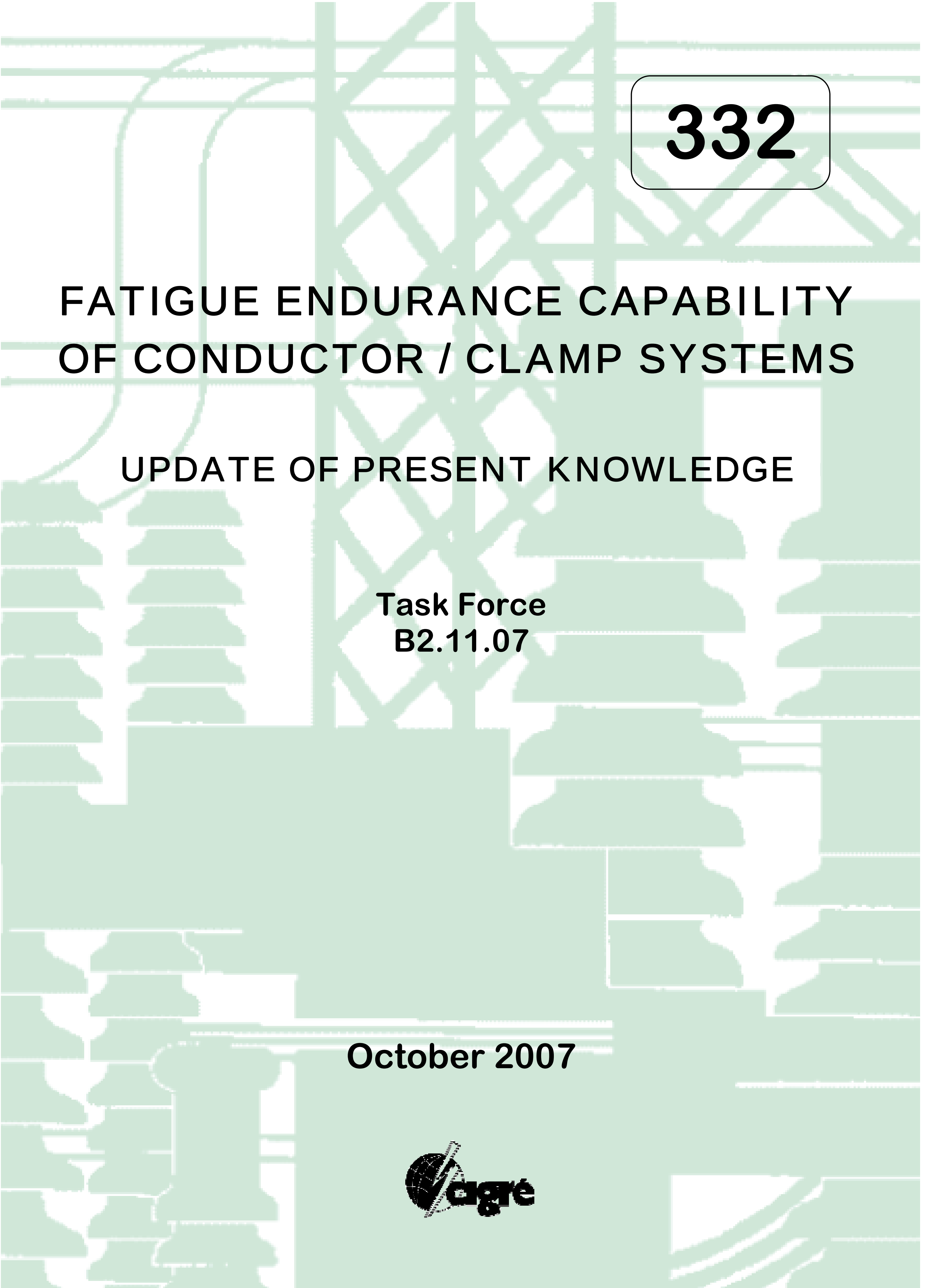

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# FATIGUE ENDURANCE CAPABILITY OF CONDUCTOR / CLAMP SYSTEMS

## UPDATE OF PRESENT KNOWLEDGE

**Task Force**
**B2.11.07**

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# FATIGUE ENDURANCE CAPABILITY OF CONDUCTOR/CLAMP SYSTEMS – UPDATE OF PRESENT KNOWLEDGE

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## B2.11.07

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# LIST OF SYMBOLS

$A_w$ : Area of wire cross-section

$B_{eq}$ : Equivalent local bending stiffness of the strand

$B_{max}$ : Maximum strand bending stiffness

$B_{min}$ : Minimum strand bending stiffness

$B_s$ : Free-field strand bending stiffness

$D$: Total fatigue damage

$d_a$ : Outer-layer wire diameter

$d_w$ : Wire diameter

$E_a$ : Young´s modulus of a outer-layer wire

$E_w$ : Young´s modulus of a wire

$EI_{max}$ : Maximum bending stiffness of the conductor

$EI_{min}$ : Minimum bending stiffness of the conductor

$K$: Slippage coefficient related to the strand bending stiffness

$n_\ell$ : Number of wires in a given layer of the conductor

$n_i$ : Number of cycles at bending stress $\sigma_i$

$N_i$ : Fatigue life at bending stress $\sigma_i$

$r_\ell$ : Layer diameter

$T$ : Conductor tensile load

$T_i$ : Wire tensile load

$x_b$ : Distance (89 mm) between the bending amplitude measurement and the last point of conductor/clamp contact

$y_{max}$ : Free-loop amplitude of vibration

$Y_b$ : Peak-to-peak amplitude of conductor motion relative to clamp at 89 mm from the last point of conductor/clamp contact (so-called bending amplitude)

$\beta$: Vibration angle

$\beta_\ell$ : Lay angle

$\kappa$: Strand curvature

$\sigma_i$ : Calculated bending stress

$\sigma_{PS}$ : Idealized bending stress in an outer layer wire (as calculated by the Poffenberger-Swart formula)

$\sigma_T$ Tensile stress in a wire

# 1. Executive Summary

## 1.1 Introduction

Conductor failure can result from many causes. One of these is the damage caused by vibration of the conductor resulting in fretting fatigue which has long been recognized as the cause of conductor strand failures. The issue faced by utility engineers at present is to understand this phenomenon to enable a better control of strand failures for a given conductor/clamp system to be realised. Although a thorough review of the issue was presented in the Electrical Power Research Instituted (EPRI) *Transmission Line Reference Book – Wind induced conductor motion* published in 1979, work has progressed and this Technical Brochure presents an update of this knowledge.

This Brochure commences with a state of the art review on the effect of fretting fatigue on the endurance capability of conductors and presents methods or means for those interested in contributing to the definition of a more precise model representing fretting fatigue of conductor strands.

The Brochure also proposes the use of a resonance type bench to better simulate this complex phenomenon.

In addition the Brochure covers the mechanism of *inner conductor mechanics* and its relation to the fretting fatigue phenomenon as a whole.

The Brochure presents the experience gained with vibration measurements to illustrate the practical situation which can be used to determine performance of existing lines. It provides practical methods to control or solve conductor fatigue problems.

## 1.2 Fretting Behaviour in Stranded Conductors

Fretting fatigue is a contact damage phenomenon that takes place when a fixed structural member is submitted to a surface micro slip associated with small scale oscillatory motion in the order of 20-100 microns.

Fretting is complex since it is influenced by a number of factors including the normal contact load, the amplitude of relative slip, friction coefficient, surface conditions, contact materials and environment.

The mechanism of fretting damage with regard to aluminium takes place in stages. Initially the surface oxide film is removed resulting in bare surfaces rubbing against each other. This forms dust between surfaces as well as plastic deformation, change in surface chemistry and formation of oxide aluminium will increase with more fretting cycles.

Started by the cracking of the thin brittle aluminium oxide layer, surface micro cracks are unavoidable. If the slip amplitudes are large enough, small cracks will be worn and it results in a fretting wear process. But if the micro cracks can propagate in the bulk material, then we get a fretting fatigue process, the cracks will grow until the complete fatigue failure of the material.

From examination of lines in service, it was found that fatigue cracks always originated through fretting marks. It is therefore considered obvious that the fretting mechanism reduces the fatigue strength of the strand relative to the unfretted strength.

In the report of Cigre WG22-04 in 1988 the basic principles of the fretting mechanism were reviewed together with its influence on the fatigue process. It was stressed that in the case of estimation of conductor short life duration, the presence of black oxide aluminium during inspection implies that there certainly has been some destruction of aluminium wires in the inner layers.

The Poffenberger – Swart formula relating bending stress and amplitude measured at a standard distance from a clamping device is commonly used as a reference for endurance test data. However, attention is brought to the research at GREMCA's laboratories which illustrated a limitation on the use of the formula at suspension clamps which indicated that wire fracture is not only induced by the alternating stress but rather by the fretting occurring at the inter-wire contact points. Observations made, clearly tended to show that part of the conductor behaves as a solid section while the rest is constituted with wires acting individually. Fatigue life prediction would become more realistic if we could link the $Y_b$ endurance limit of a clamp-conductor system to a micro-slip amplitude value which could be related to, in turn, a distinctive fretting-fatigue behaviour of the aluminium wires. This approach would use a displacement indicator which includes the effect of the tensioning load, the lay angle, and the clamp geometry instead of being only related to an idealized stress.

## 1.3 Determination of Fatigue Endurance Capability

In order to determine the fatigue characteristics of conductors, the conductor must be tested as in the past but also the conductor clamp system that is likely to be installed on the line. The type of clamp system is critical to limit the fretting damage.

In reviewing the different approaches to conduct laboratory tests simulating the actual phenomenon, it is advantageous to reproduce the phenomenon under well known parameters. The resonance test bench imposing vertical motion to the conductor with a fixed clamp position achieves this with a simpler test procedure as it avoids the difficulties associated with the interpretation of the dynamic response of a rocking clamp.

This Brochure describes a typical resonance fatigue test bench used to characterize the fatigue performance of different conductor/clamp systems and the instrumentation one needs to monitor the important test parameters. From work done by Cigre WG 22-04 it is noted that to arrive at comparable results in different laboratories an agreement on important test parameters and on an uniform method is necessary.

The fatigue life needs to be determined as a function of a measure of vibration intensity. The bending amplitude $Y_b$ (amplitude of conductor motion relative to clamp at 89mm from the last point of conductor/clamp contact), is the most widely used parameter for measuring vibration in the field and thus it is recommended to use it as well in laboratory tests. It is also advisable to measure the free loop amplitude $y_{max}$ to correlate the test results of conductors supported with clamps of different configuration and also to permit their use in establishing and endurance limit for a range of conductor sizes. However, note that conductor strand size and number of layers could lead to different fatigue curves.

The concept that there is some idealized strain or stress that can be calculated from vibration amplitude and that correlates well enough with conductor fatigue life has given the engineer a useful tool to overcome the complexity of the problem and find results that are reliable enough to be usefully applied.

## 1.4 Inner Conductor Mechanics

Due to the complex nature of the multilayer, often steel cored conductor, there is no universally accepted and applicable mechanical model to evaluate the strength and stiffness of the conductor as a mechanical element.

