sr , for different designs.

k_{s0} value (for $sr=0$) is consistent with k_{s0} value derived in § 5.7 for cores with 4 segments ; when higher segments numbers are considered, higher values are found. Experimental values do not show significant variations with the segment number. As a consequence, a single value of 0.25 is suggested.

Unfortunately, the main parameter in the formula for skin effect (the “ sr parameter”) that is connected to the inter-wire conductance appears to be difficult to evaluate, especially when considering bare wires, and moreover, this parameter is temperature dependent.

The “classical” approach based on the formula for a plain conductor connected to an empirical coefficient k_s is much easier to handle than Sugiyama’s one and leads to acceptable accuracy.

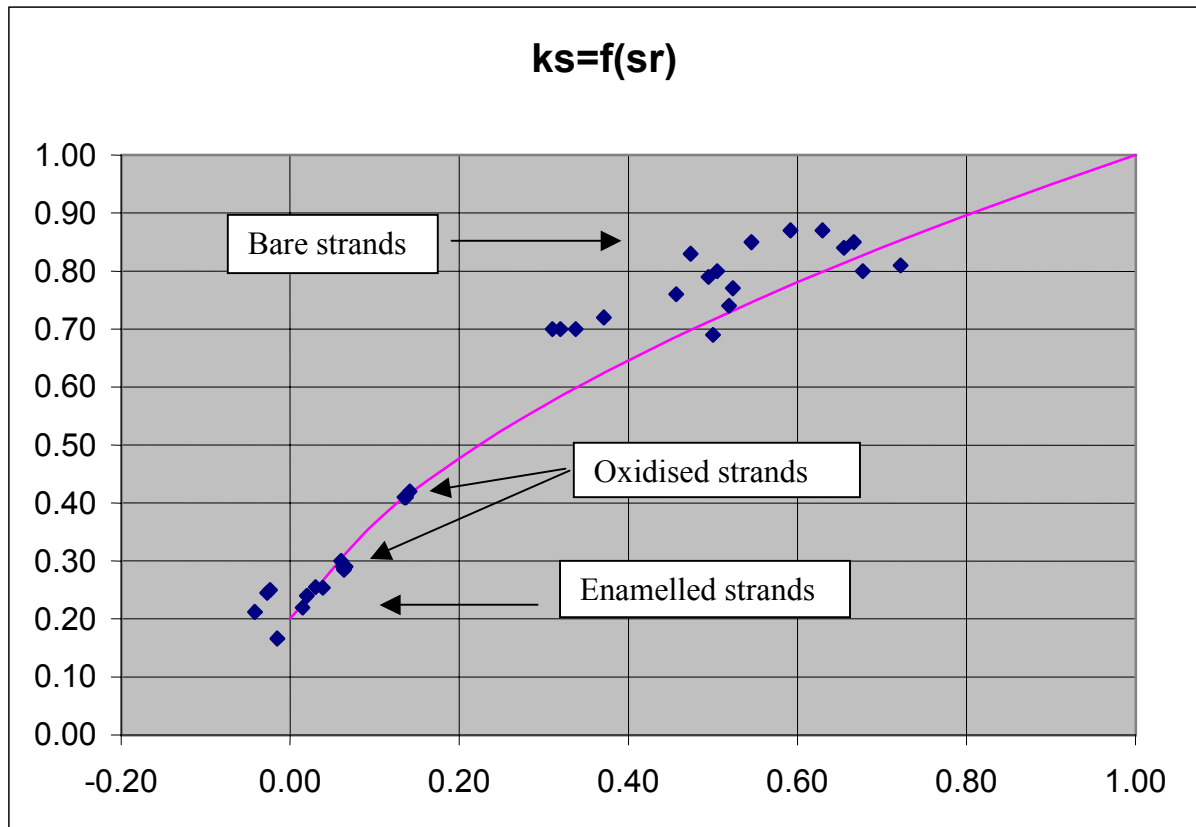


Figure 17 : k_s coefficient to apply in the classical formula as a function of the value of sr to be used in the working formulae, both to get the experimental skin effect coefficient.

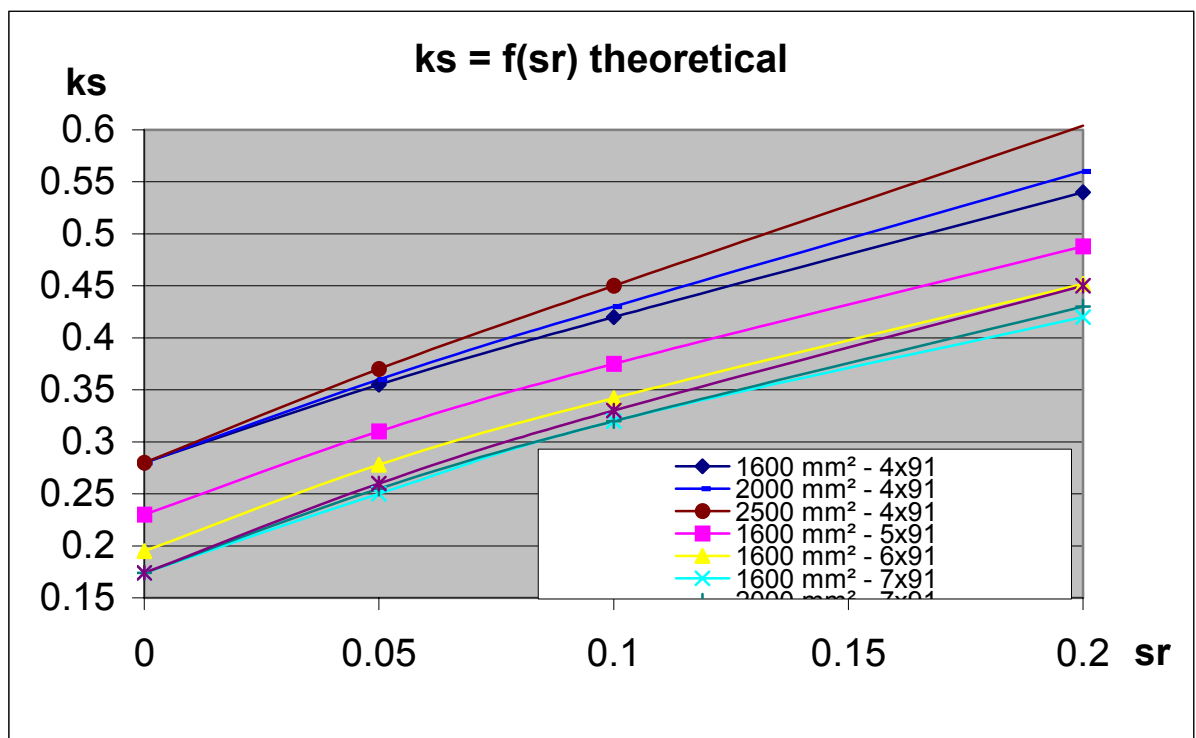


Figure 18 : theoretical k_s value to be used in classical formula as a function of sr , for different designs (cross-section – number of segments x number of wires per segment)

8 DERIVATION OF PRACTICAL FORMULA FOR PROXIMITY EFFECTS

8.1 Background

As already mentioned, the formula for the proximity effect in solid cylindrical conductors may be used for stranded cables provided that the assumed resistance of the conductor is divided by a factor k_p to cope with inter-wire conductance influence.

Arnold stated that experimental results indicate that the value of the factor k_p may vary for different stranded conductors between 0.45 and 0.8; for a well-impregnated cable, the value is likely to approach the higher limit and the lower limit is approached for dry bare conductors.

When segmental cores are concerned, the same approach may be adopted. Detailed calculations are reported in § 8.2.

8.2 Calculations

8.2.1 General

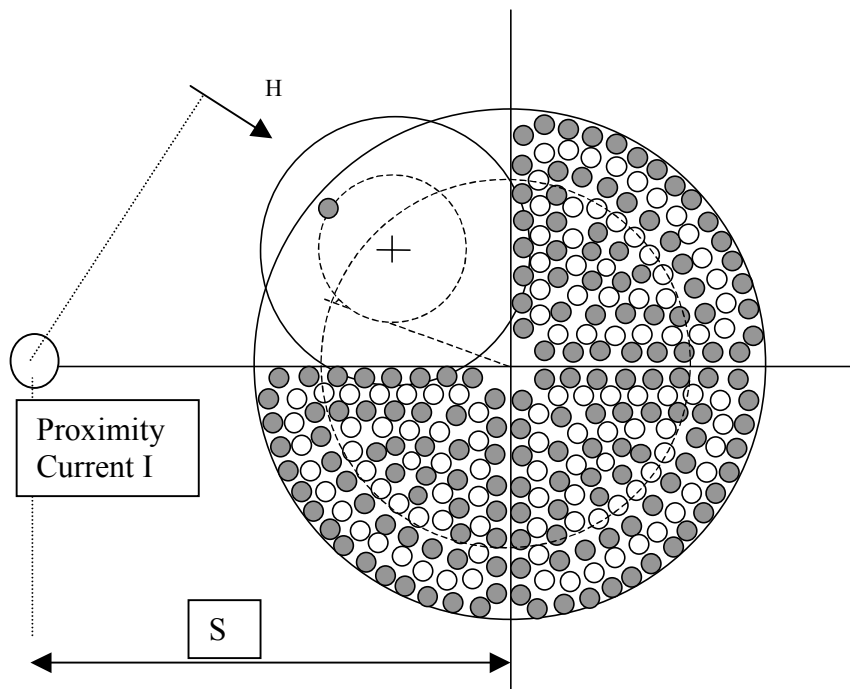


Figure 19 : Proximity effect in a segmental core where a segment is assumed circular-shaped

If inter-wire conduction is possible, circulating longitudinal currents are generated in the segments, as it would be for a solid conductor, but with an apparent conductivity taking into account the inter-wire conductivity.

If we consider a core with insulated strands, the situation is quite different :

Due to the stranding of the strands within the segments, the mean potential vector would get the same value for every wire of a given layer in a segment on a lay length, if the segments were not twisted.

The twisting of the segments within the core disturbs slightly this symmetry, but leads to identical proximity effect on every segment.

As the layers in a segment are coaxial, the mean potential vector is expected to be similar for every layers in a segment.

It follows that the mean potential vector is nearly the same for every wire in the core, and, therefore, no circulation currents occur between wires.

In that case, the proximity effect is limited to eddy currents losses generated within the strands.

8.2.2 Losses generated within the strands

Remembering that eddy currents losses in a wire (radius a) located in a magnetic field H are expressed as :

$$L = \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot H^2 \cdot a^4$$

The losses in a core due to the eddy currents within every wire are :

$$L = N \cdot n \cdot \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot a^4 \cdot \left[\frac{1}{N \cdot n \cdot \pi \cdot a^2} \sum_i H_i^2 \cdot \pi \cdot a^2 \right] \approx N \cdot n \cdot \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot a^4 \cdot \left[\frac{1}{A_c} \int_{A_c} H^2 \cdot \rho \cdot d\rho \cdot d\theta \right]$$

where H_i is the magnetic field at the location of wire i .

N is the number of segments, n is the number of strands in the segments and A_c is the core cross-section.

Expressing the local magnetic field :

$$H = \frac{I}{2\pi \cdot r} \quad \text{with : } r^2 = S^2 + \rho^2 - 2 \cdot S \cdot \rho \cdot \cos \theta$$

and using :

$$\int_0^{2\pi} \frac{d\theta}{S^2 + \rho^2 - 2 \cdot S \cdot \rho \cdot \cos \theta} = \frac{2\pi}{S^2 - \rho^2}$$

$$\int_{R_1}^{R_2} \frac{\rho \cdot d\rho}{S^2 - \rho^2} = \frac{1}{2} \cdot \ln \left(\frac{S^2 - R_1^2}{S^2 - R_2^2} \right) \approx \frac{R_2^2 - R_1^2}{2 \cdot S^2}$$

it follows :

$$\frac{1}{A_c} \int_{A_c} H^2 \cdot \rho \cdot d\rho \cdot d\theta = \left[\frac{I}{2\pi \cdot S} \right]^2$$

The following relationship is derived :

$$L \approx N \cdot n \cdot \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot a^4 \cdot \left[\frac{I}{2\pi S} \right]^2$$

Let us consider a stranded core whose cross-section is equal to the cross-section of the segmental core with N segments. For this core, we get :

$$R_{dc} = \frac{1}{\sigma \cdot \pi \cdot a_c^2 \cdot \cos \alpha} \quad \text{with} \quad a_c^2 = N \cdot n \cdot a^2$$

Defining :

$$x^2 = \frac{\omega\mu}{\pi.R_{dc}}$$

we get :

$$L \approx N.n.\frac{\pi}{4}.\frac{1}{\pi.a_c^2.\cos\alpha.R_{dc}}.[x^2.\pi.R_{dc}]^2.\left[\frac{a_c^2}{N.n}\right]^2.\left[\frac{I}{2\pi S}\right]^2$$

That is :

$$L \approx R_{dc}.I^2.\frac{1}{N.n.\cos\alpha}.\left[\frac{2.a_c}{S}\right]^2.\left[\frac{x^4}{64}\right]$$

Consequently :

$$Y_p \approx \frac{1}{N.n.\cos\alpha}.Y_p(solid)$$

Considering a 2000 mm² Cu stranded core, involving 410 wires 1.25 mm in diameter, and a parallel identical core carrying 1000 A, located at a distance of 120 mm, losses are 12.3 mW/m, corresponding to a proximity effect factor $1.4 \cdot 10^{-3}$.

For a stranded core with 372 wires 1.31 mm in diameter, imaging a 4 segment cores, losses are found to be 13.6 mW/m.

8.2.3 Losses due to circulating current between the strands

If the strands are not fully insulated, longitudinal currents are expected, and the segments behave like a solid core with an apparent longitudinal conductivity taking into account the inter-wire conductance.

Due to the twisting of the segments, same losses are expected for every segment. And they may be estimated, considering a “mean” position.

Losses in a segment (radius a) are :

$$L_s = \frac{\pi}{4}.\sigma.\omega^2.\mu^2.H^2.a_s^4$$

Losses in a core whose cross-section is equal to the cross-section of a segmental core with N segments are :

$$L_c = N^2.L_s$$

Since the radius of this core is :

$$a_c = \sqrt{N}.a$$

If losses in a segmental core are assumed to be equal to the losses in every segment :

$$L = N.L_s = \frac{L_c}{N}$$

It follows that the proximity effect factor of a segmental core is approximately :

$$Y_p(N \text{ segments}) = \frac{Y_p(solid)}{N}$$

As $Y_p(\text{solid})$ is roughly proportional to a fourth power of parameter x , this is equivalent to the situation where a coefficient k_p is introduced, with :

$$k_p = \frac{1}{\sqrt{N}}$$

Neglecting any stranding effect but taking into account an apparent longitudinal conductivity :

$$k_p \approx \sqrt{\frac{\sigma_1}{\sigma_0} \cdot \frac{1}{N}}$$

The curve k_p versus sr (ratio of conductivity) is plotted below.

8.3 Conclusion

When sr is not too small, the formula for proximity effect in a stranded core may be used, introducing a coefficient k_p .

The k_p coefficient to be introduced is plotted below as a function of the ratio of the core apparent longitudinal conductance to the strands conductance sr , for a number of segments ranging from 4 to 7.

When sr is very small, i.e. for insulated wires, proximity effect does not completely vanish.

The losses within the strands as estimated in § 8.2 are very small and could be neglected. But experimental data lead to a k_p value of about 0.14 for a 2500 mm² segmental core (6 segments) with enamelled wires.

The value of k_p stated in IEC for segmental cores, namely 0.37, appears to be a reasonable value when bare copper wires are concerned.