There has been extensive research on stranded wire ropes relating to the bending resistance (stiffness) as a function of a few parameters. This Brochure reviews these papers and apply these concepts to ACSR conductors.

It is well known that a helical strand in bending (on which a curvature is imposed) has two limiting behaviours; a) solid behaviour; b) independent wire behaviour. Within a certain set of assumptions, both cases are easily analyzed and it results in upper and lower bound values to the actual strand bending stiffness. The lay angle being small, it is often neglected. Also, wire diameter being much smaller than that of the strand, one may neglect wire bending (and twisting) stiffness for those calculations. The tensile loads in the wires cause normal loads at the crossing points of two wires in adjacent layers which, besides leading to contact stresses, determine the stick-slip behavior of the wires during cyclic bending, as this is caused by Aeolian vibrations.

At small bending amplitudes the bending stiffness can be calculated as though the wires are "welded" together and is called $EI_{max}$. At large bending amplitudes, the bending stiffness can be calculated as though the wires are totally loose and do not interact with each other; it is then called $EI_{min}$. In between these two extremes a more or less smooth transition takes place. Recent refinements of this model indicated, that $EI_{max}$ cannot be fully achieved.

At this stage, it would seem that the best available model in order to predict the static free bending stiffness of a multi-layer strand under axial tension should encompass these main hypotheses: a) Radial contact between wires; b) Contact points replaced by line contact; c) Wire torsion neglected; d) Coulomb friction assumed; e) Full pressure transmission from one layer to the next assumed; f) Tangential elastic compliance at contact points neglected; g) Incipient slip assumed to start at strand neutral axis; h) Strand centerline curvature assumed to be small enough to neglect variation of wire curvature, helix angles, strand cross section and inter layer pressures.

There is a simplified model developed in 1965 extensively used to calculate idealised conductor stresses to compare the vibration intensity of different conductors as a function of bending amplitude and to determine endurance capability with reference to accumulated stress S/N (Wohler curves).

## 1.5 Assessment of Vibration Severity on Actual Lines

Four main methods are used to assess Aeolian vibration severity : a) Computer analysis, b) Field vibration tests on outdoor spans, c) Laboratory tests, d) Measurement on actual lines. All of these are used to some extent worldwide.

Overhead conductor vibrations are caused by the alternate detachment of wind-induced vortices from the top and bottom sides of the conductor. The vortex shedding produces an alternating pressure unbalance, inducing the conductor to move up and down at right angle to the direction of airflow. The frequency of the vortex detachment, and consequently the conductor vibration frequency, depends on both the wind velocity and the overall diameter of the conductor, according to the well-known Strouhal formula. Considering that, the range of wind speed able to excite dangerous Aeolian vibrations is, in normal conditions, 1 to 7 m/s, it is possible to define the range of potentially harmful vibration frequencies of any conductor. Besides, in a conductor span, it occurs very often that two or more vibration frequencies, not too far apart, are induced simultaneously by the wind. The presence of two or more vibration frequencies determines beats, at any vibrating point of the conductor, which result in a continuous random variation of the vibration amplitude. This aspect of the conductor vibration motion is important for the correct interpretation of the data collected by a specific recorder and for the evaluation of conductor lifetime.

Measurements of Aeolian vibrations on actual lines can be made in different ways using a variety of instruments that can be classified into four groups: generic transducers, vibration detectors, optical vibration monitoring devices and specific vibration recorders (bending amplitude recorders).

Generic transducers such as accelerometers, velocity pick up, displacement transducers are connected to a ground site for data acquisition. Their use has now been limited to test stations. Vibration detectors, due to their design, are simply able to provide a quite rough relative index of vibration activity. Electro optical cameras can detect vibration from the ground, display a signal on an oscilloscope and store it in a computer system. Finally modern versions of the vibration recorders are digital recorders, microprocessor based, battery powered devices that use built in memory storage. Computer download, analysis and display are possible. They permit the direct application of the bending amplitude method proposed as a standard in 1966 by the IEEE task force on the Standardization of Conductor Vibration.

## 1.6 Evaluation of Conductor Residual Life

Prediction of the remaining life of a conductor that has been exposed to damaging fatigue stresses is of interest to guide decisions on whether repair or replacement of conductors, or upgrading vibration protection, will be necessary and, if so, when. Cumulative damage theory is used for the estimation of residual life in fatigue.

Steidel in 1960 proposed the cumulative damage theory which led to specific recommendations for its application. Laboratory fatigue tests led to the Miner's rule which has become the leading cumulative damage model.

Miner's Rule is derived from the Palmgren-Miner Law which considers that a material will fail in fatigue when the accumulated damage from all stress cycles reaches a certain value. The amount of damage each cycle causes depends on the amplitude of its stress. Under Miner's Rule, the amount of damage a cycle of stress $\sigma_i$ causes is equal to $1/N_i$, where $N_i$ is fatigue life when $\sigma_i$ is the sole fatigue-inducing stress. The Rule states that the total damage under exposure to $m$ different stress amplitudes is,

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \quad (1)$$

Where:

$n_i$ is the number of cycles at $\sigma_i$ in the exposure. Failure is expected to occur when the damage parameter $D$ exceeds 1. Equation (1) is the linear form of Miner's Rule, and is the most widely used.

Application of Miner's Rule requires two elements of information:

1. The fatigue characteristics of the conductor/clamp system where fatigue may be expected, that is, some form of the function $\sigma(N)$ or $N(\sigma)$ representing the fatigue or Wöhler curve.
2. Data on the exposure of that system to fatigue-inducing stresses in the field, that is, the distribution $n_i(\sigma_i)$ of the accumulated cycles at each of the stress levels experienced in the field. Such data are usually expressed as cycles, or megacycles, per year.

Interstrand contacts cannot be measured directly and the bending amplitude $Y_b$ is used for estimation.

Estimates of the exposure function $n_i(\sigma_i)$ can be based upon knowledge of the vibration amplitudes that have occurred up to the time the condition of a line has been brought into question. If the line under consideration has not yet been erected, the function may be based upon previous recordings from an existing similar line, or upon analytical methods such as the energy balance principle. In the latter case, reference should be made to the relative accuracy to be expected in predicted vibration levels.

Due to the complexity of the conductor system and the complex waveforms that occur in Aeolian vibration, there is an uncertainty when trying to apply cumulative damage theory to Aeolian vibration fatigue. A more thorough analysis presented in the Brochure permits to identify several other sources of uncertainty. Because of these, estimates of absolute values for the residual life of conductors in years should be viewed as indicative approximations. However, estimated residual life can serve as a parameter for expressing vibration severity and in comparing alternative damping systems. This view reflects the cumulative effect of the foregoing and therefore, until progress is made, numerical evaluation of residual life must simply be viewed with caution.

Future work encompassing notions of contact mechanics and of fracture mechanics should lead to the proposition of a model representing more closely the fretting fatigue mechanism actually causing the strand failures.

# 2. Introduction

Fatigue endurance capability of conductor/clamp systems is an important parameter taken into consideration at the design stage of a transmission line. It remains an important factor when comes the time to assess the residual service life of a particular transmission line. The fatigue behaviour of stranded conductors is highly complex. It occurs at singular points on the conductor (e.g. suspension clamps) where motion is constrained. A thorough review of this whole issue was presented by Rawlins (1979). Since publication of that review, work has progressed and this report presents an update of present knowledge.

A review of the experience gained with vibration measurements on actual lines is also made. The response of a conductor to wind excitation is far from being of constant amplitude time wise and space wise. One needs adequate instrumentation to warrant acceptable representation of the actual situation. The complex signal resulting from the measurements needs to be interpreted cautiously. It gives a portrait of a situation (vibration intensity) at a point of the conductor span. The extrapolation of the results obtained requires considerations developed in this report.

This Technical Brochure intends to present an update of actual knowledge of the fatigue endurance capability of conductor/clamp systems. The place of the mechanism of fretting fatigue is stressed in Chapter 3, an update of the laboratory determination of fatigue endurance capability is given in Chapter 4, the latest work on the inner conductor mechanics is presented in Chapter 5, the assessment of vibration severity on actual lines is given in Chapter 6 and a discussion of suitable avenues for the evaluation of conductor residual life is presented in Chapter 7.

# 3. Fretting Behaviour in Stranded Conductors

Major design aspects for high voltage transmission lines are firstly of electrical concerns, such as the power to be transmitted or the transmission losses. However, mechanical or metallurgical performance ranks high among the technical parameters in overhead transmission line design. Indeed it is important to be able to predict lifetime of conductors in order to ensure a dependable distribution of electricity at a low cost. A major cause of fatigue failures of the cables is the cyclic bending stress imposed upon conductor by Aeolian vibrations. At singular points of the conductor where motion is constrained against transverse vibration, more frequently at suspension clamps but also at spacer, damper device or hardware clamps, bending causes the strands of the conductor to slip relative to each other. The friction forces combined to the relative motion cause fretting at interstrand and clamp contacts. Once an initial crack is induced from the fretting mark surface, it may lead to the fracture of the wire and eventually to the complete break down of a conductor.

Even though the presence of fretting in conductor fatigue is a well known phenomenon, fretting fatigue is not understood enough yet to allow the prediction of the life of a conductor-clamp system by the solution of a mathematical model when knowing the mechanical and physical properties of the wires. The standard method of evaluation still remains to perform experimental tests on a case by case basis.

A state of the art on the effect of fretting fatigue on the endurance capability of conductors has recently been presented in a report prepared by GREMCA at Laval University (Dalpé et al., 2003). Considering the importance of the phenomenon for the study of fatigue endurance capability of conductor/clamp systems, the subject is presented in this Chapter. After a brief description of the mechanism of fretting, a complete review of the literature tackling its importance in cable fatigue is presented. Different hypotheses and experimental approaches that may lead to an appropriate model for the prediction of conductor fatigue life encompassing the fretting aspect of the phenomenon are also reviewed.

## 3.1 Mechanism of Fretting Fatigue

Fretting fatigue is a contact damage phenomenon that takes place when a fixed structural member is submitted to a surface microslip associated with small-scale oscillatory motion on the order of 20-100 $\mu$m. It is often responsible for unexpected fatigue failures and limits components life in aeronautical structures and common industrial machinery such as steam and gas turbines, cables, bolted plates, shaft keys and bearings.

A vast amount of research work has been published on fretting fatigue. Fretting is complex (Vincent et al., 1992, Waterhouse, 1992, Hoeppner, 1994, Mutoh, 1995) since it is influenced by a number of factors including the normal contact load, the amplitude of relative slip, friction coefficient, surface conditions, contact materials and environment. The fretting fatigue process is also recognized as the result from a competition between wear, corrosive and fatigue phenomena driven by both the microslip at the contact surface and cyclic local stresses.

The mechanism of fretting damage of aluminium material involves several stages of evolution (Hoepner, 1994). At the beginning a surface oxide film is removed and once done bare surfaces in contact start to rub against each other. At the same time the surfaces tend also to adhere to each other forming weld junctions which will be broken by the relative movement. This process forms the accumulation of wear dust between the surfaces. Surface plastic deformation, change in surface chemistry and formation of oxide aluminium and wear product will increase with more fretting cycles.

The thin and brittle layer of aluminium oxide consists in a $Al(OH)_3$ structure. As this oxide is more voluminous than the aluminium metal itself, it may provoke nucleation of grain-size crack with the help of contact stresses. Initiation of surface microcracks is then unavoidable. If the slip amplitudes are large enough the small cracks will be worn out and this will contribute to create more fret débris without any possible dangerous propagation cracks. That is a typical fretting wear mechanism. But if the microcracks can propagate below the oxide surface into the bulk material then we get a fretting fatigue process. As the crack grows deeper, the influence of the bulk stresses will then predominate until the complete fatigue failure of the material. Figure 1 shows different stages of the phenomenon.

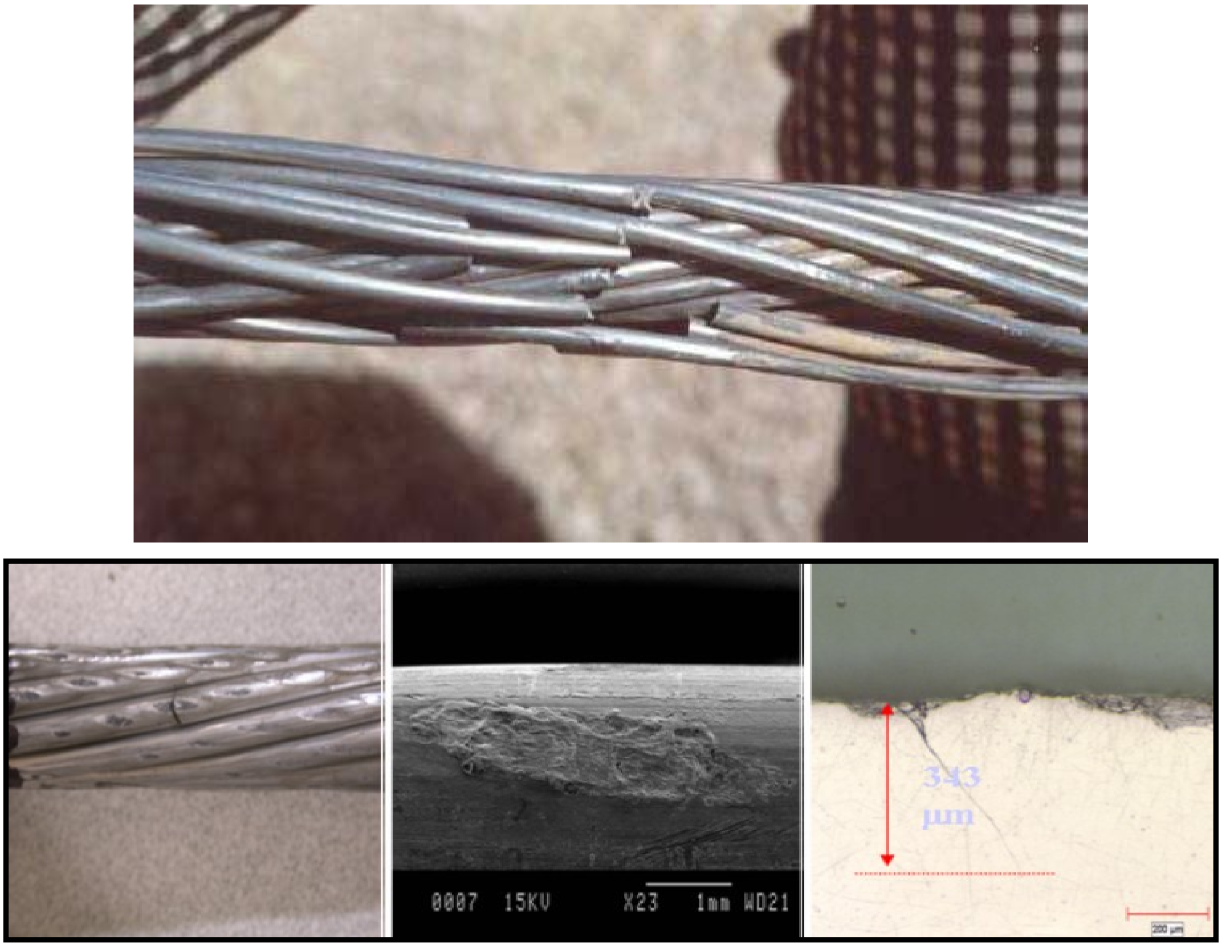

**Figure 1:** Different phases of the fretting phenomenon.

Many theories have been proposed, some well supported by laboratory tests simulating fretting fatigue, to account for the effects observed in this damage process. One may retain that the stresses reach a maximum at the edge of contact (stick-slip area) where cracks are mainly initiated. It is impossible here to attempt a complete study of the mechanism of fatigue. It suffices to mention a few researchers in that field: Waterhouse (1992), Nowell and Hills (1987), Mutoh (1995) and Johnson (1985).

In addition to the stress analysis and calculations in the bodies in contact, the fretting fatigue problem is often approached empirically. For example several researchers (Mindlin, 1949, Kuno et al., 1989, Vingsbo and Soderberg, 1988, Zhou and Vincent, 1995) used fretting maps concepts for controlling the fretting fatigue damage in practice.

## 3.2 Conductor Fatigue

The effect of wind excitation on overhead electrical conductors has been studied for a long time (Varney, 1926, Nefzger, 1933). But it is more recently that the effect of fretting fatigue has been recognized. In 1968, Fricke and Rawlins (1968) highlighted the importance of fretting in vibration of overhead conductors. The conductor wire failures were examined from lines in service and from laboratory tests and showed that fatigue cracks originated through fretting marks in all cases. These authors were amongst the first to evaluate the fretting damage by metallographic means and to present pertinent photos of broken surfaces. In addition they suggested the use of some surface coatings as a remedial approach.

Lutchansky (1969) described the stresses in armor wires of a submarine cable when bent over a sheave. In a discussion of this paper, Poffenberger and Komenda (1971) developed some of the complexities involved in fatigue problem which are mainly illustrated by the rub marks on the aluminium wires as shown by Lutchansky (1969). They also understood that the calculation of the flexural stiffness implied the consideration of the situation represented by some wires acting individually and other wires sticking together.

Analysis of a large number of fatigue breaks from field and laboratory tests have been done by many authors. Numerous photos of metallographic analysis pertaining to fretting marks and wire breaks made by Moecks (1970) confirmed that “frictional corrosion” had a substantial influence on the fatigue resistance of stranded conductors. Dulhunty (1971) showed visual results of wires broken near vibration dampers location. These photos have shown clearly that the fretting scars were the crack starting points.

In EPRI’s reference book, Rawlins (1979) reviewed various ways of relating fatigue of conductors to measurable vibration data, illustrated by some of the available data and field results. But it is a complicated matter because first, the stresses that cause failure are complex and do not relate in a simple way to the gross motions of the conductors and second, the failures originate at fretting marks. It is obvious that this mechanism of fretting reduces the fatigue strength of the strand relative to its unfretted strength.

The CIGRE SC22 WG04 (1979) presented recommendations for the evaluation of the fatigue strength of transmission line conductors. It is established graphically that any fretting decreases the fatigue strength of metals drastically with a comparison of S-N curves of separate wires with those of conductors.

Ramey and Townsend (1981) stated that fretting is the single most important parameter in fixing strand break locations. It is also a major parameter in determining the number of cycles at a given amplitude level to cause these breaks.

In a guide for endurance tests of conductors inside clamps written by the CIGRE SC22 WG04 (1985), it is explained that fretting occurs between the wires of adjacent layers of the conductor and between the conductor external wires in direct contact with the line accessories.

A final report of CIGRE SC22 WG04 (1988) again states the basic principles of the fretting mechanism in the stranded cables and its dangerous effect on the fatigue process in transmission line conductor. Of course it is pointed out that the fret débris produced can become an important sign of vibration danger in particular situations. For example in the case of estimation of conductor short life duration, the presence of black oxide aluminium during inspection indicates that there are certainly some contact points in the inner layers where fretting fatigue conditions are at work, even though the black powder comes from other points in the fretting wear regime.

In general, conductor endurance test data are analyzed in the same manner as those obtained from bending tests on solid specimens. The P-S formula (Poffenberger and Swart, 1965) which yields a bending stress value from amplitude measured at a standardized distance from a clamping device is commonly used as a reference. In the early nineties, Cardou et al. (1990, 1994) presented several ACSR conductor fatigue test results at suspension and spacer clamps. These authors explained an important limitation on the use of this formula because we cannot consider that wire fracture is only induced by the alternating stress but rather by the fretting occurring at inter-wire contact points. Also, observations made from fatigue tests clearly show that part of the conductor behaves as a solid section while the rest is constituted with wires acting individually.

At GREMCA’s laboratory of Laval University, a careful dissection of the test specimens in areas where wire breaks occurred is systematically made. It has been found (Cardou et al., 1990, 1994) that fracture location patterns at high and low amplitudes definitely differ. This suggests that more fracture pattern data should be generated at amplitudes near endurance limit level. Moreover more work remains to be done to relate fatigue data obtained on full-size conductors with single wire fretting fatigue behaviour. This will require:

- more experimental data on conductor bending fatigue behaviour;
- experimental data on such single wire behaviour;
- a theoretical or numerical model describing more rigorously conductor bending and conductor-clamp mechanical interaction.

This approach has already been proposed by Hobbs and Raoof (1994) for steel cable applications. They suggest that single (or twin) wire fretting experiments should be carried out under realistic modes of interwire movement representative of the conditions inside steel cables. Some formulations are developed to estimate the appropriate levels of normal loads and relative displacements to be established in such fretting fatigue experiments.

With the same perspective, Zhou et al. (1993, 1994, 1994a, 1995, 1995a, 1996, 1996a) present an extensive analysis of static marks and of fretting patterns under cyclic loading. Following their fatigue test fracture analyses, they were able to corroborate the results obtained in (Lanteigne et al., 1986 and Cardou et al., 1994).

Several bending fatigue tests with different types of clamps have been performed on several types of ACSR conductors in GREMCA's laboratories. Clamp systems and lubricant have been studied more specifically by Zhou et al. (1995a, 1996, 1996a). The clamp design has a strong influence, in its vicinity, on wire-wire and wire-clamp contact conditions. It appears that the mixed fretting regime may be reduced drastically for spacer clamp with pre-twisted rods and elastomeric cushion. The application of the lubricant can greatly increase the conductor service lifetime. It is noted that fretting is not entirely prevented but fretting crack nucleation can be delayed and that usual remedies for fretting cracking from the classical fretting point of view cannot be applied.

In parallel to the analysis of the fretting patterns, some tests were undertaken to gain a better understanding of the characteristics of inter wire contacts (Dalpé, 1999). Leghzaouni (1995) studied also the fretting behaviour in a wire-wire contact but this time under a cyclic axial tensile force.