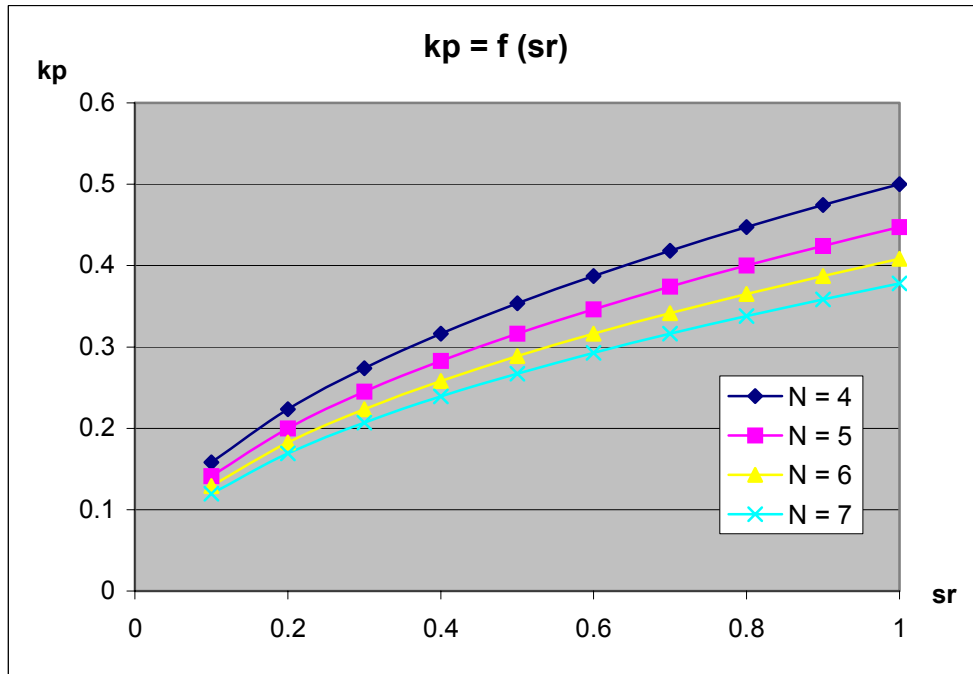


Figure 20 : Proximity effect factor as a function of the ratio of the core apparent longitudinal conductance to the strands conductance, for different numbers of segments.

9 MEASUREMENT TECHNIQUES

9.1 General

The best way to assess the a.c. resistance of conductors is probably to carry out direct measurement, but, unfortunately, there is no standard method to measure a.c. resistance of conductors.

The main methods that have been used are reviewed below. This includes thermal measurement methods [14,16] as well as electric measurement methods : conventional ones, using powermeter, potentiometer [9,17,18,19], or a.c. bridge [20, 21], and recently developed methods, using phase blocking amplifier [11] or digitizer.

An outline of each method is given below together with details of the reference document where this information came from.

9.2 Electric Methods

9.2.1 Introduction

Electric methods that are used to determine the ac resistance of large conductor size cables are based on the measurement of voltage and current at the nominal power frequency and maximum design temperature. Potentiometers and bridges have both been used to measure ac resistance and all use the four point measurement method.

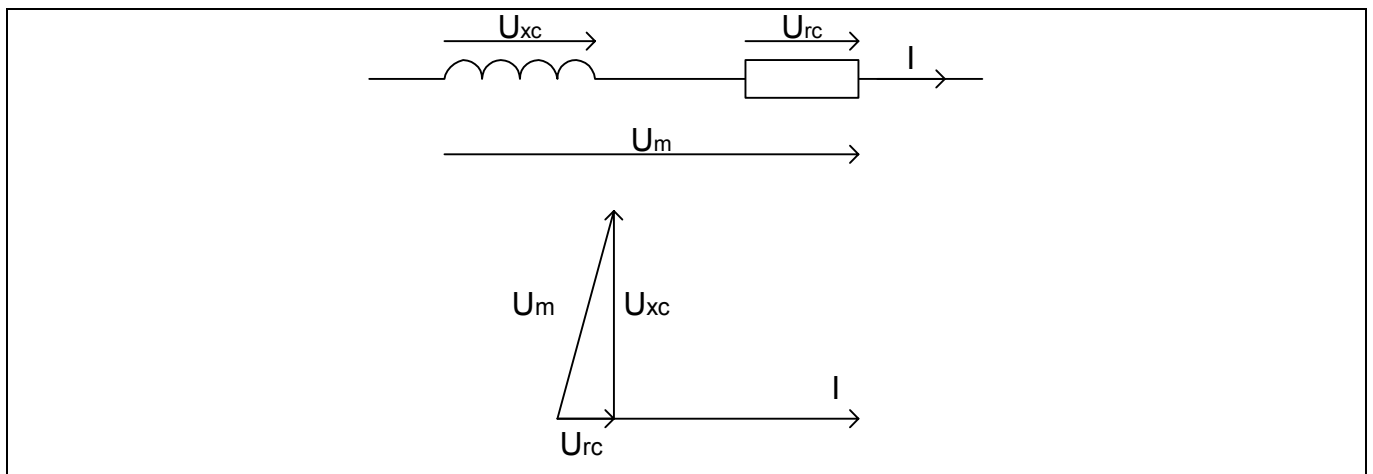


Figure 21: Equivalent circuit for R_{ac} measurement loop for large conductors together with the vector diagram.

Where:-

U_m = measured voltage.

U_{xc} = Voltage dropped across inductive component.

U_{rc} = Voltage dropped across resistive component.

The voltage dropped across the inductive component is caused by the measurement loop itself. The voltage U_{xc} has a 90° phase shift from U_{rc} and the magnitude of U_{xc} can far exceed U_{rc} (i.e. typically $U_{xc} \approx 40 \times U_{rc}$). If the magnitude of U_{xc} is reduced or eliminated then U_{rc} can be measured with greater precision.

There are several different measurement methods that are briefly discussed below.

9.2.2 Measurement of conductor ac resistance using the sheath as the return conductor.

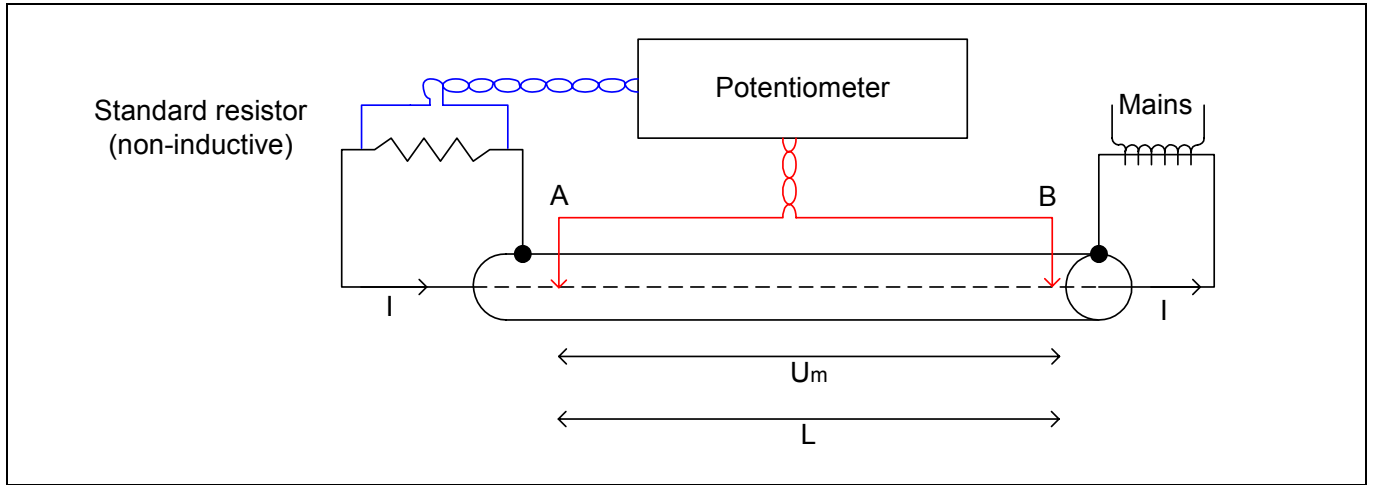


Figure 22: ac resistance using the sheath as the return conductor

By using the sheath as the return conductor this measurement method eliminates any proximity effect due to possible interaction of magnetic fields produced by currents flowing in other conductors. For this coaxial arrangement there is no measurement error due to induced voltages in the leads used to carry out the measurements. Also by using the sheath as the return path for the current flowing in the conductor the reactance of the loop is reduced and hence the magnitude of the reactive voltage component U_{xc} .

The measurement of voltage U_m between points A and B is carried out by high precision ac coordinate potentiometer. The current that flows through the standard resistor (with an accurately known resistance) also produces a voltage drop that is measured by the potentiometer. Hence the magnitudes of U_m (for a length of cable L) and I are obtained together with the phase displacement (θ).

The ac resistance per unit length is as follows.

$$R_{ac} = \frac{U_m \times \cos \theta}{I \times L}$$

Reference [9] suggest that potentiometers have limited use in measuring ac resistance due to the requirements for highly skilled operators and the length of time taken for tuning the potentiometer

9.2.3 Measurement of ac resistance using an ac bridge with a current transformer.

In the bridge arrangement shown in figure 23 and described in reference [20], the conductor under test is between points a and b. The key features to note are as follows:-

- The resistors R_2 and R_4 are precision non-inductive resistors.
- R_2 is a decade resistor.
- C_4 is decade capacitance.
- The use of the current transformer (CT) within the test arrangement means that the bridge impedances do not have to carry the high test currents that the main conductor of the power cable has to carry.
- The bridge is suitable to compare two 4-terminal impedances with a large difference in currents.

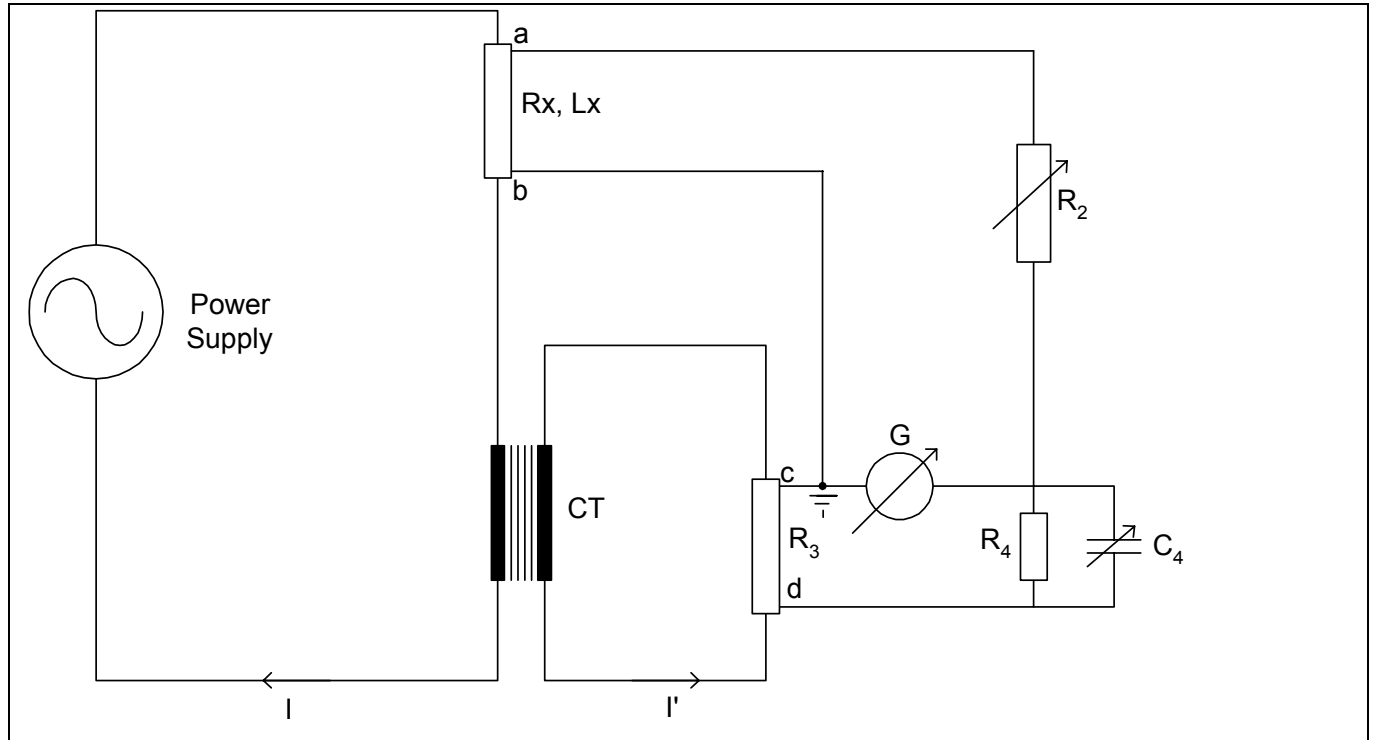


Figure 23 : ac resistance using an ac bridge with a current transformer.

Following relationships are obtained :

$$R_x = \frac{R_2 R_3}{\zeta_n R_4} (1 + \alpha) [1 - \tau \sin(\gamma - \delta)]$$

$$L_x = \frac{R_2 R_3 C_4}{\zeta_n} (1 + \alpha)$$

Where:

ζ_n = Nominal current transformer ratio.

α = Ratio error of current transformer CT

$$\tau = \omega R_4 C_4 \approx \omega \frac{L_x}{R_x}$$

$$\gamma = \varphi_2 + \varphi_3 + \varepsilon$$

$\varphi_2 \varphi_3$ = phase angle of resistors R_2 and R_3 respectively.

ε = phase angle of current transformer CT

δ = loss angle of C_4

9.2.4 Methods that could be used to give greater accuracy in measurement.

As stated previously in this report the magnitude of the voltage across the inductive component within the loop can be approximately 40 times higher than the resistive component. By annulling or compensating for this reactive component greater precision and accuracy can be achieved in measuring the voltage dropped across the resistive component.

To gain this greater accuracy in the measurement of the voltage across the resistive component the following techniques have been used.

- Superposition of a compensating voltage signal.
- Removal of the reactive voltage using mathematical means.