## 3.3 Current Work

From all the above research material it is clear that fatigue phenomenon of overhead cables can no longer be considered as a classical material fatigue problem. If any progress is to be made in conductor fatigue prediction, efforts should be pursued in order to define more clearly the influence of micro-slip between wires and clamping device in the transmission line conductor design criteria. However, a deeper analysis of the phenomenon shows that it is difficult to relate precisely the stresses at the location of wire breaks to vibration amplitudes of the conductor (Lutchansky, 1969, Lanteigne et al., 1986, Papailiou, 1995, Cardou et al., 1997). One should keep in mind that the P-S formula (Poffenberger and Swart, 1965) does not yield the actual bending stress value. A short review of fatigue endurance of conductors by Cloutier et al. (1999) gives a good summary of this problem. It is suggested that, instead of the P-S stress, the bending amplitude $Y_b$ (IEEE Report, 1966) be used as the indicator of the micro-slip amplitude which is imposed to the strands at critical points like the edges of the clamp. Fatigue life prediction would become even more realistic if we could link the $Y_b$ endurance limit of a clamp-conductor system to a micro-slip amplitude value which could, in turn, be related to a distinct fretting-fatigue behaviour of the aluminium wires.

In that perspective, Cardou et al. (1997) studied the Drake ACSR bending fatigue at a suspension clamp under various test conditions. They have shown that a fretting-fatigue approach could be used to predict conductor fatigue endurance limit, where conductor bending amplitude can be related to wire-wire conditions. Using Dalpé's fatigue test results (Dalpé, 1999) a numerical application led to a rational prediction of the endurance limit of the conductor-clamp system evaluated.

In a recent paper, Ouaki et al. (2003) applied fracture mechanics to conductor wires on the basis of fatigue test results on a Bersfort ACSR 48/7 conductor. With some simplifying assumptions and using strain values measured from several wire cross-sections, stress intensity factors were computed at the tip of cracks located at the edge of inter wire contact areas. The computed stress intensity factor ranges were consistent with the experimental results obtained from the bending fatigue tests.

Fretting fatigue has a decisive influence on overhead conductor fatigue strength. While this complex damage process has already been the subject of numerous studies (Hoepner et al., 2000), work continues in order to achieve a better understanding of the phenomenon (Kinyon et al., 2003).

# 4. Determination of Fatigue Endurance Capability

## 4.1 Background

Either at the design stage or for an evaluation of the residual life of a line, there is a need to relate the potential level of vibration of an overhead conductor to the likelihood of fatigue of its strands. For an endurance assessment, as well as for an improvement of clamp design, fatigue tests are advantageous. The exact modelling of the actual system and of the field conditions is a complicated matter. The failures originate at interlayer strand contacts or at contacts between the outer strands and the line accessories where conditions for fretting are present. The definition of a more appropriate model than the one presently proposed (IEEE Report, 1966) to represent the actual phenomenon remains to be completed. Thus, it is still necessary to note not only that fatigue characteristics of conductors must be determined by fatigue tests of conductors themselves (Rawlins, 1979), but also that these tests should be conducted with clamps having similar characteristics to those of the conductor/clamp system being characterized. A guide for endurance tests of conductors inside clamps was prepared by CIGRE SC22 WG04 (1985) where it is stated that to arrive at comparable results in different laboratories an agreement on important test parameters and on an uniform method is necessary.

## 4.2 Laboratory Conditions

Different systems have been developed to simulate conductor motion (Elton et al., 1959, Goudreau et al., 2003, Cardou et al., 1994, Philipps et al., 1972), each presenting specific advantages. However, a test bench of the resonance type imposing a conductor motion in a vertical plane should be preferred. It is important indeed to reproduce as closely as possible the actual situation and to be able to control the parameters evaluated in the tests. Thus only this approach will be described in detail, applied to the case of a conductor supported in a *standard metallic clamp,* normally a short clamp. The limitations of that approach relate to the ability to reproduce the actual conditions experienced by the conductor, the clamp supports, and the range of bending amplitudes associated with the phenomenon reproduced.

Figures 2a and 2b show a typical installation of a resonance type test bench. The active length of the conductor specimen must be long enough, at least 5 m between the clamp and the point of excitation (vibrator), to ensure a good distribution of the load within the strands at the test end where the clamp is held. It is placed in a position to reproduce the static bending angle of the conductor. Typically, this angle is 5° to 10° for suspension clamps, and 0° for spacer clamps. The length of the conductor specimen is chosen to ensure that the length of the clamp is still small relative to the wave length induced. The minimum distance between the clamp under test and the back dead end of the conductor should be at least 2 m, again to ensure an adequately uniform load distribution in the conductor strands. That section of the conductor experiences no motion.

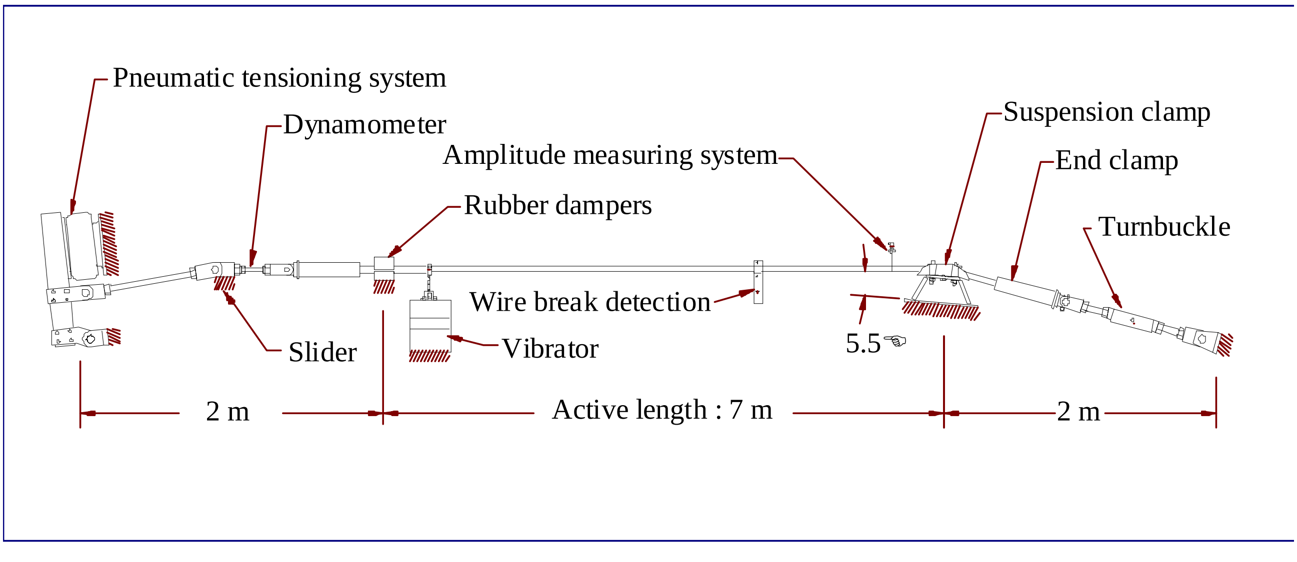

**Figure 2a:** Typical resonance fatigue test bench.

**Figure 2b:** Resonance type test benches, GREMCA's laboratories.

Although suspension clamps in actual lines are generally free to rock, holding the clamp in a fixed position results in a simpler test procedure, because it eliminates the difficulties associated with the dynamic response of a rocking clamp and the ensuing complex motion that remains to be adequately interpreted (Cardou et al., 1990). Of course, the transverse pressure between the clamp keeper and the conductor should be evaluated by a proper measuring device and controlled during the tests.

At the other end of the test bench, a load is applied through some tensioning device. The load is held constant within ±2.5 % during the test. The load may be applied in different ways, such as dead weight loading with a cantilever, hydraulic piston, or with a pneumatic tensioning device, as shown in Figure 2a. It is advisable to introduce a dynamometer to be able to continuously monitor the tension applied or to have its value checked periodically. It also simplifies the process for the initial setting. The tension level in the conductor should be representative of the actual prevailing line conditions, in order to induce a somewhat similar mean static stress in the system (CIGRE SC22 WG04, 1985). However, according to results reported by Rawlins (1979), this parameter apparently has little effect on the fatigue test data, given a conductor and its supporting clamp. It is noteworthy to add, though, that it is a question that is not yet settled. An attempt was made to include the conductor tension as a conductor fatigue parameter (Lanteigne et al., 1886). The large scatter of test results makes it difficult to arrive at a conclusive statement. The actual knowledge of the fretting phenomenon and of the conditions of contact favouring microwelds and crack initiation, however, warrants the requirement for adequate control of a constant tension during a test campaign.

An electrodynamic shaker is a good choice as a device to impose conductor vibration in the system because of characteristics well suited for such tests, particularly when they last for several months. Most tests are carried out at constant amplitude and frequency. A frequency in the range of 10 to 50 Hz best fits the field experience and thus simulation of the actual field conditions. But it is optional. However, one should avoid much higher frequencies to avoid strong possibilities of altering the interlayer strand contact conditions responsible for the initiation of the microcracks and their propagation leading to strand failures. Fretting fatigue is a contact phenomenon resulting in wear, as seen in the areas where cracks are initiated. Wear produces debris, which can modify the tribological conditions, and is a function of the sliding velocity at the contact, and hence of the frequency of excitation. In fatigue tests it is important to try to reproduce as closely as possible the actual conditions. Frequencies normally chosen, within that range, are those corresponding to a resonant mode of the taut-conductor system. It makes it easier to achieve constant amplitude conductor vibration for long-duration tests.

## 4.3 Test Parameters

In such tests, the fatigue life of the conductor must be determined as a function of some measure of vibration intensity. The stresses or stress combinations that would characterize the conditions favouring strand failures are not easily accessible to direct measurement.

Several measures of vibration have been employed: vibration bending angle $\beta$, dynamic strain in an outer-layer strand in the vicinity of the clamp, free-loop amplitude of vibration $y_{max}$, and bending amplitude $Y_b$ (amplitude of conductor motion relative to clamp at 89 mm from the last point of conductor/clamp contact).

Bending amplitude $Y_b$ is the most widely used parameter for measurement of vibration in the field, and it is recommended to use it as well in laboratory tests to avoid the necessity of converting this bending amplitude into any of the other parameters. That conversion depends strongly on the proper choice of the bending stiffness of the actual conductor (Rawlins, 1979). It is advisable to also measure the free loop amplitude $y_{max}$ to facilitate the correlation of the test results of conductors supported with clamps of different configuration and also to permit their use in establishing an endurance limit for a range of conductor sizes. However, results from tests on one conductor size are not necessarily applicable to all the others of the same size. Two conductors of similar size but of different geometry e.g., two layers of coarse strands as compared to three layers of finer strands, could lead to different fatigue curves.

The concept that there is some idealized strain or stress that can be calculated from vibration amplitude and that correlates well enough with conductor fatigue life has given the engineer a useful tool to overcome the complexity of the problem and find results that are reliable enough to be usefully applied.