Superposition of a compensating voltage (reference [11])

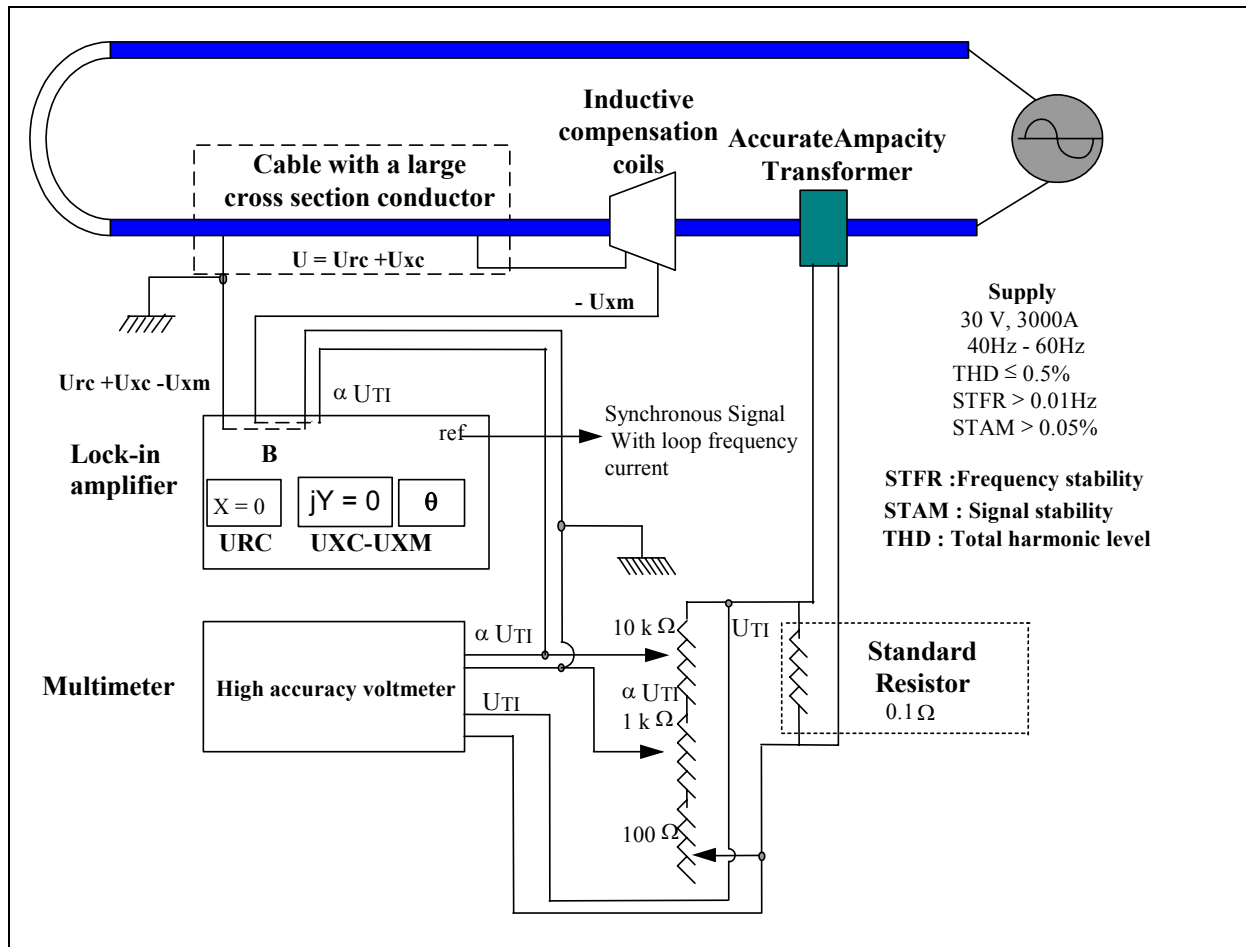


Figure 24 : Superposition of a compensating voltage signal

For the conductor being tested the voltage U_m is measured and will consist of the following.

$$\vec{U}_m = \vec{U}_{rc} + \vec{U}_{xc}$$

Using instrumentation the voltage U_m is analysed and U_{rc} and U_{xc} are determined, to do this the active reference phase (phase = 0°) must be known. The annulling of the reactive voltage is achieved by adding another signal that is equal and opposite to U_{xc} (i.e U_{xm}). The remaining signal is equal to U_{rc} .

The lock-in amplifier is not accurate enough to measure U_{rc} . Therefore the lock-in amplifier is used in its differential mode, the active signal U_{rc} is added to the purely active variable signal αU_{TI} , this is the voltage that is generated at the terminals of the standard resistor caused by the flow of current measured by the current transformer (i.e the CT measurement of the current flowing in the main conductor). Annulling of the active voltage U_{rc} is carried out until the measured signal can not be separated from the noise level (i.e. less than 100 micro-volts at 50 Hz).

The voltage αU_{TI} is measured accurately using the multi-meter.

With this method it is important to maintain the stability of the frequency of the supply and to minimise the harmonic content within the supply. This is because to annul the measured inductive signal a phase shifted sinusoidal wave is added at the relevant assumed frequency, if the supply is non-sinusoidal and not at the assumed frequency then clearly there could be significant measurement error. The following limits are suggested

- Frequency stability > 0.01 Hz
- Signal stability $> 0.05\%$
- Total harmonic content $\leq 0.5\%$

Removal of the reactive voltage using mathematical means. [22]

This method also tries to improve accuracy of the measurement by removing the reactive voltage U_{xm} from the measured voltage U and focus on the active voltage U_{rc} . This method analyses the voltage and current signals using a digitiser. After measurement of one cycle the various currents and voltages can be analysed using Fast Fourier Transforms (FFT). The disturbing induced voltage can be removed mathematically.

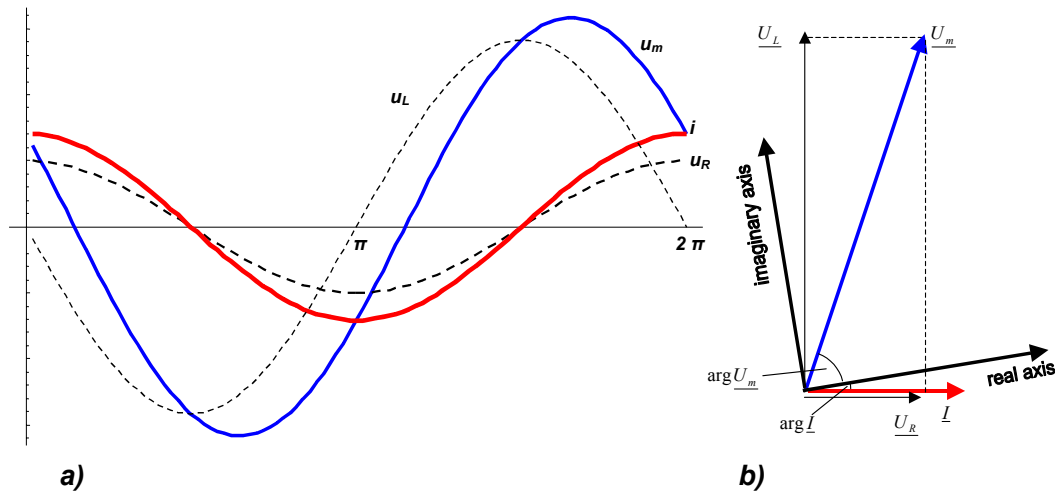


Figure 25 : The measured current, resistive component and an inductive component of the measured voltage in time domain a) and as vector diagram in the complex space b).

After the measurement the voltage and current signals of one time period are available as lists of samples $(u_1 \cdots u_n \text{ and } i_1 \cdots i_n)$. This signals can then be transformed into the frequency domain, e.g. by FFT. The only component of interest is the power frequency component. The transformation procedure is:

$$\underline{U}_m = \frac{\sqrt{2}}{n} \sum_{r=1}^n u_r \cdot e^{j \cdot 2\pi(r-1)/n} \quad \text{and} \quad \underline{I} = \frac{\sqrt{2}}{n} \sum_{r=1}^n i_r \cdot e^{j \cdot 2\pi(r-1)/n},$$

where

j is the imaginary number $\sqrt{-1}$,

$u_1 \cdots u_n$ list of voltage signals of one time period,

$i_1 \cdots i_n$ list of current signals of one time period,

$\underline{I}$ is the complex rms value of the measured current of the base frequency and

$\underline{U}_m$ is the complex rms value of the measured voltage of the base frequency

The AC resistance can easily be calculated: $R_{AC} = \frac{|\underline{U}_m|}{|\underline{I}|} \cos(\arg(\underline{U}_m) - \arg(\underline{I}))$

The measurement circuit for the experimental investigations is shown below. The current was measured with a 1000:1 current transformer parallel to a 0.2 Ω resistance.

The transformation into the complex space is carried out by a computer.

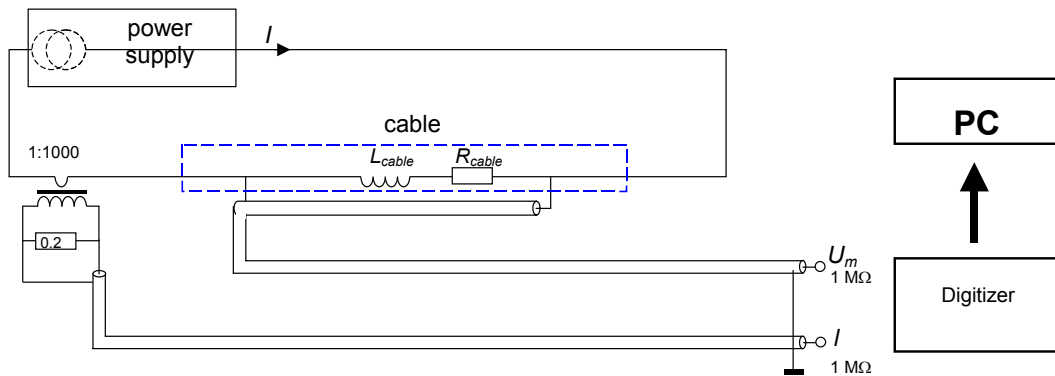


Figure 26 : Test arrangement

9.3 The Thermal Measurement Method

The basic test arrangement for this method is shown below in Figure 27.

The principle of the test arrangement is that an insulated conductor is installed in the centre of a tube with known dimensions and thermal properties. The test assembly should be located in a room with a constant temperature and it should have limited air movement in the room. The conductor of the cable would then be heated by circulating current in the main conductor. This circulating current would cause an increase of temperature (due to the $I^2 R_{ac}$ losses). By measuring the temperature drop across the tube it is possible to calculate the watts loss generated by the cable.

The thermal resistance of the tube T_{tube} can be determined using the following well-established formula.

$$T_{tube} = \frac{\rho}{2\pi} \cdot \ln\left(\frac{Do}{Di}\right)$$

Where ρ is the thermal resistivity of the tube ($^{\circ}\text{C.m/W}$); Do is the outer diameter of tube and Di is the inner diameter of tube

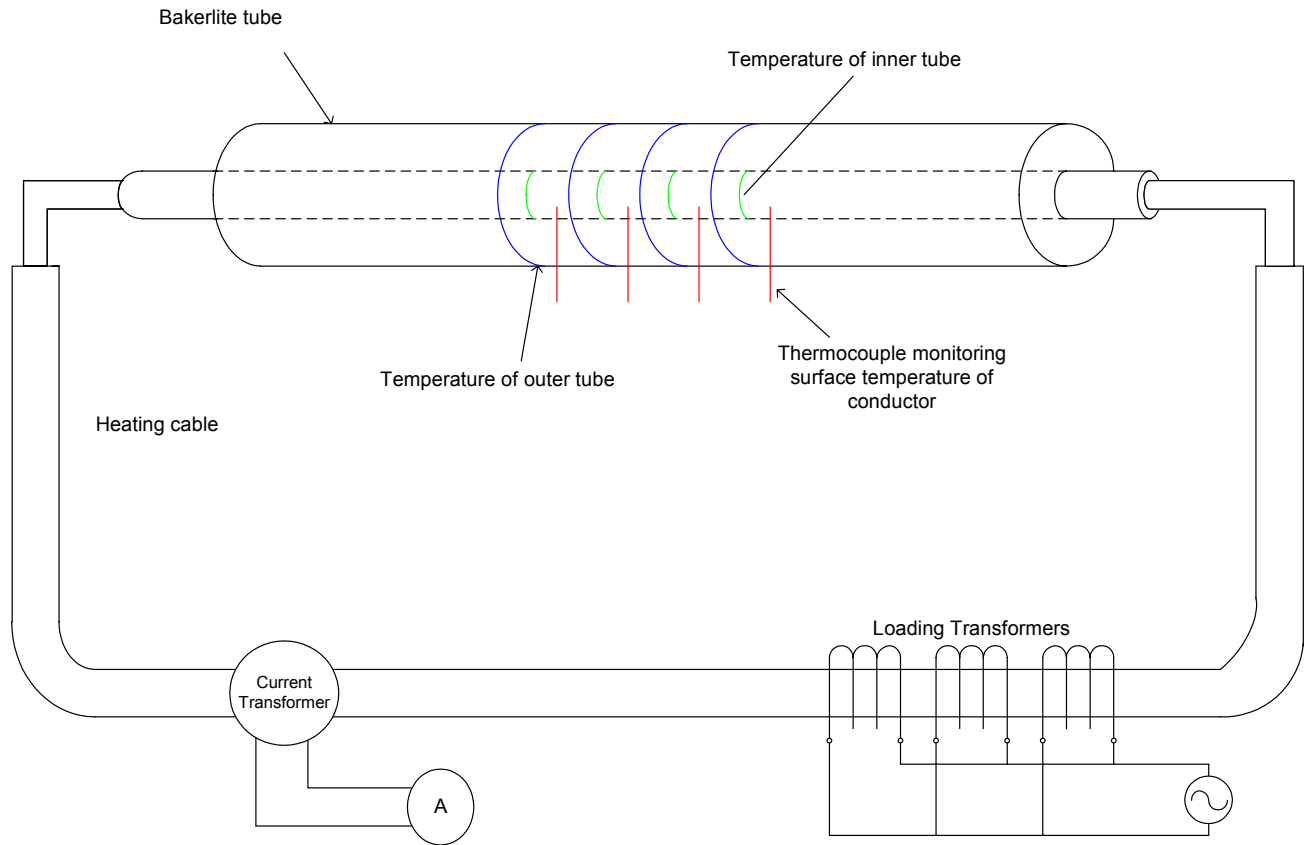


Figure 27. Basic test arrangement for the Thermal Test Method

The watts losses generated by the conductor can be determined using :

$$W = \frac{\Delta T}{T_{tube}}$$

Where:

W = Power loss of conductor (W/m)

ΔT = Temperature drop across tube (K)

T_{tube} = Thermal resistance of tube (K.m/W)

The ac resistance can be determined from :

$$R_{ac} = \frac{W}{I^2}$$

Where:

R_{ac} = resistance

I^2 = Loading current (measured load) (A)

The surface temperature of the conductor must also be known so that the ac resistance is known at a relevant temperature.

To get sufficient accuracy, the longitudinal heat flux has to be negligible. The longitudinal heat flux would be due to the conductor temperatures at both ends of the test assembly operating at a lower temperature. Negligible longitudinal heat flux can be achieved by:-

- Ensuring test assembly is of sufficient length.
- Setting a thermal insulating material at both ends of the tube.
- By comparing measurements performed on several rings on the tube.

With the method shown above the resistance can be calculated but the conductor temperature is unknown. Hence unless the conductor temperature is known accurately effect is still unknown. Therefore it would be necessary to monitor the conductor temperature as well.

10 RECOMMENDATIONS

Skin effect in very large stranded conductors.

For very large conductors (over 1600 mm²) : the expression of y_s stated in IEC 60287-1-1 should be modified since, in this case, parameter x_s is beyond the limit of validity of the approximation of Bessel's functions ($x_s < 2.8$). Improved formulae are [2b] and [2c] .

Segmental conductors.

Considering that the accuracy of calculation methods is not possible to determine, specially for copper bare wires, the Working Group recommends that the a.c. resistance of large conductor size cables be measured when the cables are being type tested.

The Working Group recommends that the classical IEC formula be retained with the values of the k_s and k_p coefficients revised as follows for copper conductors, when measurements are not available.

Type of conductor	k_s Value	k_p Value
For copper enamelled wires and aluminium wires	0.25	0.15
For copper oxidised wires (value based on study for uni-directional only)	0.35	0.20
For inter layer insulated copper.	0.50	0.37
For uni-directional stranding of copper bare wires	0.62	0.37
For bi-directional stranding of copper bare wires	0.80	0.37

To go further with promising Sugiyama's approach, additional measurements together with constructional detail records have to be gathered and investigation is necessary to evaluate the influence of the inter-wire conductivity and to derive consolidated values of the "sr parameter".

PART 2 - SHORT-CIRCUIT PERFORMANCES OF COMPOSITE SCREENS

11 INTRODUCTION.

In IEC Standards, no method is available yet to **calculate the short-circuit current performances** of composite screens.

When designing composite screens, generally, only wire shielding is counted for short-circuit performances. These performances may be worked out using IEC 60949 standard which gives a method for calculating the short-circuit rating of any current carrying component of a cable.

Taking into account the sheath would reduce the number and/or the diameter of wires to be used, and, so, would result in some savings.

12 CALCULATION METHOD.

The basis of IEC 60949 is a formula linking the temperature rise of the component to the current and duration of the short-circuit.

When composite screens are considered, the short-circuit current flows partly through wires and partly through the sheath. The temperature rises depend upon the way the current is shared between the 2 components of the screen (and upon thermal exchange coefficients).

Short-circuit rating of composite screen may be carried out using IEC approach through an iterative procedure :

- A short-circuit current is assumed.
- This current is supposed to share according to the resistances of these two components in parallel.
- The final temperature of each component is derived with the IEC formula and compared to the permissible temperature.
- A revised short-circuit current is assumed, lower if the final temperature of a component exceeds the allowable temperature, and so on.

13 INVESTIGATING TEST DATA.

From short circuit tests carried out on cables with aluminium foil and copper or aluminium wires [23], it has been observed that the current distribution between the wires and the foil is approximately proportional to the resistances of these two components. But there is no indication on temperature rises.

It seems that the distribution of the short circuit current is not so simple when considering designs involving a stainless steel sheath and copper wires.

Following results were reported :

Screen component	Constructional Data	Fault current (kA) sharing	Measured temperature (°C) Before / after	Calculated temperature (°C) (*)
Stainless steel	0.8 mm thick 149 mm diam.	11.6	19.7 / 42.9	57.1
Copper wires	80 wires diam. 2.0 mm	44.5 (all wires) 0.56 (one wire)	19.0 / 63.0	77.5
(*) Neglecting heat dissipation				
Earth fault condition 55.7 kA for 0.2 s				

While the ratio between the resistance of the stainless steel and the resistance of copper wires is approximately 25, the short-circuit current sharing follows a ratio of about 4 : the current flowing in the steel sheath is much larger than expected.

When considering a short-circuit current sharing based on the impedances, i.e. taking into account inductances of the two screen components, the current flow in the steel sheath increases from about 7 % (assuming that the test arrangement is performed so that loops with significant different areas are not managed).

The current sharing depends strongly on the way the current arc passes inside the cable.

A possible explanation is that, during the test, the arc moved preferentially from the core to the sheath and – at least at the beginning - to only some of the wires in the bundle (due to “large” clearance between the wires in the bundle).

When designing composite screen, generally, only wire shielding is counted for short-circuit performances.

14 RECOMMENDATIONS.

If a thin metallic foil is used as water radial barrier only, the sole wire shielding should be counted for short-circuit performances.

If a thick metallic sheath is applied, wires and sheath should be counted for short-circuit performances. Taking into account the sheath would turn into cheaper designs.

Performing calculations considering a current sharing according to the resistances – which is quite simple – leads to a design on the safe side.

Use of more sophisticated design rules – to be assessed – are not recommended, as expected savings would probably be not significant on the cable cost.

PART 3 - EDDY CURRENT LOSSES IN COMPOSITE SCREENS.

15 INTRODUCTION.

The metallic screens must be designed to have a suitable cross-section area capable to sustain the short-circuit current that will occur during fault conditions.

For HV and VHV cables installed in systems with very high earth fault currents, the use of traditional lead sheaths would turn into very heavy cables. That is the reason why the metallic screen for extruded cables is more and more designed in some countries as a combination of aluminium or copper wires and thin metallic sheath or laminated foil acting as water barrier.

When using a metallic sheath as a screen, its large thickness implies that the eddy current losses can reach values of 10 - 30 % and can not be eliminated, so reducing the conductor rating by 5 - 15 %.

For wires screens, there is no proper calculation method in IEC Standard 60287 and normally these types of screens are assumed to be similar to an equivalent tubular sheath of the same size.

This assumption is not completely valid : in fact, a wire screen is a set of small cross-section individual elements and cannot be modelled by a single element even if it has the same global cross-section. Exactly as the metal sheets in the magnetic circuit of a transformer have a different behaviour from the one of an « equivalent » bulk nucleus.

There is some experimental evidence that for a metallic screen constituted by wires having an electrical cross-section area equivalent to the metallic sheaths, eddy currents are practically negligible.

Some formulas are proposed in [24] for various constructions.

In this paper, we present the general considerations which make possible evaluating screen losses in composite screens when special bonding is applied.

The effect of a magnetic field on a bundle of wires has to be considered. It is demonstrated that the inter-wire conductivity is the leading parameter.

Formula and calculation results are given for screen losses.

16 LOSSES IN CABLE SCREEN WIRES.

For a 3-phase link, the derivation of the losses in screen wires of the cables requires a lot of calculations.

Considering screen wires inside any cable :

- The magnetic field from the current in the core generates the same eddy-currents in every wire.
- The magnetic fields from currents flowing in parallel cables generate eddy-currents in every wire, which are different from one wire to another.
- They may also generate circulating currents inside wires, as 2 wires may be associated to form a loop influenced by the flux of these magnetic fields.

The situation is as illustrated on figure 28.

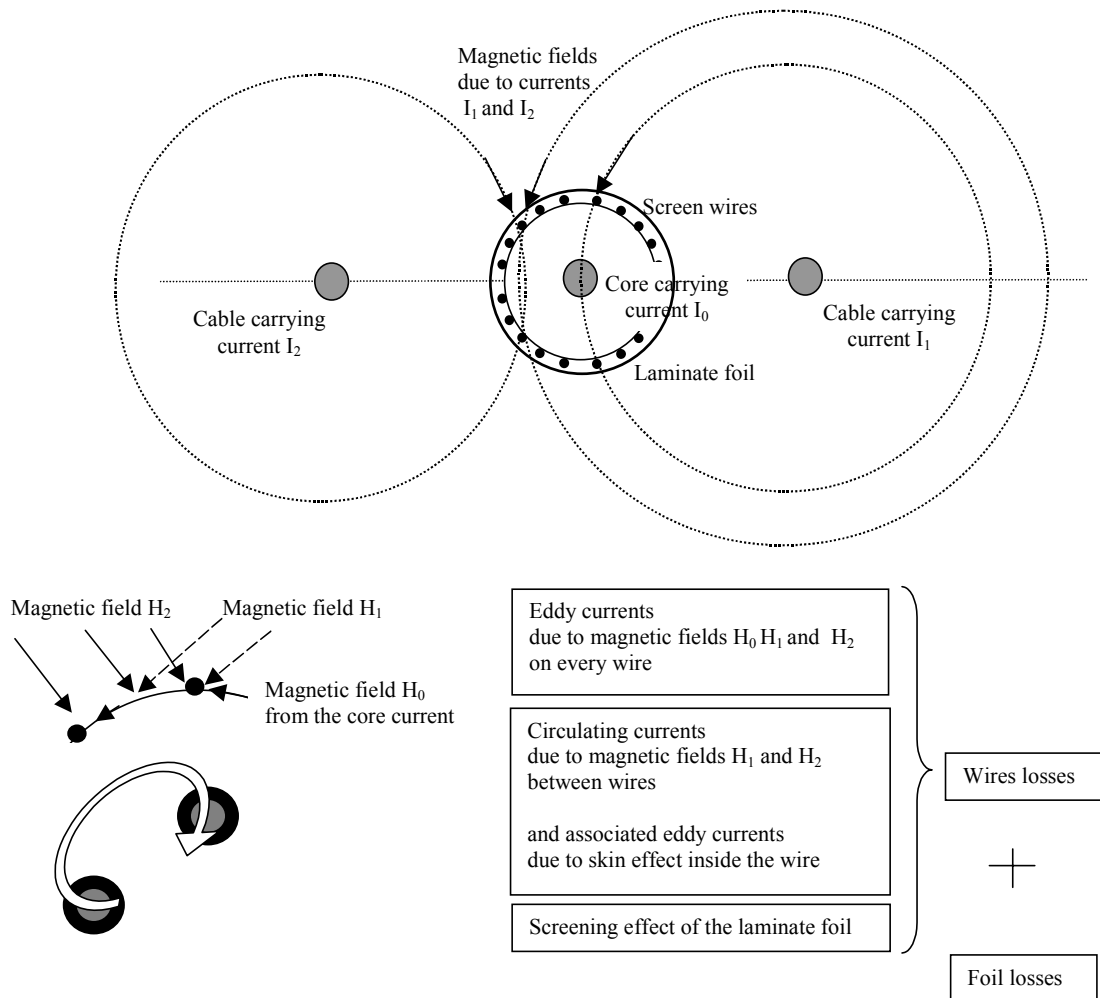


Figure 28: losses in a composite screen

17 CIRCULATING CURRENTS IN A BUNDLE OF WIRES DUE TO A MAGNETIC FIELD.

17.1 General.

First, a bundle of straight wires, located in a space where exists a magnetic field, is considered. Although not realistic, this situation is interesting since it allows understanding involved phenomena : the impact of the inter-wire medium conductivity is stated.

Then, the practical case of wound wires is addressed.

17.2 Wires are assumed to be laid straight.

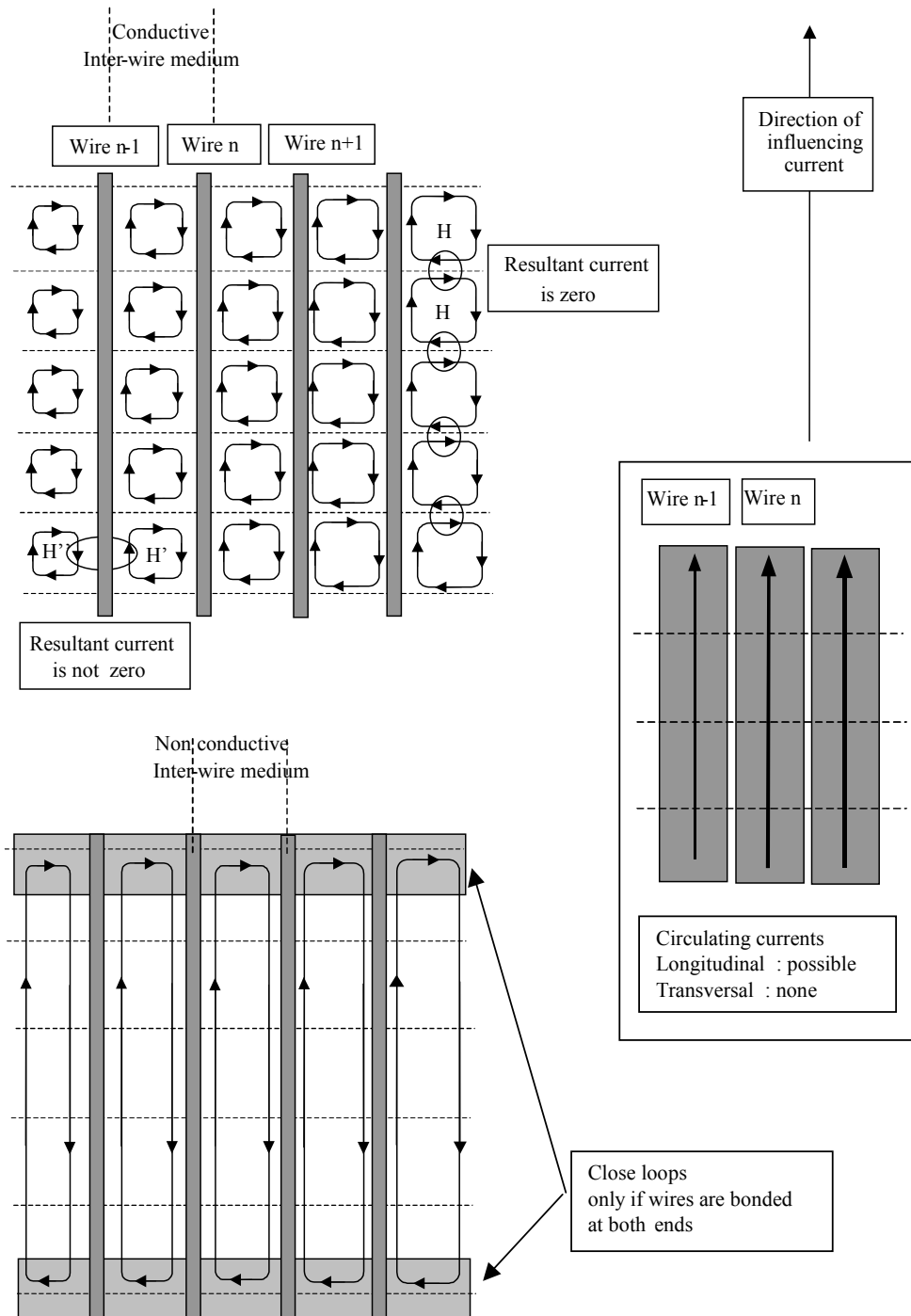


Figure 29 : Elementary loops in a bundle of straight wires.

- If the inter-wire medium is supposed to be conductive, elementary loops may be formed. As the flux of the external magnetic field varies with the distance to the influencing current, therefore from a loop to another in the direction perpendicular to the influencing current, different currents will be generated in these loops.
- If the wires are insulated, the inter-wire medium is supposed to be non-conductive. In that case, it is not possible to get close loops and circulating currents, except if wires are bonded at both ends.

17.3 Wires are wound.

If the wires are insulated or not touching, the magnetic field acts the same way on every wire over a lay length. Inter-wires currents will not be generated, whatever the end bonding is.

In that case, the losses will be similar to the losses when wires are straight and not bonded at both ends.

If inter-wire conduction is possible (touching and not insulated wires), then elementary loops are possible with circulating currents due to the (small) variations of the external magnetic field with the position.

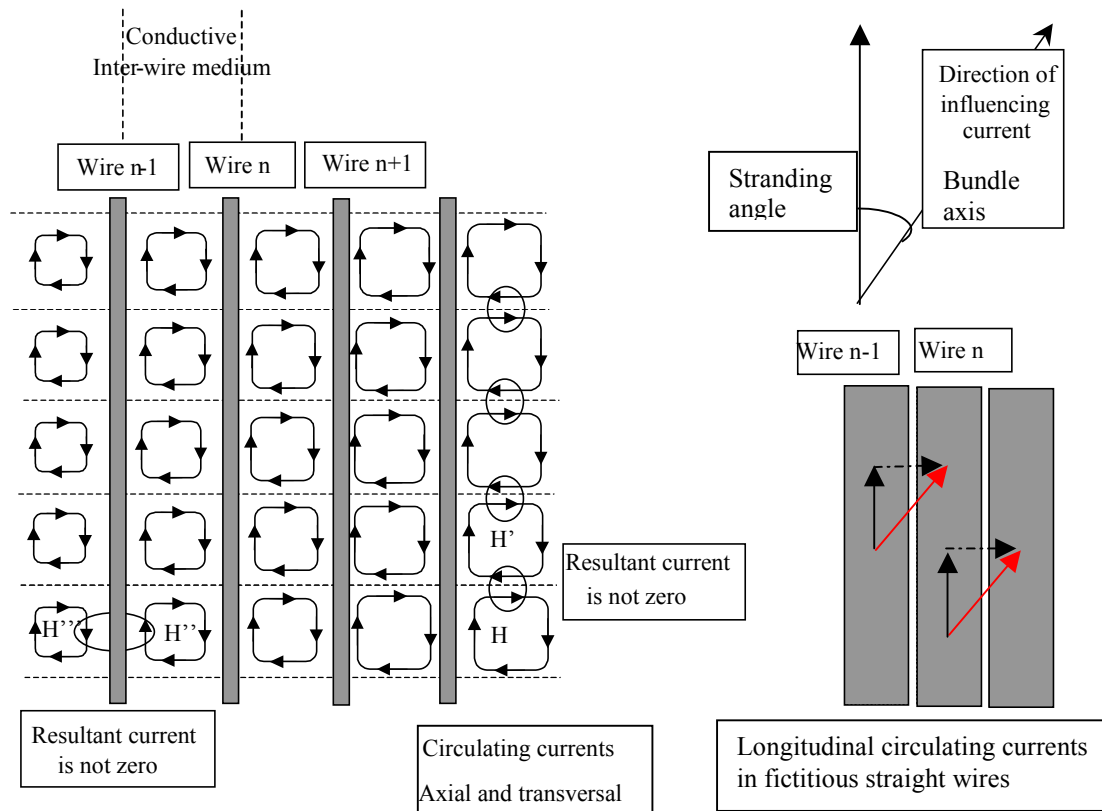


Figure 30 : Elementary loops in a bundle of stranded uninsulated wires.