The number of cycles to failure *N* generally refers to failure of the first strand (Rawlins, 1979). However, in (CIGRE SC22 WG04, 1985), one reads that "three broken wires or 10% of the aluminium wires – whatever is smaller – should be used as the damage criterion in respect of the relationship between the stress amplitude and the number of cycles." In practice, suffice to indicate clearly to what situation one refers to when reporting test results. Due consideration should be given to that point when comparing results from different laboratories.

Detection of failures by periodic visual inspection of the conductor outer surface was made in some early tests. It is well established that failures often occur at inner-layer strands, so that this practice is certainly not preferred. The strand failure detector is a solution to this problem. A simple method developed at Alcoa Laboratories (Rawlins,1979) has been extensively used at GREMCA's Laboratories (Cloutier et al., 1999). It consists of a small arm attached to the conductor in order to amplify its relaxation in torsion when a strand failure occurs. The rotational motion of the arm is detected by any suitable sensor (LVDT, proximity sensor, optical sensor), and it results in a step signal that may be associated with *N*, the number of cycles applied. Tests conducted until three and more strand failures have occurred are providing much more useful information, taking into account the inherent scatter of such test results (Hardy and Leblond, 2001).

Tests should be carried out with different values of the vibration parameters to obtain fatigue endurance curves (similar to the so-called S/N curves or Wöhler curves for a material). Those curves also provide a value for the endurance limit, the amplitude of bending below which a particular clamp-conductor combination will endure almost indefinitely. The endurance limit is defined, as currently accepted for aluminium, as the highest amplitude with no break at 500 Mc. In practice a test is interrupted when N failures are observed, it is a choice left to the laboratory responsible of the tests as stated above, or else when 500 Mc are reached. Because of the scatter of fatigue data, three tests per level of vibration amplitude should be considered as a minimum, and four amplitude levels are barely enough to define the fatigue diagram with sufficient accuracy.

## 4.4 Analysis of Results

After completion of a test, the clamp region of the conductor should be submitted to a dissection process that will permit correlating the strand failures observed with those indicated by the strand failure detector and to produce a map of the failures in the transverse plane as well as in the longitudinal plane (the position of the failure relative to the clamp support). This information is very helpful to improve our comprehension of the complex mechanism responsible for conductor fatigue. In several instances laboratories conducting such tests will further their analysis by a closer examination of the interlayer strand contact area where fretting occurred. This is particularly useful when tests are performed to compare or improve the design of clamps and to evaluate the use of lining materials.

The most common form to present conductor fatigue test results is the semi logarithmic fatigue endurance curve mentioned previously as the S/N curve. It is possible to superpose, on the same graph, points indicating the first, second, third, and $k^{th}$ strand failures for a series of tests. It then shows the dispersion of the results and certain particular anomalies when, for instance, an early first failure occurs but is not followed by a second one within the 500-Mc duration of the test.

To assist in the interpretation of available data on fatigue endurance of certain conductor/clamp system, a statistical analysis (Hardy and Leblond, 2001) was presented that led to the determination of various S/N curves on a sound probabilistic basis.

## 4.5 Fatigue Testing with Other Supporting Devices

It is indeed important to be able to use the database available for the cases of the fatigue performance of conductors supported in standard short metallic clamps when evaluating the performance of special supporting devices.

The temptation to rapidly define an *equivalent* $Y_b$ is strong but not necessarily easy. To illustrate the point, let us consider the evaluation of the fatigue endurance characteristics of a special clamp lined with a resilient material between the clamp itself and the conductor. One can see easily that the last point of contact between the supporting clamp and the conductor defined to establish a reference length (89 mm) at which one measures the bending amplitude $Y_b$ does not exist in the way that it was defined for the standard case of a conductor supported in a short metallic clamp. Moreover, the resilient lining supporting the conductor is likely to affect the profile of deformation of the conductor being flexed and hence the conditions of fretting fatigue. The analysis of such cases requires specific tests that will respect conditions such as the appropriate modelling of the actual situation on the line and the choice of test parameters that could be related to the situation in the field. To compare the performance of these special supports to the one of a standard short metallic clamp, the ($fy_{max}$) amplitude parameter is likely to be the best choice.

Some special supporting devices use armor rods, which give a longer equivalent contact length of the conductor with the support. This point has to be taken into consideration when designing a test bench for those devices. An active length of 7 m was indicated as a typical value in Figure 2a. It is possible that the length of any special support (e.g. armor rods) imposes a longer active length to satisfy the requirement of a minimum of 5 m between the end of the support and the point of excitation at the vibrator.

It may be worth noting here that for Aeolian vibration test set up, different active lengths could be recommended: e.g. IEEE Std 1138-1994, proposes a minimum of 30 m for vibration tests of Composite Fiber Optic Overhead Ground Wire (OPGW).

# 5. Inner Conductor Mechanics

## 5.1 Introduction

Overhead line conductors normally consist of alternating lays of wires – quite often of a steel based core and one or more layers of aluminium (Figure 3) – and look as having a simple construction.

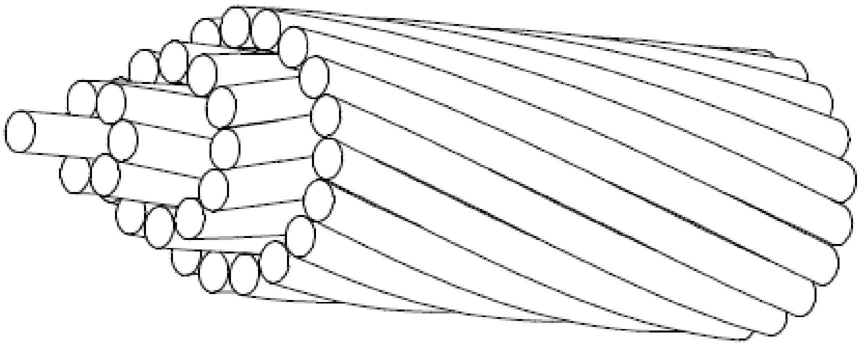

**Figure 3**: Typical construction of overhead line conductors.

This first impression is deceiving. In reality such conductors are quite complicated structures and pose demanding problems as far as their analytical description under mechanical loads is concerned. The task to calculate the stresses and strains in the individual wires of a conductor submitted to external loads (or deformations) will be called "inner conductor mechanics". Science is far from having created a universally accepted and applicable mechanical model in order to fulfill the above-mentioned task. At the time of preparing this report, more or less crude approximations and assumptions are necessary for the mechanical modeling of conductors. It is the intention of this Chapter of the report to review the situation in this field.

## 5.2 Definition of the Problem

It is well known – and extensively treated in the previous parts of this Technical Brochure – that Aeolian vibrations lead to conductor fatigue. The fatigue mechanism of vibrating conductors is a complicated chemo-mechanical process called fretting fatigue (Rawlins, 1979) as described in Chapter 3.

Fretting fatigue depends on many factors. The following are – possibly – the main ones (see Figure 4):

a) the macroscopic or bulk stresses in the individual wires of the conductor;

b) the relative movement between the wires;

c) the force acting between two adjacent wires and the resulting contact stresses at the crossing "points".

A better understanding of the inner conductor mechanics, should lead to a reasonably accurate prediction of the parameters a) to c) above, which in turn may enable a quantitative approach to conductor fatigue. The ultimate "vision" could be (Lanteigne et al., 1986), to reduce full size conductor fatigue tests to fatigue tests of individual conductor wires and thus reduce the complexity of the problem significantly. However a lot remains to be understood and explained before reaching that situation.

In particular there is a demand to bridge via an adequate modeling of a vibrating conductor, the difference existing today between the industry standard for vibration measurements (see Chapter 6), which is based on bending amplitudes $Y_b$ and the endurance limit of the conductors, which is based on stresses or strains, $\sigma_b$ or $\varepsilon_b$ (CIGRE SC22 WG11,1995) (see Figure 5).

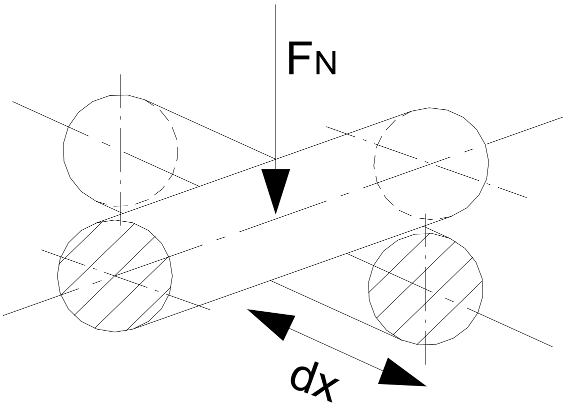

**Figure 4**: Wires of two adjacent layers subjected to a normal load $F_N$ and relative movement dx.

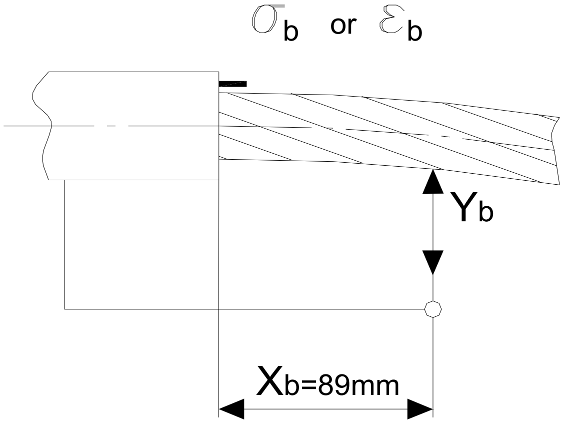

**Figure 5**: Definition of the bending amplitude $Y_b$.

Progress in inner conductor mechanics could lead in a better analytical description of conductor self-damping and of the damping properties and thus the modeling at the dynamic behavior of Stockbridge dampers.

## 5.3 Literature Review

There is an abundance of literature on the mechanical modeling of stranded ropes, but only few papers have been presented specifically for bending of overhead line conductors. An excellent and extensive review has been published by Cardou and Jolicoeur (1997) and Cardou (2006). Only the papers covering specifically conductor bending will be quoted from there.

Lanteigne (1985) proposed a model in which the bending moment contribution comes mostly from the fiber effect ($T_j$), calculated using the Bernoulli-Euler hypothesis, and also from independent wire bending. Wire twisting is neglected and the bending moment is taken parallel to the strand neutral axis. A model is also proposed for stick-slip transition. In a multi-layer strand, all layers are initially in the no-slip state (wire bending with respect to the strand neutral axis). As the imposed curvature increases, the outer layer reaches a point where it is in a full-slip state, with the wires bending independently with respect to their own axis. The slip criterion is based on the $T_i$ difference, for a given curvature, in contacting wires of adjacent layers, and on interlayer pressure due to axial load. In this model, the slip starts at the wires lying furthest from the neutral axis. The axial force on the wire $T_i$ (due to bending) vanishes as soon as a layer enters the full-slip regime, and the variations in global strand bending stiffness with imposed curvature yield a staircase-like curve.

Similar wire bending equations were used by Papailiou (1997), except that the slip criterion is based on the $T_i$ variation along a given wire and partial slip can be considered. Slip initiation occurs at wire sections located on the strand neutral axis. Wire tension $T_i$ is assumed to keep its initial value (before bending) at such points. This hypothesis yields a simple $T_i(\psi)$ function for each wire. After slip is initiated at position $\psi$ for a local critical curvature, $T_i$ remains constant at that point when the imposed curvature increases. This model is then extended to a multi-layer strand, with further hypotheses regarding interlayer pressure transfer. Application to the four-layer 1/6/12/18/24 ACSR Cardinal shows a gradual transition of bending stiffness $B_s$ from $B_{max}$ to $B_{min}$ as the imposed curvature is increased. The transition curve is found to depend slightly on the applied axial load.

In the preceding models, it is assumed that contact conditions between a wire and the layer underneath are either stick or slip (fully or partial). Another contact mode was analyzed by Goudreau and Cardou (1993): a wire may roll on the core, with or without slipping, when constant curvature bending is imposed on the core. End effects are taken into account and the free field conditions obtained far away from the ends.

The free-field strand bending stiffness $B_s$ was calculated in the no-slip and no-friction conditions.

To evaluate different models, the following experimental controls exist:

a) wire strain measurements;
b) equivalent strand bending stiffness;
c) lateral vibration damping.

A number of dynamic tests, performed on electrical conductors, were reported by Poffenberger and Swart (1965). These tests try to correlate outer layer wire strain with bending vibration amplitude measured at a standard distance from a suspension clamp. Axial load effects were also reported. An elementary bending theory is also presented in the same paper, in which the conductor is assumed to behave like a beam whose stiffness $B_{min}$ is calculated as if all wires were parallel and independent. The maximum wire dynamic bending strain and stress in the wire is calculated as a function of strand curvature. Very good agreement was reported between calculated and experimental values. it should be noted that, in this theory, $B_s$ intervenes only through a power of 1/2. Thus, a slightly higher $B_s$ value would not change the results appreciably. It is also interesting to note that the outer layer seems to behave as a collection of independent wires.

Claren and Diana (1969) reported the same kind of measurements made on eight different types of conductors. Specimens were 25 m long and several axial load levels were used. Strain gage measurements were made both near to the supports and in the free field (at antinodes). Strain amplitudes were compared with those obtained from solid beam models vibrating at the same amplitude. The ratio of the two values is called a *slippage coefficient* and is smaller than one. In the clamp region, the slippage coefficient is not as small as assumed by Poffenberger and Swart (1965). In the Discussion of the Claren and Diana (1969), Poffenberger suggests that this could be due to the low vibration amplitudes which have been used (bending amplitudes less than 0.36 mm). The large scatter in the results is also underlined by the authors.

On the experimental side, global strand behavior in bending has been studied both statically and dynamically. Scanlan and Swart (1972) studied the first vibration mode of a 1.5 m (60 in.) long, clamped-clamped specimen of the Pheasant electrical conductor, a 35.1 mm outer diameter conductor made of 19 steel and 54 aluminum wires. Deflections were measured at 15 equally spaced points between one clamp and the mid-span. These were used to calculate the first and second derivatives of the deflection curve. Using beam equations and calculated local curvature, an equivalent local stiffness $B_{eq}$ was obtained. Results show that $B_{eq}$ decreases from a maximum value in the clamp region of about $0.4B_{max}$ to $0.2B_{max}$ at mid-span. These are just average values. Values from a maximum of $0.57B_{max}$ near the clamp going down to less than $0.05B_{max}$ at the center ($B_{min} = 0.014B_{max}$) were obtained. Similar vibration tests conducted by Seppä (1971) on a 36 m Drake conductor specimen, indicated a global $B_s$ much closer to $B_{max}$: from about $0.7B_{max}$ to $0.9B_{max}$ respectively at low and high axial loads. Here, $B_s$ was determined from the measured resonant frequencies. Seppä also calculated the equivalent stiffness in the clamp region and found that, over the same load range, $B_{eq}$ varied from about $0.54B_{max}$ to $0.7B_{max}$ which are lower values than in the free field.

McConnell and Zemke (1980) studied the static bending stiffness of electrical conductors by subjecting 7 m specimens under tension to a transverse load. Bending was applied to a small part of the specimen, consisting of a flexure-test length of 914 mm for the larger conductors and 508 mm for the smaller ones, with this length bounded by heavy clamps which were bolted after the specimen had been put under tension, and after the torque had been removed. The transverse load was then applied and the cross-loads and deflections recorded. The equivalent global stiffness $B_s$ was calculated from the clamped-clamped axially loaded beam bending equation under a center load. It was found that it increased with imposed axial load from a low value around $B_{min}$ to a maximum value only a small fraction of $B_{max}$ (typically, about 5%).

Similar tests (both static and dynamic) have been reported by Hadj-Mimoune et al. (1993) and Hadj-Mimoune and Cardou (1994). These tests were performed on a generic strand consisting of a hollow aluminum tube and a layer of aluminum helical wires. $B_s$ was also found to vary with applied axial load from about $B_{min}$ to $0.8B_{max}$. Although the theoretical model, based on a slightly modified Orthotropic Sheet Model, does take into account both imposed axial load and transverse deflection, this last parameter does not seem to influence the actual behavior, at least over the range of imposed amplitudes.

Goudreau and Cardou (1993) have studied the static bending behavior of oversized epoxy 1/6 strand models under axial load. $B_{eq}$ was calculated from the mid-span deflection. Its value was found to vary with applied axial load and boundary conditions (clamped ends and pivoted ends). A slippage coefficient $K$ was defined as the ratio $(B_{eq}-B_{min})/(B_{max}-B_{min})$. It was found that, for pivoted ends, and for the range of axial forces investigated, $0.3 \leq K \leq 0.65$, while for clamped ends, $0.25 \leq K \leq 0.30$. The experimental findings did not support a gradual change of $B_{eq}$ with increasing transverse load, thus contradicting predictions of several partial slip theoretical models. Perhaps the data did not cover a sufficiently low load range, where a gradual change might be expected before the full-slip condition was reached.

Papailiou (1997) has performed static bending tests on 1.0 m and 2.65 m clamped-clamped, axially loaded, specimens of the ACSR Cardinal, a conductor made of 7 steel and 54 aluminum wires. A transverse center load was applied. The paper shows the deformed shape of the specimen in a fixed end region and, also, the center deflection vs. applied force. Experimental results were compared with $B_{min}$ and $B_{max}$ beam solutions and with calculations based on the theoretical bending model described previously. Since this model applies only to constant curvature bending, the clamped-clamped problem has been replaced by a finite element model (100 two-dimensional beam elements). The transverse load is applied incrementally. Each beam element bending stiffness is obtained from the theoretical model, assuming a local constant curvature, and updated after each load increment. Very good agreement is found between measured and calculated values. The observed slightly nonlinear loading-unloading behavior is closely matched by the numerical model.

Hardy and Leblond (2003) have developed a realistic taut, stranded cable model which took into consideration not only the helicoïdal construction of the cable but also the actual tangential compliance of the constituent wires at the contact interfaces, assuming an elastic material behavior. In accordance with the theory of contact mechanics, the paper shows that as bending increases, shearing and micro-slipping occur in the contact interfaces and this translates into a slowly decreasing cable flexural rigidity. Ultimately, adhesion is lost at most contact interfaces and the cable flexural rigidity then degenerates into the sum of the wire individual rigidities. On account of these phenomena, it was concluded that actual cable flexural rigidity is significantly lower than $B_{max}$, however small is the bending. Finally, it was shown that the predicted flexural rigidity for vibrating cables agreed reasonably well with experimental results reported in the literature.

## 5.4 Description of the Conductor Bending Phenomenon

Aeolian vibration causes the tensioned conductor to undergo vertical movements, i.e. to bend. Prior to bending the conductor has obviously to sustain an external tensile load. This load is distributed in the individual conductor wires, causing tensile stresses. Even this, at first sight a simple loading case (i.e. conductor under pure tension), is a complicated mechanical problem, in particular where temperature effects, manufacturing process, creep etc. are or have to be considered (Helms, 1964), (Ziebs, 1970), (Rawlins, 1997). In order not to complicate matters more, simplified formulas are used to calculate these tensile stresses in the individual wires, which are given in Appendix. In the calculation of these stresses, it is possible to include the Poisson effect and also since the tensile load causes the helix of the individual wires to change its shape, in particular its curvature and torsion, there occur further bending and torsional stresses in the wires, all these effects having a relative small impact on the wire stresses for the case under study here.

Finally the tensile loads in the wires cause normal loads at the crossing points of two wires in adjacent layers (Figure 4), which, besides leading to contact stresses, determine the stick-slip behavior of the wires during cyclic bending, as this is caused by Aeolian vibrations (Papailiou, 1997).

Qualitatively when a conductor is bent, the movement of its wires is suppressed by the friction forces acting between the wires and mainly between the wires of two adjacent layers. Mechanically this situation is described by the axial force equilibrium of a differential wire element, Figure 6, where Z is the tension in the wire, dZ the tension differential over the differential wire length dx, dR the friction force and dN the normal force differential at the crossover point with the underlying wire.

This way the wires can develop a bending strain, as though the conductor would be a solid body; the wires stick to each other. From a certain bending conductor curvature (or corresponding conductor deflection) onwards, the interlayer friction forces are not enough to hinder interwire movement, the wires slip relative to each other and their bending strain (and related stress) develops as though they bend around their own neutral axis. Additionally they maintain the maximum strain (and stress) value just before slip, which is constantly distributed over the wire cross-section and causes a secondary tensile stress. This process leads to a variation of the conductor bending stiffness during bending. At small bending amplitudes the bending stiffness can be calculated as though the wires are "welded" together and is called $EI_{max}$. At large bending amplitudes, the bending stiffness can be calculated as though the wires are totally loose and do not interact with each other; it is then called $EI_{min}$. In between these two extremes a more or less smooth transition takes place as indicated in Figure 7 (Papailiou, 1997). Recent refinements of this model indicated, that $EI_{max}$ cannot possibly be fully achieved (Hardy and Leblond, 2003).

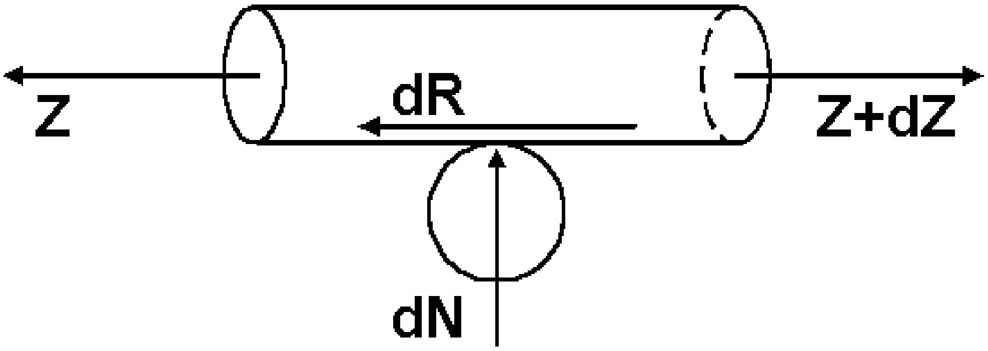

**Figure 6**: Axial force equilibrium of a differential wire element.

Formulas for calculating $EI_{max}$ and $EI_{min}$ are given in Appendix (Chapter 10). This fact, i.e. that the conductor bending stiffness varies during bending, becomes also evident in the non-linearity of the (static) load-deflection curve of a conductor, Figure 8, and is also observed and is actually the base for power dissipation in the messenger cable of the Stockbridge dampers, Figure 9.