It is possible to consider that longitudinal currents flow, as for straight wires, but as they have to pass from a wire to another, the apparent conductivity of the fictitious straight wires has to be modified to cope with inter-wire medium conductivity.

18 CALCULATION OF LOSSES IN CABLE SCREEN WIRES.

18.1 Eddy current losses due to the magnetic fields acting on every wire.

When a straight wire is placed in a magnetic field H , perpendicular to its axis, eddy currents are generated, flowing parallel to the wire axis. Corresponding losses may be expressed as :

$$W = \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot H^2 \cdot a^4$$

where H is the rms value of the magnetic field, and a is the wire radius.

When the magnetic field acting on a wire is generated by the currents flowing in several conductors, the superposition principle has to be used, to get losses due to the resultant eddy currents.

It is found that :

- Losses due to the internal current (flowing in the core) are similar for a bundle of wires and an “equivalent” sheath (with same resistance and same mean radius).
- Losses due to proximity effects, for usual spacing, are similar to losses due to the internal current for a bundle of wire whereas they are quite larger (several orders of magnitude) for the “equivalent” sheath.

This is illustrated below for a flat formation.

The coefficient Φ_M to derive the losses in the bundle of wires of the middle cable when considering 3 cables in flat formation is calculated as indicated hereafter.

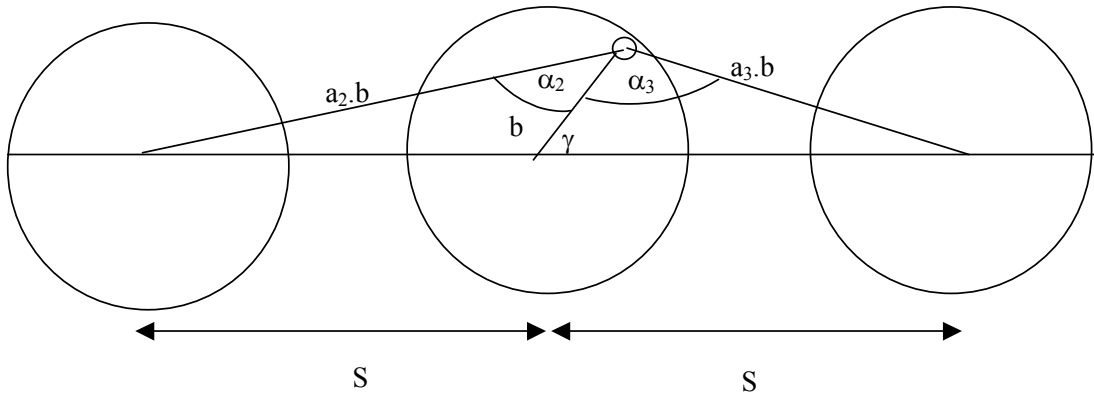


Figure 31 : Screen wire influenced by core current and proximity currents

$$\Phi_M(\gamma) = 1 + \frac{1}{a_2^2} + \frac{1}{a_3^2} - \left[\frac{\cos(\alpha_2)}{a_2} + \frac{\cos(\alpha_3)}{a_3} + \frac{\cos(\alpha_2 - \alpha_3)}{a_2 \cdot a_3} \right]$$

with :

$$a_2^2 b^2 = b^2 + S^2 + 2 \cdot b \cdot S \cdot \cos(\gamma)$$

$$a_3^2 b^2 = b^2 + S^2 - 2 \cdot b \cdot S \cdot \cos(\gamma)$$

$$S^2 = b^2 + a_2^2 \cdot b^2 - 2 \cdot a_2 \cdot b^2 \cdot \cos(\alpha_2)$$

$$S^2 = b^2 + a_3^2 \cdot b^2 - 2 \cdot a_3 \cdot b^2 \cdot \cos(\alpha_3)$$

The average value is approximately :

$$\overline{\Phi}_M \approx \frac{1}{2} \left[\Phi(0) + \Phi\left(\frac{\pi}{2}\right) \right] \approx \frac{1}{2} \left[\frac{1 + 3 \cdot \left(\frac{b}{S}\right)^2}{\left[1 - \left(\frac{b}{S}\right)\right]^2} + 1 \right] \approx 1 + \frac{5}{2} \cdot \left(\frac{b}{S}\right)^2$$

When a straight wire is placed in a magnetic field H, perpendicular to its axis, eddy currents are generated, flowing parallel to the wire axis. Corresponding losses may be expressed as (see above) :

$$W = \frac{\pi}{4} \cdot \sigma \cdot \omega^2 \cdot \mu^2 \cdot H^2 \cdot a^4$$

where H is the rms value of the magnetic field, and a is the wire radius.

Accordingly, losses due to the magnetic field generated by the current in the conductor are :

$$W = \frac{\omega^2 \cdot \mu^2}{16 \cdot \pi^2 \cdot R_s} \cdot \left(\frac{a}{b}\right)^2 \cdot I^2$$

where R_s is the resistance (p.u. length) of the bundle and b its mean radius.

Losses within an “equivalent” sheath (i.e. with same resistance and mean radius) may be expressed [1,23] :

$$W_s = \frac{\omega^2 \cdot \mu^2}{48 \cdot \pi^2 \cdot R_s} \cdot \left(\frac{t}{b}\right)^2 \cdot I^2$$

where t is the sheath thickness.

With the relationship :

$$2\pi \cdot b \cdot t = n \cdot \pi \cdot a^2$$

Hence :

$$\frac{W}{W_s} = \frac{12}{\pi^2} \cdot \frac{1}{\left(\frac{n \cdot a}{\pi \cdot b}\right)^2}$$

The order of magnitude is the same : close to 1.2 when wires are touching.

For a flat formation, losses in the screen of the middle cable due to currents in the adjacent conductors may be expressed, when considering a bundle of wires, as :

$$W \approx R_s \cdot m^2 \cdot \left(\frac{a}{b}\right)^2 \cdot \frac{5}{2} \cdot \left(\frac{b}{S}\right)^2 \cdot I^2$$

where :

$$m = \frac{\omega \mu}{4\pi \cdot R_s}$$

For a sheath, they are approximately [1] :

$$W_s \approx R_s \cdot m^2 \cdot \frac{6}{1+m^2} \cdot \left(\frac{b}{S}\right)^2 \cdot I^2$$

The ratio is :

$$\frac{W}{W_s} \approx \left(\frac{a}{b}\right)^2$$

which is usually very small.

As a conclusion :

Losses due to the internal current are similar for a bundle of wires and a sheath (with same resistance and same mean radius).

Losses due to proximity effects are very large for usual spacing, compared to losses due to the internal current for a sheath whereas for a bundle of wire they have the same order of magnitude.

18.2 Losses due to longitudinal currents.

18.2.1 General

The current distribution in the bundle of wires is the same as within the “equivalent” sheath if wires are supposed straight. This is confirmed by calculations using circuit theory and finite element method; It is worth noting that the eddy-currents in the core turn into a reduction of the losses in the bundle (as for a sheath) which is not taken into account by the IEC formula for eddy-currents in cable sheaths.

Considering an actual bundle of wound wires, as stated above, the situation is basically the same, considering an “apparent” conductivity of the wires, taking into account the inter-wire medium conductivity. If the wires are insulated – inter-wire conductivity assumed to be zero - no circulating currents are generated.

18.2.2 Calculations using circuit theory

A bundle of screen wires is considered, located in an area where exists the magnetic field generated by an influencing current.

The wires are bonded at both ends, but, at least at one end, there is no connexion to ground. That means the total current in the bundle is zero.

When considering any 2 wires, they make a loop, and the flux of the magnetic field on this loop generates an electro-motive force which produces a circulating current.

To derive the losses in the bundle, the distribution of the currents in the wires has to be worked out.

This is achieved using circuit theory, assuming that wires are straight and that the inter-wire medium is non-conductive.

Currents in a bundle of screen straight wires

According to circuit theory, the longitudinal voltage drop in any wire may be expressed as :

$$\Delta V_i = \sum_1^n Z_{ij} \cdot i_j + j\omega \cdot A_i$$

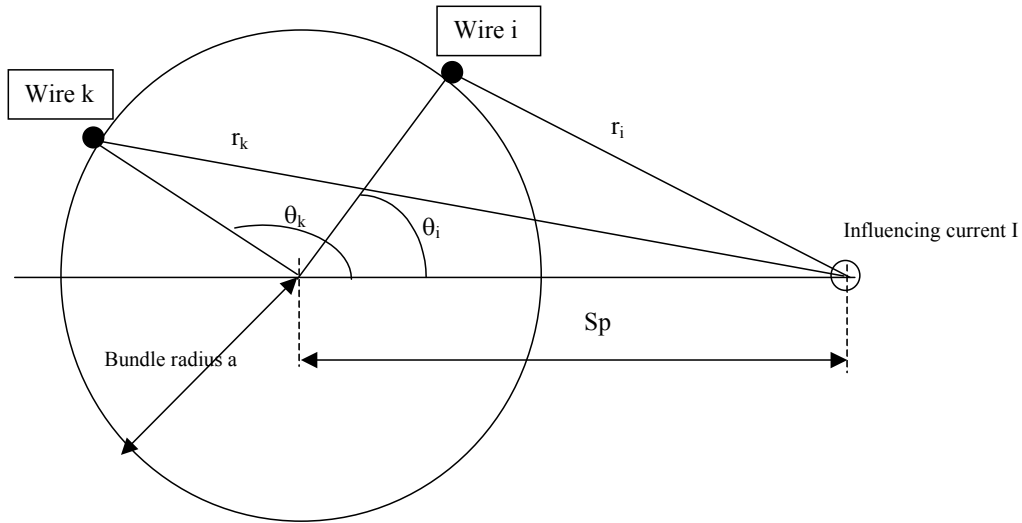


Figure 32 : Bundle of wires influenced by parallel current.

With :

$$A_i = \frac{\mu}{2\pi} . I . Ln[r_i]$$

$$r_i^2 = a^2 + S_p^2 - 2.a.S_p . \cos(\theta_i)$$

$$Z_{ii} = R + j \frac{\omega\mu}{2\pi} . Ln(0.778.r)$$

$$Z_{ij} = j \frac{\omega\mu}{2\pi} . Ln[d_{ij}]$$

$$d_{ij}^2 = 2.a^2 . (1 - \cos(\theta_i - \theta_j))$$

$$R = \frac{1}{\sigma . \pi . r^2}$$

r being the wires' radius.

Where the wires are bonded at both ends, the voltage drop is the same for every wire.
If there is no connexion to ground at one end, the total current in the bundle must be zero.

$$\sum i_i = 0$$

The distribution of the currents may be computed from this set of (n+1) equations ; unknown variables being the n currents and the voltage drop.

Note : If we consider loops as illustrated below, neglecting mutual influences between loops, the currents derived are only slightly different.

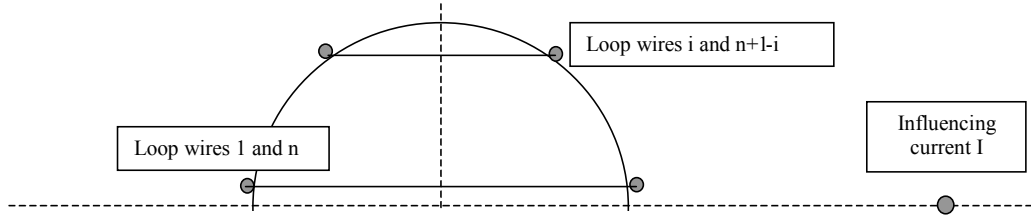


Figure 33 : Loops in a bundle of wires influenced by parallel current

Model for current generation.

The voltage drop may be derived as :

$$\Delta V = \frac{1}{n} j\omega \cdot \sum_1^n A_i \approx \frac{1}{2\pi} \cdot j\omega \frac{\mu I}{2\pi} \int_0^{2\pi} \frac{1}{2} \ln(Sp^2 + a^2 - 2 * Sp * a * \cos(\theta)) d\theta = j\omega \frac{\mu I}{2\pi} \ln(Sp) = j\omega \cdot \bar{A}$$

In practical situations, where the induced currents in the wires are small and therefore the mutual influences between wires may be neglected :

$$R.i_i \approx -j\omega \cdot (A_i - \bar{A})$$

Each wire forms a loop with a fictitious return conductor located at the axis of the bundle.

Let us consider 2 adjacent wires :

The current in these wires may be expressed as the resultant of a mean current i and a go and return current j linked to the flux of the external magnetic field on the loop formed by the 2 wires.

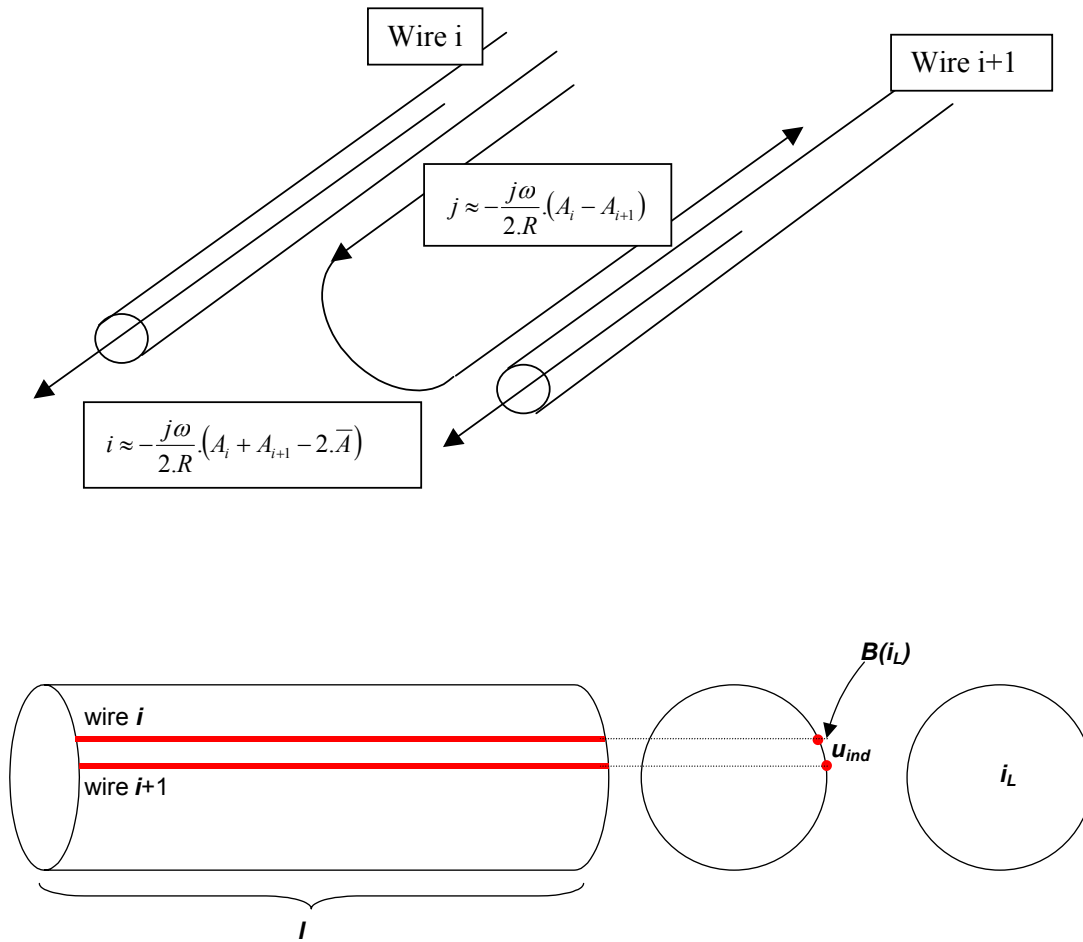


Figure 34 : 2 wires influenced by parallel current.