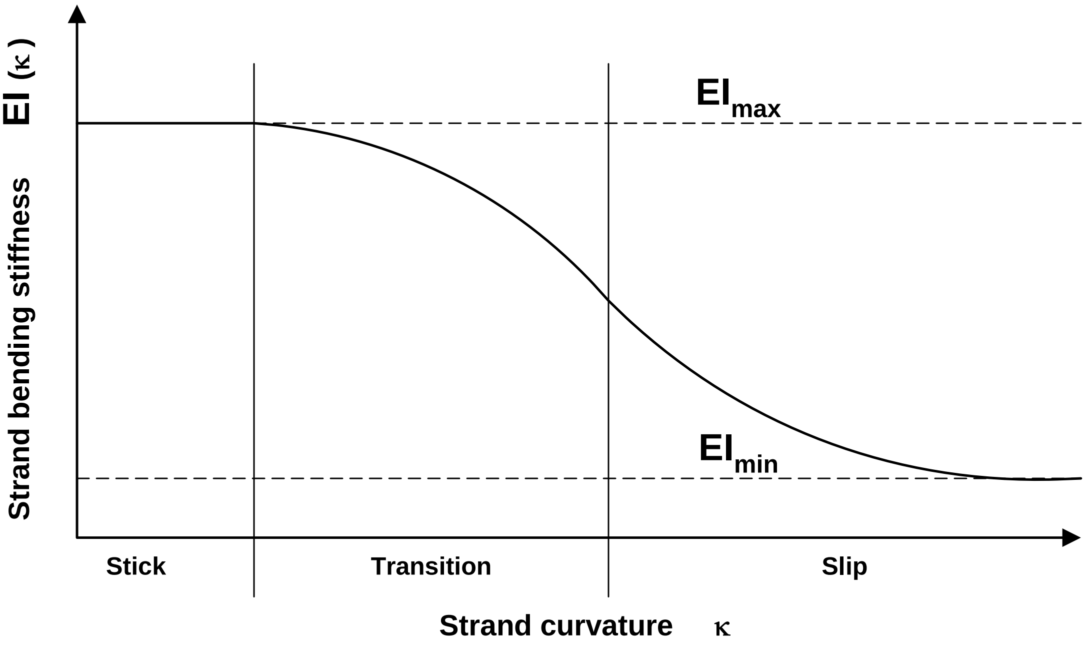

**Figure 7**: Bending stiffness as a function of conductor curvature.

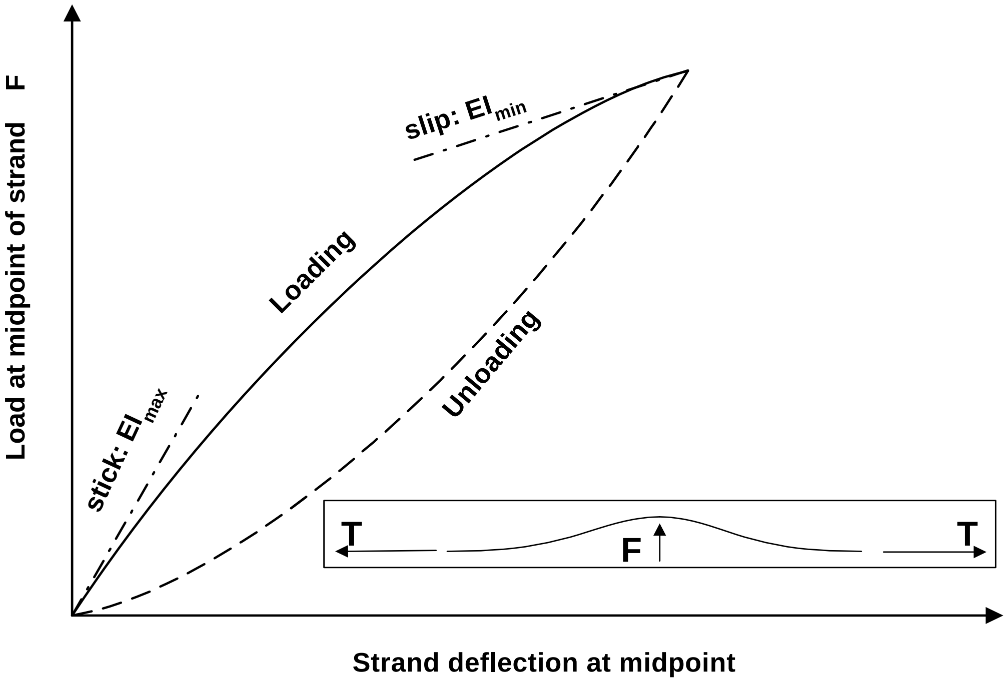

**Figure 8**: Load-deflection curve of a conductor at midpoint.

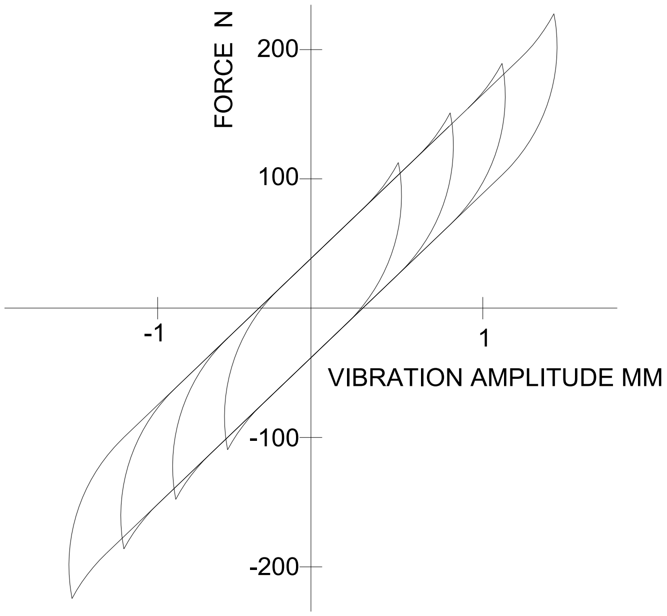

**Figure 9**: Hysteresis loops obtained at 16 Hz on 19-strand messenger cable of a Stockbridge damper with four different values of shaker vibration amplitudes.

## 5.5 Calculation of Idealized Stresses

Because of the complexity of the bending process of a conductor under tension as described above, a simplified model has been developed in 1965 (Poffenberger and Swart, 1965) and is since then, almost exclusively and extensively used in order to calculate “idealized” conductor stresses. These are used as a sort of “figure-of-merit” or reference stresses, in order to compare the vibration intensity of different conductors – as determined by bending amplitude measurements in the field – and also to put them in perspective with the so-called safe stress limits or fatigue endurance limits (accumulated stress or S/N (Wöhler) curves) (CIGRE SC22 WG11, 1995), a comparison which enables a statement about the conductor endurance capability.

The Poffenberger-Swart approach assumes that the vibrating conductor near the clamp acts as a fixed cantilever beam under tension, with an imposed deflection (half the bending amplitude) at the free end. The bending stiffness of this beam is taken as the sum of the bending stiffness of the individual (considered to be parallel) wires, $EI_{min}$, i.e. with the assumption "wires loose", no interstrand friction. Using the classic Bernoulli-Euler theory, the deflection curve of the conductor near the fixed end (clamp) and, from it, its curvature at that location and with that, the stresses in the wires of the outer layer – which are evidently assumed to bend around their own neutral axis (coinciding here with their center of gravity) – are calculated. It can be shown, that the formula for the wire stress (or strain) gained this way, is a good approximation for the same stress, if instead of the quasi-static approach, the differential equation of a taut vibrating string with constant bending stiffness is used (Rawlins, 1979), (Scanlan and Swart, 1968). The so-called Poffenberger-Swart (PS) formula which ultimately relates (measured) bending amplitudes with (calculated) wire stresses in the outer conductor layer is given in Appendix (Chapter 10).

The PS formula has been a valuable tool for the assessment of vibration severity of overhead line conductors since more than 30 years. Because of its relatively easy and straightforward application, it has been adopted by practically all researchers in this field and has become the *de facto* standard for the calculation of a "nominal" conductor stress on the outer layer for given (measured) bending amplitude. Because of this "standardization", its main contribution has been to enable comparative statements, very valuable if not absolutely exact, on the effects of a certain vibration level to the (mechanical) safety level (limit stress) of a conductor.

Since the very beginning of the introduction of this formula, there has been a certain "uneasiness" of its "universal" application, i.e. without considering the approximations underlying its development. In particular for small vibration amplitudes – which in the field accumulate the highest number of cycles and have thus a significant effect on conductor endurance – even Poffenberger and Swart mentioned in the closure of their seminal paper, that there is a "cloud" of uncertainty in that region.

The main reason for this statement has been that one would expect intuitively, that for small bending amplitudes, the individual strands of the conductor would "stick" together, and thus the conductor would behave as a "solid rod", responding to the bending load with its maximum bending stiffness. This should lead theoretically in significant higher stresses in the wires for small bending amplitudes, than those predicted by the PS formula. With increasing bending amplitudes more and more wires slip, the conductor bending stiffness comes closer to $EI_{min}$ and thus the PS formula can becomes a good approximation for the wire stresses in the outer layer, Figure 10.

There have been various approaches to overcome this problem, such as using empirical factors to the bending stiffness etc., but none of them achieved wide acceptance. Also there has been some publications (Lanteigne et al., 1986), (Ramey, 1987) presenting strain measurement results on conductors not in good agreement with the PS formula, but again, they remained on the long range unnoticed. Finally, one of the crudest discrepancies "produced" by the application of the PS formula, is the fact, that it leads to two different "safe" vibration stress levels (limit stresses) for multi-layer and simple-layer conductors, differing by almost a factor of three (8.5 MPa vs. 22.5 MPa) (Rawlins, 1979). It is however an additional indication that we are facing a problem related to wire contact conditions, very different with those two types of conductors.