18.3 Losses in the foil and screening effect of the foil.

Losses in the foil may be calculated using standard IEC formulae for sheath losses.
The correction to take into account eddy currents in the core turns into a significant reduction.

The magnetic field from outside the cable is reduced, under the foil, by eddy currents in the foil.
The screening effect of the foil may be dealt with, introducing a corrective reduction factor, which, according to Heyda [25] is expressed as :

$$F = \frac{1}{\sqrt{1 + m^2}} \quad \text{with} \quad m = \frac{\omega \mu}{4\pi.R}$$

where R is the resistance per unit length of the foil.

18.4 Calculation results.

Some calculation results are displayed in the following table 2.

A representative cable (2500 mm² Cu – 420 kV) with a bundle of 180 copper wires (1.8 mm in diameter) and an aluminium laminate foil (0.5 mm thick) was considered.

Calculations for a flat formation (spacing 200 mm) were carried out, assuming circulation currents in the bundle (C and D) or not (A and B), and including the laminate foil (A and C) or not (B and D).

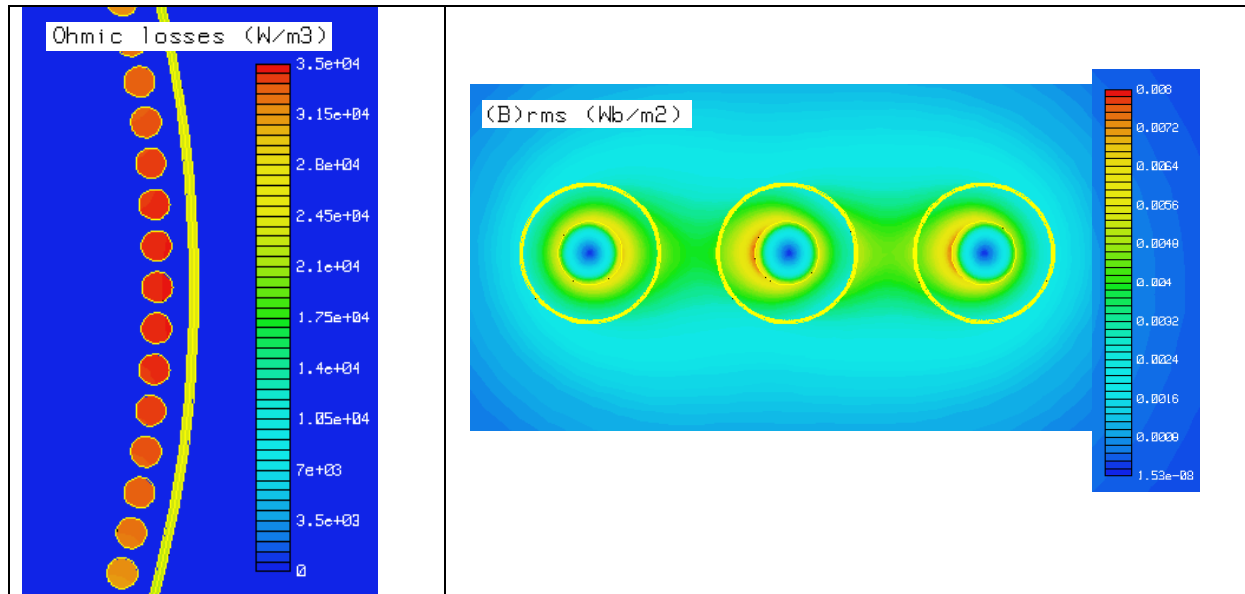


Figure 35 : 2D Finite Element Calculation output

1 2 3 $I_1=1000A \sqrt{2} \sin(\omega t)$, $I_2=1000A \sqrt{2} \sin(\omega t-120^\circ)$, $I_3=1000A \sqrt{2} \sin(\omega t-240^\circ)$, $\omega=2\pi 50\text{Hz}$, phase distance 200 mm,									
	2D Finite Element Method				Analytical calculations				
case	A	B	C	D	A	B	C	D	
conductor 1	10.79	10.83	10.67	10.71					
screen wires 1	0.0044	0.0045	1.70	2.04					
screen wires 2	0.0055	0.0056	7.63	9.17	0.056	0.0060	11.55 / 8.51*	12.25 / 9.05*	
screen wires 3	0.0045	0.0045	2.58	3.00					
Al laminate 1	1.04	X	0.61	X					
Al laminate 2	4.46	X	2.72	X	6.10 / 4.60*	X	/	X	
Al laminate 3	1.31	X	0.92	X					
sum	40.16	33.47	48.81	47.09					
* standard IEC formula for sheaths / corrected									

Table 2 : Losses (W/m) in a typical configuration.

18.5 Stranding effect.

To cope with stranding of the wires, correcting factors may be introduced, but as, usually, the lay length is long, compared to the bundle radius, this does not lead to significant change. [23]

18.6 Inter-wire medium and counter-helix.

Semi-conducting tapes may be associated to the bundle of wires and wires may be touching : there are possible paths for the current to flow from one wire to another. This issue may be dealt with, considering an “apparent” longitudinal conductivity of the wires, as suggested above.

As a matter of fact, there is no use to proceed that way for typical materials and designs : calculations in **appendix 3** show that the eddy currents that may be generated are negligible.

In some designs, a copper tape is laid as a counter-helix on the wires bundle. This turns into building up meshes with the screen wires and the counter helix where circulating currents may be induced. From the study detailed in appendix 3, corresponding losses are negligible.

19 RECOMMENDATIONS.

The Working Group recommends that eddy current losses in the bundle of wires, as component of a composite screen, be considered negligible, when single point bonding or cross-bonding is applied. Losses to be taken into account are only losses in the sheath, and the tapes if any.

REFERENCES

1 - IEC 60 287-1-1 standard [1]

Electric cables – Calculation of the current rating – Part 1 : current rating equations (100 % load factor) and calculation of losses – Section 1 : General
1994

2 – Eddy-current losses in cylindrical conductors, with special applications to the alternating current resistances of short coils.

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APPENDIX 1 : FROM MAXWELL'S EQUATIONS TO EDDY CURRENTS IN A WIRE.



Starting from Maxwell's equations, the general expression of the electromagnetic field in a wire, carrying some current, located in a place where exists a constant magnetic field is established.

1 MAXWELL'S EQUATIONS.

Let us consider a wire placed in a magnetic field (the peak value being noted as H).

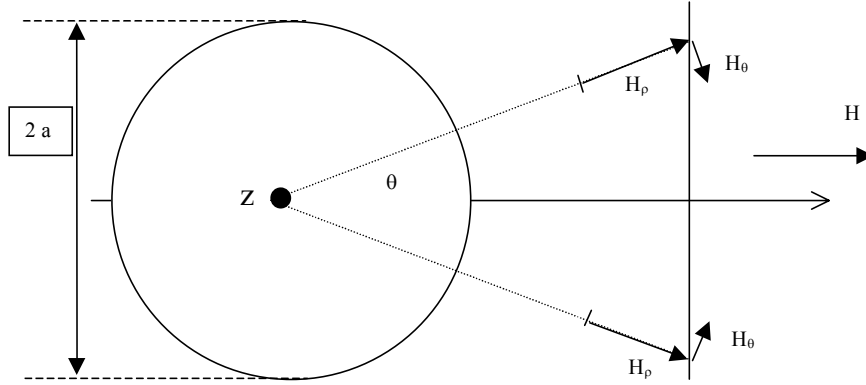


Figure A 1 – 1 : Magnetic field acting on a wire

From Maxwell's equations, the general expression for the longitudinal electric field and the radial and transversal magnetic fields may be found, assuming no variation with respect to z coordinate.

For sinusoidal forcing conditions :

$$\frac{1}{\rho} \frac{\delta}{\delta \theta} E_z = -j\omega\mu \cdot H_\rho \quad (1)$$

$$-\frac{\delta}{\delta \rho} E_z = -j\omega\mu \cdot H_\theta \quad (2)$$

$$\frac{1}{\rho} \frac{\delta}{\delta \rho} (\rho H_\theta) - \frac{1}{\rho} \frac{\delta}{\delta \theta} H_\rho = (\sigma + j\omega\epsilon) E_z \quad (3)$$

Expressing in (3), H_ρ and H_θ from (1) and (2) :

$$\frac{1}{\rho} \frac{\delta}{\delta \rho} \left(\rho \frac{\delta}{\delta \rho} E_z \right) + \frac{1}{\rho^2} \frac{\delta^2}{\delta \theta^2} E_z = j\omega\mu (\sigma + j\omega\epsilon) E_z$$

Standard solving method is the separation of variables method.

$$U(\rho, \theta) = R(\rho) * \Theta(\theta)$$

It follows :

$$\frac{1}{\Theta} \frac{d^2 \Theta}{d\theta^2} = -n^2$$

$$\frac{1}{R} \cdot \left[\frac{d^2 R}{d\rho^2} + \frac{1}{\rho} \frac{dR}{d\rho} \right] + \left(m^2 - \frac{n^2}{\rho^2} \right) = 0$$

with :

$$m^2 = -j\omega\mu \cdot (\sigma + j\omega\varepsilon)$$

Taking into account the symmetry of the problem, n is an integer and the general solution of the first equation is

$$A_n \cdot \sin(n\theta) + B_n \cdot \cos(n\theta)$$

As regards the second equation :

For $m \neq 0$:

$$\frac{1}{R} \cdot \left[\frac{d^2 R}{d\tau^2} + \frac{1}{\tau} \frac{dR}{d\tau} \right] + \left(1 - \frac{n^2}{\tau^2} \right) = 0$$

with : $\tau = m\rho$

This is a Bessel's equation ; the general solution is :

$$R(\rho) = [C_{n1} \cdot J_n(m\rho) + C_{n2} \cdot Y_n(m\rho)]$$

where J_n is first type Bessel's function of rank n and Y_n is second type Bessel's function of rank n.

For $m=0$:

$$\text{if } n \neq 0 : \quad R(r) = \left[C_{n1} \cdot \rho^n + C_{n2} \cdot \frac{1}{\rho^n} \right]$$

$$\text{if } n=0 : \quad R(\rho) = C_{01} \cdot \ln(\rho) + C_{02}$$

2 FIELDS EXPRESSIONS.

Inside the wire.

Since the field has to be finite on wire axis, Bessel's functions of second kind have to be excluded and only Bessel's functions of first kind J_n remain in the general expressions.

$$E_z = A_0 \cdot J_0(m\rho) + \sum_1^{\infty} A_n \cdot J_n(m\rho) \cdot \sin(n\theta + \psi_n)$$

$$H_\rho = -\frac{1}{j\omega\mu} \cdot \sum_1^{\infty} \frac{n}{\rho} \cdot A_n \cdot J_n(m\rho) \cdot \cos(n\theta + \psi_n)$$

$$H_\theta = \frac{m}{j\omega\mu} \cdot \left[A_0 \cdot J'_0(m\rho) + \sum_1^{\infty} A_n \cdot J'_n(m\rho) \cdot \sin(n\theta + \psi_n) \right]$$

Where :

$$m = j^{3/2} \cdot \sqrt{\omega\mu \cdot (\sigma + j\omega\epsilon)}$$

In the free space.

Due to the symmetry, only the even circular function has to be kept in the expression of H_ρ

$$E_z = [C_{01} \cdot \ln(\rho) + C_{02}] + \sum_1^{\infty} [C_{n1} \cdot \rho^n + C_{n2} \cdot \rho^{-n}] \sin(n\theta)$$

$$H_\rho = -\frac{1}{j\omega\mu} \cdot \sum_1^{\infty} n \cdot [C_{n1} \cdot \rho^{n-1} + C_{n2} \cdot \rho^{-n-1}] \cos(n\theta)$$

$$H_\theta = \frac{1}{j\omega\mu} \cdot \left[\frac{C_{01}}{\rho} + \sum_1^{\infty} n \cdot [C_{n1} \cdot \rho^{n-1} - C_{n2} \cdot \rho^{-n-1}] \sin(n\theta) \right]$$

3 BOUNDARY CONDITIONS.

The constants A_n , C_{ni} and ψ_n are determined by expressing fields continuity at the boundary of the wire.

For large distances from the cylinder, the magnetic field is H. Therefore :

$$C_{n1} = 0 \quad \text{for } n > 1$$

$$C_{11} = j\omega\mu \cdot H$$

In the close vicinity of the cylinder, the magnetic field H is disturbed by the magnetic field generated by current I in the cylinder :

$$C_{01} = j\omega\mu \cdot \frac{I}{2\pi}$$

Expressing continuity of the magnetic field on the cylinder surface, for every θ and for $\rho=a$, we get

for n = 0

$$A_0 = \frac{I}{2\pi.a} \cdot \frac{j\omega\mu}{m} \frac{1}{J'_0(m.a)}$$

for n = 1

$$H \cdot \left[1 + \frac{C}{a^2}\right] \cdot \cos(\theta) = \frac{1}{j\omega\mu} \cdot \left[\frac{1}{a} \cdot A_1 \cdot J_1(ma)\right] \cdot \cos(\theta + \psi_1)$$

$$H \cdot \left[1 - \frac{C}{a^2}\right] \cdot \sin(\theta) = \frac{m}{j\omega\mu} \cdot [A_1 \cdot J'_1(ma)] \sin(\theta + \psi_1)$$

where :

$$C = \frac{C_{12}}{j\omega\mu.H}$$

It follows :

$$\psi_1 = 0$$

$$A_1 = 2 \cdot \frac{j\omega\mu.a}{J_1(ma) + m.a.J'_1(ma)} \cdot H = 2 \cdot \frac{j\omega\mu}{m.J_0(ma)} \cdot H$$

$$2.C.H = \frac{a.A_1}{j\omega\mu} \cdot [J_1(ma) - ma.J'_1(ma)] = -\frac{m.a^2.A_1}{j\omega\mu} \cdot J_2(ma)$$

$$C = -a^2 \cdot \frac{J_2(ma)}{J_0(ma)}$$

for n > 1

$$\psi_n = 0$$

$$A_n \cdot J_n(ma) = C_{n2} \cdot a^{-n}$$

$$m.A_n \cdot J'_n(ma) = -C_{n2} \cdot a^{-n-1}$$

$$A_n = 0$$

$$C_{n2} = 0$$

resulting from :

$$A_n \cdot [J_n(ma) + m.a.J'_n(ma)] = 0$$

It is worth recognizing that the vector potential in the free space from which H may be derived is :

$$A = -\mu \frac{I}{2\pi} \ln\left(\frac{\rho}{S}\right) - \mu H \cdot \left[\rho - \frac{a^2}{\rho} \cdot \frac{J_2(ma)}{J_0(ma)} \right] \cdot \sin(n\theta)$$

4 FIELDS AND CURRENT DENSITY WITHIN THE CYLINDER.

From calculations above :

$$E_z = \frac{I}{2\pi \cdot a} \cdot \frac{j\omega\mu}{m} \frac{J_0(m\rho)}{J'_0(ma)} + 2 \cdot H \cdot \frac{j\omega\mu}{m} \cdot \frac{J_1(m\rho)}{J_0(ma)} \cdot \sin(\theta)$$

$$H_\theta = \frac{I}{2\pi \cdot a} \cdot \frac{J'_0(m\rho)}{J'_0(ma)} + 2 \cdot H \cdot \frac{J'_1(m\rho)}{J_0(ma)} \cdot \sin(\theta)$$

As :

$$m = j^{3/2} \cdot \sqrt{\omega\mu(\sigma + j\omega\epsilon)} \approx j^{3/2} \cdot \sqrt{\sigma\omega\mu} = j^{3/2} \cdot k$$

following expressions may be deduced :

$$E_z = -j^{3/2} \cdot \frac{k}{\sigma} \cdot \left[\frac{I}{2\pi \cdot a} \frac{J_0(m\rho)}{J'_0(ma)} + 2 \cdot H \cdot \frac{J_1(m\rho)}{J_0(ma)} \cdot \sin(\theta) \right]$$

$$j_z = -m \cdot \left[\frac{I}{2\pi \cdot a} \frac{J_0(m\rho)}{J'_0(ma)} + 2 \cdot H \cdot \frac{J_1(m\rho)}{J_0(ma)} \cdot \sin(\theta) \right]$$

$$j_z = \frac{\sqrt{\sigma\omega\mu}}{j^{1/2}} \cdot \left[\frac{I}{2\pi \cdot a} \frac{J_0(m\rho)}{J'_0(ma)} + 2 \cdot H \cdot \frac{J_1(m\rho)}{J_0(ma)} \cdot \sin(\theta) \right]$$

APPENDIX 2 : SUGIYAMA'S CALCULATION OF SKIN EFFECT IN SEGMENTAL CORES

1 GENERAL

This appendix gives some details on K.Sugiyama's calculations

2 EQUIVALENT CONDUCTANCE.

Longitudinal currents may be considered as the resultant of currents flowing within the wires and “transversal” currents passing from a wire to another.

Let us consider 2 wires in the same layer.

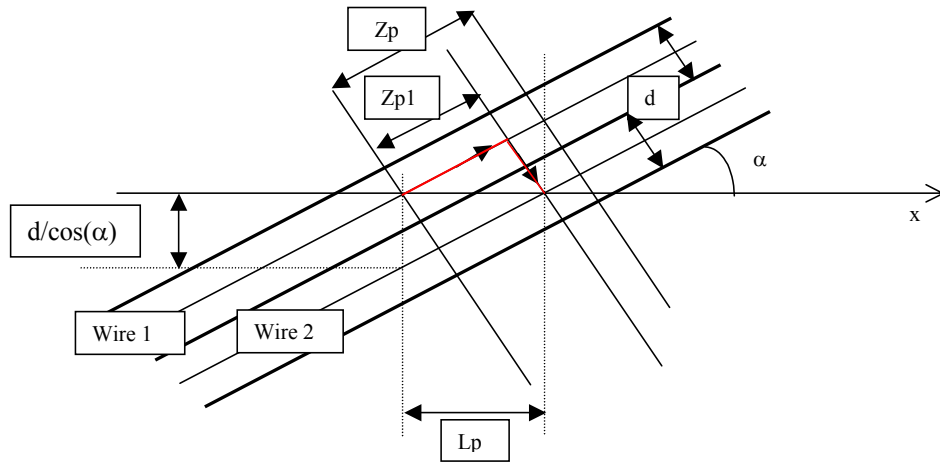


Figure A2-1 : 2 adjacent wires in the same layer

The apparent resistance between 2 touching wires axes is :

$$\Delta R = \frac{L_p}{\sigma_1 \cdot \left(\frac{\Delta s}{\cos \alpha} \right)}$$

where σ_1 is the apparent conductivity and Δs is the wire cross-section.

Considering the current flowing path, ΔR may also be expressed as :

$$\Delta R = R_0 \cdot Z_{p1} + \frac{1}{Y_\theta \cdot Z_p}$$

where R_0 is the resistance of strands (per unit length) and Y_θ is the conductance in the circumferential direction (p.u.)

Recognizing that :

$$\cos \alpha = \frac{L_p}{Z_p} = \frac{Z_{p1}}{L_p}$$

it follows :

$$\Delta R = \frac{1 + R_0 \cdot Y_\theta \cdot L_p^2}{Y_\theta \cdot L_p} \cdot \cos \alpha$$

Identifying with the first expression for ΔR , and introducing the d.c. resistance, it comes :

$$\sigma_1 = \sigma_0 \cdot \left[\frac{R_0 \cdot Y_\theta \cdot L_p^2}{1 + R_0 \cdot Y_\theta \cdot L_p^2} \right]$$

Currents passing through adjacent strands in different layers may be taken into account, using the same approach.

Y_r being the p.u. conductance in radial direction, the final result is :

$$\sigma_1 = \sigma_0 \cdot \left[\frac{R_0 \cdot L_p^2 \cdot (Y_\theta + Y_r)}{1 + R_0 \cdot L_p^2 \cdot (Y_\theta + Y_r)} \right]$$

3 SKIN EFFECT IN THE STRANDS OF A SEGMENT ALONE.

The current distribution is derived from Maxwell's equations, linking the current density and the magnetic field.

In a plain conductor, the tangential magnetic field at a distance ρ from the segment centre is assumed to be :

$$H_i = \frac{1}{2\pi \cdot \rho} \cdot \int_0^{2\pi} \int_0^\rho j_z(r) \cdot r \cdot dr \cdot d\theta$$

where $j_z(r)$ is the current density, at a distance r from the segment centre.

From Maxwell's equations, we get :

$$\frac{1}{\sigma_0} \cdot \frac{\partial j_z}{\partial \rho} = j\omega\mu \cdot H_i$$

From these 2 relationships, the equation governing the current distribution is deduced as :

$$\frac{\partial^2 j_z}{\partial \rho^2} + \frac{1}{\rho} \cdot \frac{\partial j_z}{\partial \rho} + m^2 \cdot j_z = 0 \quad \text{with} \quad m = \sqrt{-j\omega\mu\sigma_0}$$

On a lay length, the length of the strand is increased by the coefficient $1/\cos\alpha$ (α being the stranding angle). So that, j_z being the current flowing in wires :

$$\frac{1}{\sigma_0 \cdot \cos \alpha} \cdot \frac{\partial j_z}{\partial \rho} = j\omega\mu \cdot H_i$$

Assuming that the expression of the magnetic field is unchanged, it follows that the j_z current distribution is the same as in a plain conductor, with a conductivity $\sigma_0 \cdot \cos \alpha$.
It will be noted as :

$$J_{s0} = -m \cdot \frac{I_s}{2\pi \cdot a} \cdot \frac{J_0(m \cdot \rho)}{J_0(m \cdot a)} \quad \text{with} \quad m = \sqrt{-j\omega\mu\sigma_0 \cdot \cos \alpha}$$

where a is the segment radius.

4 FINAL EXPRESSION FOR SKIN EFFECCT IN SEGMENTAL CORES

4.1 Introduction.

The a.c./d.c. resistance ratio is expressed as :

$$F_s = F_{s0} + F_{e0} - F_{es} + \Delta F$$

$$F_{s0} = \left[\frac{A_s}{I_s^2} \cdot \int_{As} |J_{s0}|^2 \cdot ds - 1 \right]$$

$$F_{e0} = \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c}{I_c^2} \cdot \frac{\Delta d^2}{12} \cdot \int_{Ac} H_c^2 \cdot ds$$

$$F_{es} = N \cdot \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c}{I_c^2} \cdot \frac{\Delta d^2}{12} \cdot \int_{As} H_s^2 \cdot ds$$

$$\Delta F = \frac{\sigma_1}{\sigma_0} \cdot \frac{\omega^2 \cdot \mu^2 \cdot \sigma_0^2 \cdot A_c^2 \cdot \cos \alpha}{4\pi \cdot N \cdot I_c^2} \cdot \int_{Ac} H_c^2 \cdot ds$$

4.2 Dealing with H_c

Current density is assumed to be constant :

$$\frac{dJ_c(\rho)}{2\pi \cdot \rho \cdot d\rho} = \frac{I}{\pi \cdot (R_2^2 - R_1^2)}$$

$$J_c(\rho) = I_c \cdot \frac{\rho^2 - R_1^2}{R_2^2 - R_1^2}$$

It follows :

$$H_c(\rho) = \frac{I_c}{2\pi \cdot \rho} \cdot \frac{\rho^2 - R_1^2}{R_2^2 - R_1^2}$$

$$\int_{A_c} H_c^2 ds = \frac{I_c^2}{8\pi} \cdot G$$

with :

$$G = \frac{D_2^2 - 3 \cdot D_1^2}{D_2^2 - D_1^2} + \frac{4 \cdot D_1^4}{(D_2^2 - D_1^2)^2} \cdot \ln\left(\frac{D_2}{D_1}\right)$$

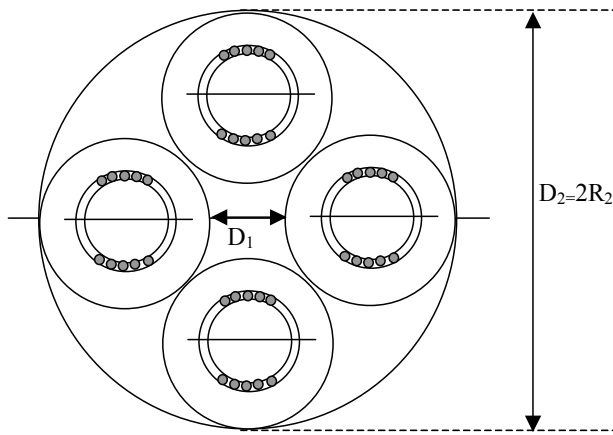


Figure A2 – 2 : Core geometry

4.3 Dealing with Hs

Considering a cylinder (radius a) carrying the current I (rms value), the following relationship is derived in appendix 1 :

$$\frac{1}{\sigma} \cdot \int_S |J|^2 dS + j\omega \cdot \mu \cdot \int_S |H|^2 dS = \frac{I^2}{\pi \cdot a^2} \cdot \frac{1}{\sigma} \cdot [1 + F_{s0} + j \cdot B_s]$$

with :

$$F_{s0} = \frac{x}{2} \cdot \frac{ber(x) \cdot bei'(x) - bei(x) \cdot ber'(x)}{ber'^2(x) + bei'^2(x)} - 1$$

$$B_s = \frac{x}{2} \cdot \frac{ber(x) \cdot ber'(x) + bei(x) \cdot bei'(x)}{ber'^2(x) + bei'^2(x)}$$

$$x = \sqrt{\sigma \omega \mu} . a$$

Therefore :

$$\int_{As} H_s^2 . ds = I^2 \frac{Bs}{\pi . x^2}$$

For typical values of x, ranging between 1 and 1.9, error is less than 3 % when considering :

$$\frac{8.B_s}{x^2} \approx 1$$

In the case we are dealing with (considering a segment) :

$$\int_{As} H_s^2 . ds \approx \left(\frac{I_c}{N} \right)^2 . \frac{1}{8\pi}$$

4.4 Consolidation

Expressing the integrals according to §§ 4.2 and 4.3. and introducing :

$$\delta_0^2 = \frac{2}{\omega . \mu . \sigma_0}$$

it follows :

$$F_{e0} = \Delta d^2 . A_c . \frac{G}{24 . \pi . \delta_0^4}$$

$$F_{es} \approx \Delta d^2 . A_c . \frac{1}{24 . \pi . \delta_0^4 . N}$$

$$\Delta F = \frac{\sigma_1}{\sigma_0} . \frac{A_c^2}{N} . \frac{G . \cos \alpha}{8 . \pi^2 . \delta_0^4}$$

4.5 Simplifications

The formula for the skin effect coefficient within segments F_{s0} may be approximated as in IEC 60287.

Simplification of F_{e0} and F_{es} is achieved when introducing the number n of wires in a segment.

$$n.\Delta d^2 = A_s = \frac{A_c}{N}$$

Simplification of ΔF is obtained :

- Introducing x as :

$$x^2 = \frac{\omega\mu}{\pi.N.R_{dc}}$$

- Using :

$$R_{dc} = \frac{1}{\sigma_0.A_c.\cos\alpha}$$

Note : assuming $\cos\alpha \approx 0.95$, this formula is the relationship used in IEC 60228 between the d.c. resistance and the nominal cross-section

- Recognising that :

$$\cos\alpha = \frac{\pi.N}{2.A_c}.\delta_0^2.x^2$$

4.6 Working formulae

The working formulae, as presented in the main part, are derived from these simplifications, together with the introduction of a correcting factor for ΔF , leading to :

$$\Delta F = \frac{\sigma_1}{\sigma_0} \cdot \frac{A_c.x^2.G}{16.\pi.\delta_0^2} \cdot \left(\frac{N-1}{N} \right)^2$$

APPENDIX 3 : LOSSES IN A BUNDLE OF SCREEN WIRES

1. TAKING INTO ACCOUNT INTER-WIRE CONDUCTIVITY

An equivalent circuit as shown below is used to describe the losses between and within two neighbouring screen wires.

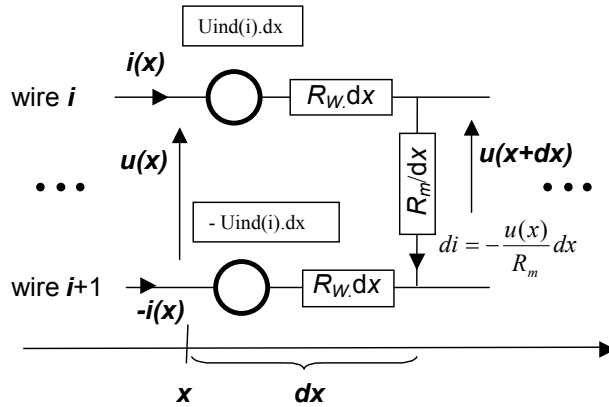


Figure A3 – 1 : Equivalent circuit for 2 wires influenced by parallel current.

In this equivalent circuit :

$$U_{ind}(i) \approx -\frac{j\omega}{2} \cdot (A_i - A_{i+1})$$

R_w is the specific resistance of the wire [Ω/m] and R_m is the specific inter wire resistance [Ωm].

From this equivalent circuit, the following equations are derived :

$$du = -\left(2R_w^* \cdot i(x) + \Delta U_{ind}\right) dx \quad (1)$$

$$di = -\frac{u(x)}{R^*} dx \quad (2)$$

$$\Delta U_{ind} \approx -j\omega \cdot (A_i - A_{i+1})$$

The combination of (1) and (2) leads to:

$$R^* \frac{d^2 i}{dx^2} - 2 \cdot R_w^* i(x) - \Delta U_{ind} = 0 \quad (3)$$

Where the wires are wound, the induced voltage varies periodically along the cable axis.

$$\Delta U_{ind} = \frac{\omega\mu}{2\pi} . I . Ln \left(\frac{\rho^2 + S^2 - 2.\rho.S.\cos(\theta + \Delta\theta)}{\rho^2 + S^2 - 2.\rho.S.\cos(\theta)} \right)$$

$$\theta = \frac{2.\pi}{L} . x$$

$\Delta\theta$ is the angular spacing between 2 adjacent wires.

ΔU_{ind} may be approximated as :

$$\Delta U_{ind} \approx U . \sin(\alpha.x) \quad \text{with} \quad U \approx \frac{\omega\mu}{2\pi} . I . \frac{2\rho}{S} . \sin(\Delta\theta) \quad \alpha = \frac{2.\pi}{L}$$

General solution is :

$$i = -\frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U}{2.R_w} . \sin(\alpha.x) - \frac{C}{2.\gamma} . ch(\gamma.x) \quad \text{with :} \quad \gamma^2 = \frac{2.R_w}{R_m}$$

Which, for a long wire, reduces to :

$$i = -\frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U}{2.R_w} . \sin(\alpha.x)$$

Losses for the 2 wires are :

$$\frac{dLos}{dx} = 2.R_w . i^2 + R_m . \left(\frac{di}{dx} \right)^2 = (2.R_w . \sin^2(\alpha.x) + \alpha^2 . R_m . \cos^2(\alpha.x)) \left(\frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U}{R_w} \right)^2$$

For a lay length :

$$Los = \frac{1}{2} . (2.R_w + \alpha^2 . R_m) \left(\frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U}{R_w} \right)^2 = R_w . \left(1 + \frac{\alpha^2}{\gamma^2} \right) \left(\frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U}{R_w} \right)^2 = \left(\frac{\gamma^2}{\alpha^2 + \gamma^2} \right) . \frac{U^2}{R_w}$$

For a lay length and per wire :

$$Los = \frac{1}{2} . \frac{\gamma^2}{\alpha^2 + \gamma^2} . \frac{U^2}{R_w}$$

Calculation of R_m

The value of R_m is determined by the electric field distribution between two neighbouring screen wires as a function of the resistivity ρ of the material between the wires.

In order to calculate R_m a finite element program was used. Selected field plots are shown in figure A3 - 2.

The results for R_m are, with ρ expressed in $\Omega.m$:

$$R_m = 0,24.\rho$$

This is consistent with analytical calculation which gives for 2 wires isolated (without foil) :

$$R_m = \frac{\rho}{\pi} \cdot \ln \left(\frac{D}{2.a} + \sqrt{\left(\frac{D}{2.a} \right)^2 - 1} \right)$$

where D is the axial spacing between wires and a is the wires' radius.

That is : $R_m = 0.264$ for $\rho = 1 \Omega.m$

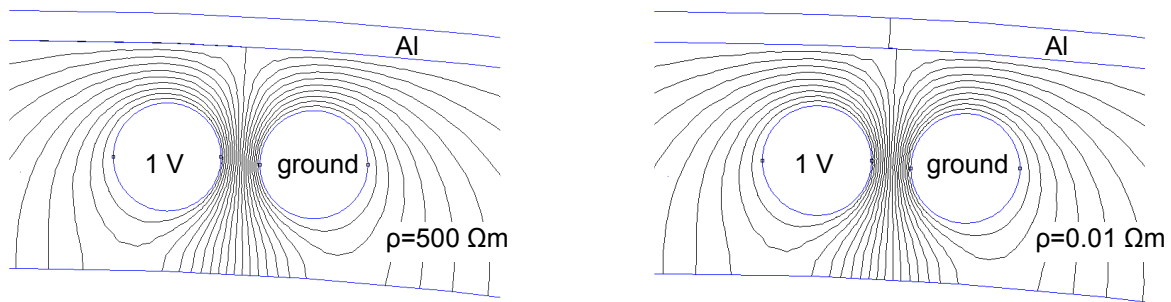


Figure A3-2 : Field plots of the electric field around the screen wires of the model cable for 2 conductivities of the material between the wires.

The following figure plots the ratio as a function of the inter-wire resistivity, for a bundle of copper wires (1.8 mm diameter - lay length : 2m)

$$ratio = \frac{1}{2} \cdot \frac{\gamma^2}{\alpha^2 + \gamma^2}$$

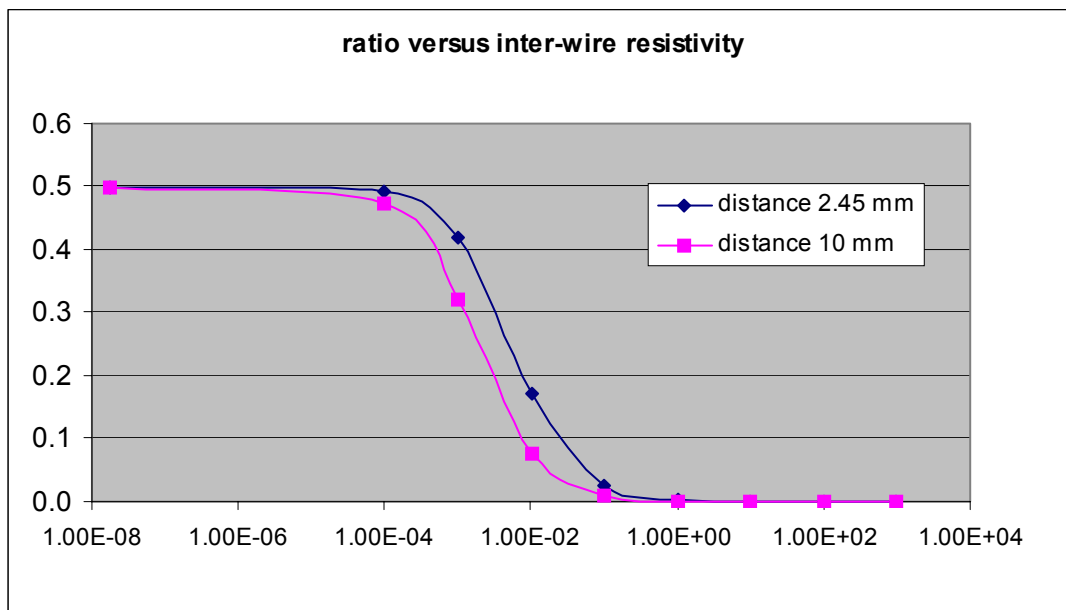


Figure A 3 – 3 : Losses Ratio as a function of inter-wire resistivity

Example

Losses in the bundle (mW/m) – Proximity current = 1 kA				
Inter-wire Resistivity (Ohm.m)	180 wires Dmb= 140.6 mm Sp=140 mm	180 wires Dmb= 140.6 mm Sp=90 mm	90 wires Dmb= 140.6 mm - Sp=90 mm	90 wires Dmb= 70.3 mm Sp=90 mm
1.72E-08	64.57	156.25	312.13	78.03
1.00E-04	63.35	153.30	300.53	76.56
1.00E-03	54.15	131.03	225.21	65.43
1.00E-02	22.07	53.41	64.23	26.67
1.00E-01	3.19	7.71	7.88	3.85
1.00E+00	0.33	0.81	0.81	0.40
1.00E+01	0.03	0.08	0.08	0.04
1.00E+02	0.00	0.01	0.01	0.00
1.00E+03	0.00	0.00	0.00	0.00
Copper wires – diameter 1.8 mm - pitch of the screen wires : 2 m Dmb is the mean diameter of the bundle – Sp is the axial spacing bundle / proximity				

Conclusion

The losses due to currents through the semiconducting material between the screen wires can be neglected.

2. LOSSES INDUCED BY THE COUNTER HELIX

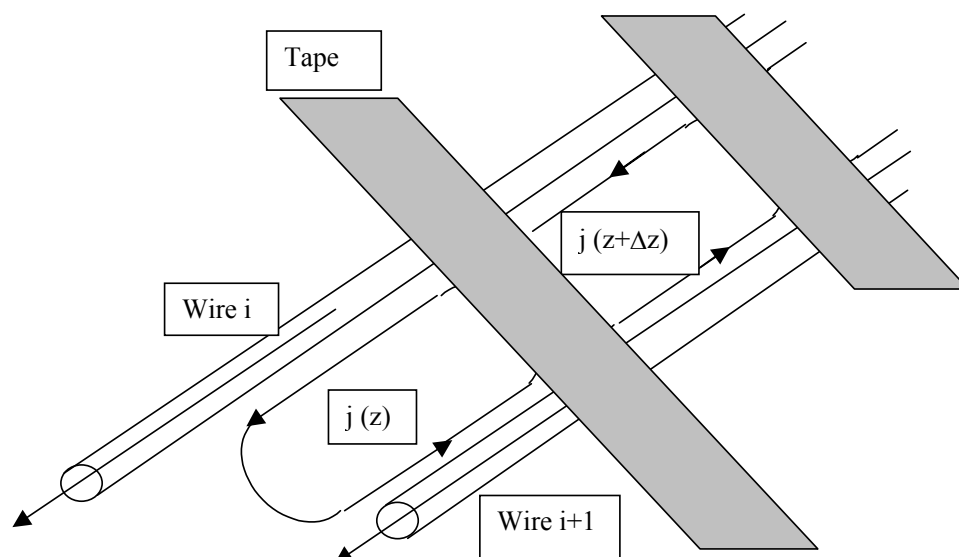


Figure A3 – 4 : Loops formed by 2 wires and the tape, with circulating currents

We will work out the order of magnitude for the losses induced by the circulating currents in the meshes built up by the screen wires and the counter helixes. These currents are a consequence of the closure of an electrical circuit around the varying magnetic induction B , created by the cables laid beside the considered cable (please refer to schematic diagram here under for understanding the following). The calculation is made for one mesh and will be derived for the whole screen.

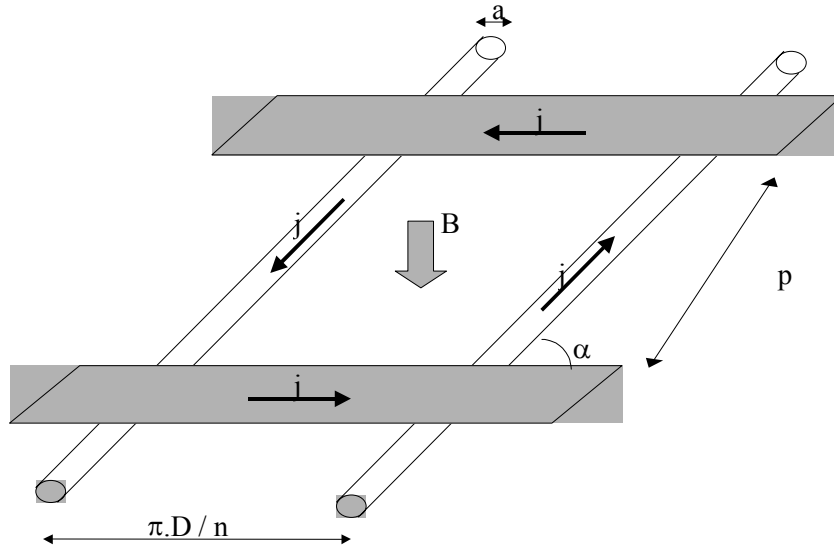


Figure A 3 – 5 : Loop formed by 2 wires and the tape

Variables used

- D screen mean diameter
- d representative distance between the inducing current and the considered mesh
- I rms value of the inducing current
- μ magnetic permeability
- f frequency
- σ copper electrical conductivity
- R electrical resistance of the “loop” around the mesh
- R_{tape} electrical resistance of the counter helix between two adjacent contact points with the wires.
- R_{wire} electrical resistance of one wire on one laying pitch of the counter helix
- j induced current around the mesh
- S area of the mesh
- Φ magnetic flux in the mesh
- n wire number in the screen
- a wire diameter
- α angle between wires and counter helix
- p counter helix lay length
- b counter helix tape width
- s counter helix tape thickness

Losses induced by current in the central conductor

The magnetic induction due to this current is parallel to the mesh surface and so no e.m.f. will be generated in the loop. As a consequence: no “counter helix losses” induced by the cable in its proper screen.

Losses induced by currents in the adjacent conductors

In this case we will simplify the study considering just one resulting current in the vicinity of the cable and generating the magnetic induction **B**. The screening effect of the metallic sheath of the adjacent cores is not taken in account. We will calculate the induced current in a mesh perpendicular to the magnetic induction and consider the same value for all meshes around the cable. In this way we will get an upper limit for the losses amount.

E.m.f. in the loop

The e.m.f. **E** in the loop around the mesh can be expressed as :

$$E = \omega \cdot \Phi = 2 \cdot \pi \cdot f \cdot \Phi$$

$$\Phi = B \cdot S$$

$$\text{with } S = \frac{\pi \cdot D}{n} \cdot \frac{p}{\sin \alpha}$$

A further simplifying assumption will be made: the external inducing current is the current in one conductor only (no vectorial composition of the currents in two adjacent conductors) and this external cable is in close contact (trefoil arrangement) with the cable under study (maximum magnetic interference). Again we will get an upper limit for these losses.

$$B = \frac{\mu \cdot I}{2 \cdot \pi \cdot d}$$

With the above mentioned assumption : **d = D**

As a result:

$$E = \frac{2 \cdot \pi \cdot f \cdot \mu \cdot I \cdot \pi \cdot D}{2 \cdot \pi \cdot d \cdot n} \cdot \frac{p}{\sin \alpha}$$

$$E = \frac{\pi \cdot \mu \cdot f \cdot I \cdot p}{n \cdot \sin \alpha}$$

Electrical resistance of the loop

The resistance of the loop can be expressed as $R = 2 R_{\text{tape}} + 2 R_{\text{wire}}$. We assume no skin effect in the mesh and no contact resistance between wire and tape, as a consequence the actual resistance is higher than this one (we stay on the safe side for induced current calculation).

$$R_{\text{tape}} = \frac{1}{\sigma} \cdot \frac{1}{b \cdot s} \cdot \frac{\pi \cdot D}{n} \quad R_{\text{wire}} = \frac{1}{\sigma} \cdot \frac{4}{\pi \cdot a^2} \cdot \frac{p}{\sin \alpha}$$

As a consequence :

$$R = \frac{2}{\sigma} \cdot \left(\frac{\pi \cdot D}{b \cdot s \cdot n} + \frac{4 \cdot p}{\pi \cdot a^2 \cdot \sin \alpha} \right)$$

Losses in one loop

$$p'' = \frac{E^2}{R} = \frac{\sigma}{2} \left(\frac{\pi \cdot \mu \cdot f \cdot I \cdot p}{n \cdot \sin \alpha} \right)^2 \left/ \left(\frac{\pi \cdot D}{b \cdot s \cdot n} + \frac{4 \cdot p}{\pi \cdot a^2 \cdot \sin \alpha} \right) \right.$$

Losses per meter of cable

The losses per meter of cable are (n wires around the core) : $P'' = \frac{n \cdot p''}{p}$ that is to say:

$$P'' = \frac{\sigma}{2} \cdot \left(\frac{\pi \cdot \mu \cdot f \cdot I}{\sin \alpha} \right)^2 \left/ \left(\frac{\pi \cdot D}{b \cdot s \cdot p} + \frac{4 \cdot n}{\pi \cdot a^2 \cdot \sin \alpha} \right) \right.$$

Numerical application

- D = 130 mm
- I = 1000 A
- $\mu = 4 \cdot \pi \cdot 10^{-7}$ H/m
- f = 50 Hz
- $\sigma = 58 \cdot 10^6$ S/m
- n = 180
- a = 1.8 mm
- $\alpha = 45^\circ$
- p = 80 mm
- b = 20 mm
- s = 0.1 mm

$$P'' = 22 \text{ mW/m} \quad (I=1000 \text{ A})$$

Conclusion

The order of magnitude of the losses induced by the counter helix is some tens of mW/m. They can be neglected in the general calculation of the wire screen losses.