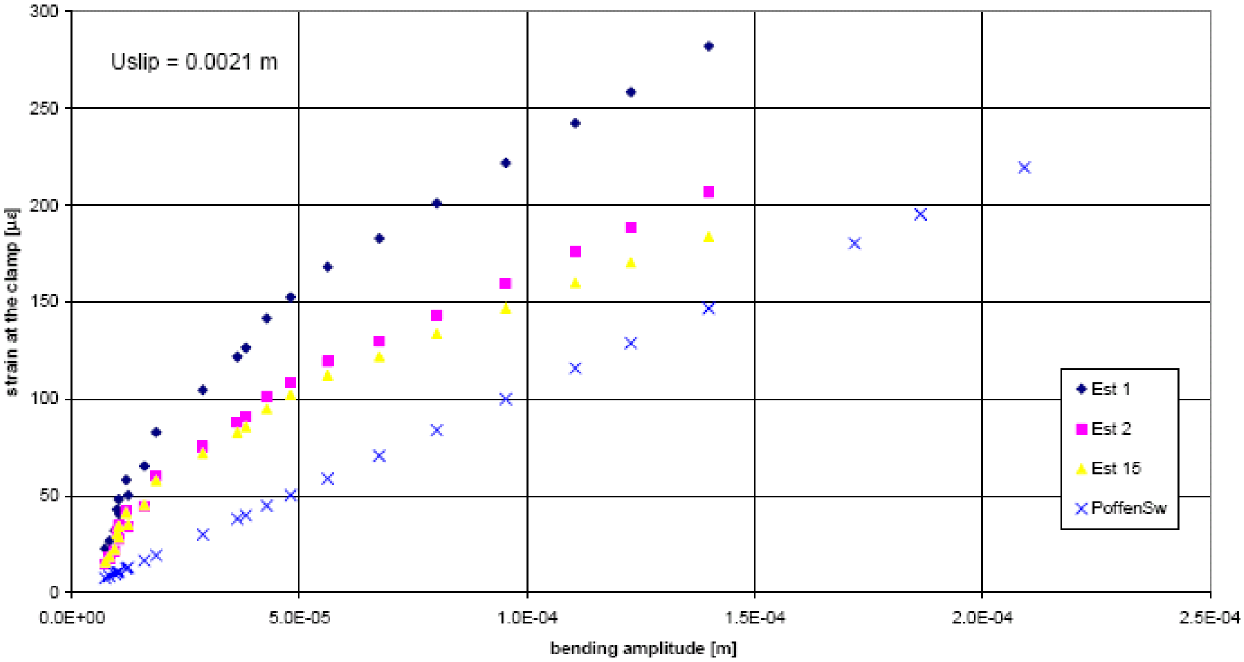

**Figure 10**: Bending strain vs. bending amplitude: comparison between strains measured at the clamp (Est 1, Est 2, Est 15, see also Figure 11)) and strains calculated by the Poffenberger-Swart formula (PoffenSw) for a Drake conductor at 20% RTS.

There are several reasons why the PS formula when “checked” in the laboratory by simultaneous bending amplitude/stress measurements give reasonable results, although obviously based on crude approximations, Figure 11:

a) The formula has been initially and is often verified at conductors in commercial or custom made suspension clamps, which obviously strongly deviate from a fixed end, as the analytical development of the formula assumed. This means, that the measured stress – which is compared with the formula – will very much depend on what lateral distance the strain gauge is placed in respect to the fixed end for the actual clamp – i.e. the location where the tangent to the deflection curve is horizontal (first derivative is there zero). Even for small distances away from that location, the stresses will decline quasi exponentially with distance, showing values closer to the PS formula.

b) The maximum stress in the wires will not be necessarily on the wire top, where the strain gauges are normally placed, because this stress depends not only on the change of magnitude of the strand curvature vector but also on the change of its direction. Depending on conductor geometry, this stress will be shifted along the conductor and the wire perimeter, i.e. again, measured values will tend to be smaller than the actual maximum wire stress values, coming thus closer to stresses calculated with the PS formula.

c) The tensile stresses before bending in individual wires, tend in the short –compared to the field – laboratory spans, to differ from each other by factors of up to 100%, although the sum of these stresses over the conductor cross-section will evidently lead to the external tensile load. Since the bending stresses depend on the tensile stresses, there is a good chance that measured stress will show in some wires much lower values – i.e. closer to the PS stress values – than expected by the stick-slip model.

It is worth noting, that the above statements are not to be understood as a criticism to the Poffenberger-Swart formula -which value cannot be overemphasized-, but as an indication of the complexity of the matter, the limits of the simple conductor model and possibly also on areas of future research.

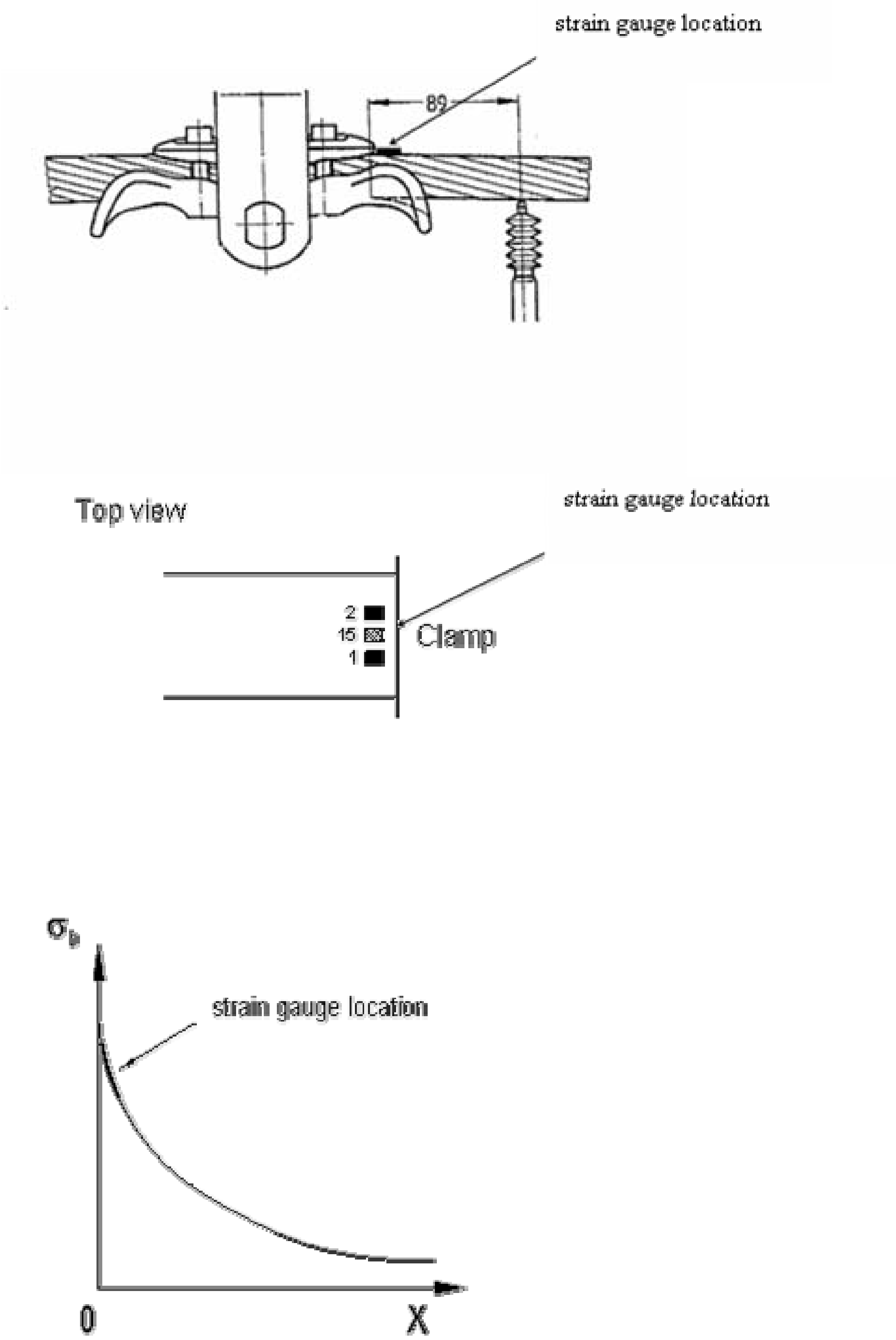

**Figure 11:** Sources of possible errors when checking the Poffenberger-Swart formula showing from top: a) Bending amplitude measurement at a typical suspension clamp; b) strain gauge measurements on some conductor wires; c) bending stress $\sigma_b$ curve as a function of distance x from the fixed end.

# 6. Assessment of Vibration Severity on Actual Lines

## 6.1 General

Overhead transmission line conductors can be subjected to various wind induced motions (Docy et al., 1979). Those called "Aeolian vibrations", are the most recurrent and the most dangerous for conductor integrity and may cause fatigue failure of the conductor strands and of other line components.

For this reason, great efforts are devoted to the assessment of Aeolian vibration severity. Four main methods are available for this task:

1. Computer analysis of the conductor vibrations
2. Field vibration tests on outdoor experimental spans
3. Vibration test on laboratory spans
4. Vibration measurements on actual lines

Computer analysis (CIGRE SC22 WG11, 1998), (Claren et al., 1976), (Claren and Diana, 1977) is mainly used during the design of the line to anticipate the performance of single and bundled conductors under Aeolian vibration and to evaluate, when necessary, the amount of additional damping required to maintain the vibration amplitudes within safety limits.

Outdoor experimental spans exposed to natural wind have been built in several countries world-wide for research purposes and for the comparative evaluation of conductor damping systems proposed for important projects (Cloutier et al., 1974), (Houle et al., 1987). Some test stations have been used also for the evaluation of new damping systems and for the assessment of the vibration behaviour of new line configurations.

These test stations represent the most accurate mean for the investigation of vibration phenomena as they can be extensively instrumented with a large use of recording and monitoring systems. However, the considerable costs involved can be afforded only by government financed Power Authorities and Research Institutes or justified in some major transmission project.

Vibration tests on laboratory spans, generally 30 to 90m long, can provide important information on the conductor dynamic characteristics such as self-damping (CIGRE SC22, 1970) and dynamic bending stiffness. Moreover intensive test are performed to assess the fatigue behaviour of various conductor–clamp systems (CIGRE SC22 WG04, 1985) and effectiveness of vibration dampers (IEEE, 1993).

Vibration measurements on overhead lines are commonly performed as final acceptance test of the conductor damping system, at the end of the line construction, and, on lines in operations, for assessment of vibration intensity of the conductors. For these measurements, a variety of instruments have been employed. Among the different test procedures, the “bending amplitude method” involving specific live-line vibration recorder has been widely used for almost 40 years.

## 6.2 Aeolian Vibrations

Overhead conductor vibrations are caused by the alternate detachment of wind-induced vortices from the top and bottom sides of the conductor. The vortex shedding produces an alternating pressure unbalance, inducing the conductor to move up and down at right angle to the direction of airflow. The frequency of the vortex detachment, and consequently the conductor vibration frequency, depends on both the wind velocity and the overall diameter of the conductor, according to the well-known Strouhal formula (Doocy et al., 1979). Considering that, the range of wind speed able to excite dangerous Aeolian vibrations is, in normal conditions, 1 to 7 m/s, it is possible to define the range of potentially harmful vibration frequencies of any conductor. Besides, in a conductor span, it occurs very often that two ore more vibration frequencies, not too far apart, are induced simultaneously by the wind. The presence of two or more vibration frequencies determines beats, at any vibrating point of the conductor, which result in a continuous random variation of the vibration amplitude as shown in the records of Figure 12.

This aspect of the conductor vibration will be considered for the correct interpretation of the data collected by a specific recorder and for the evaluation of conductor lifetime.

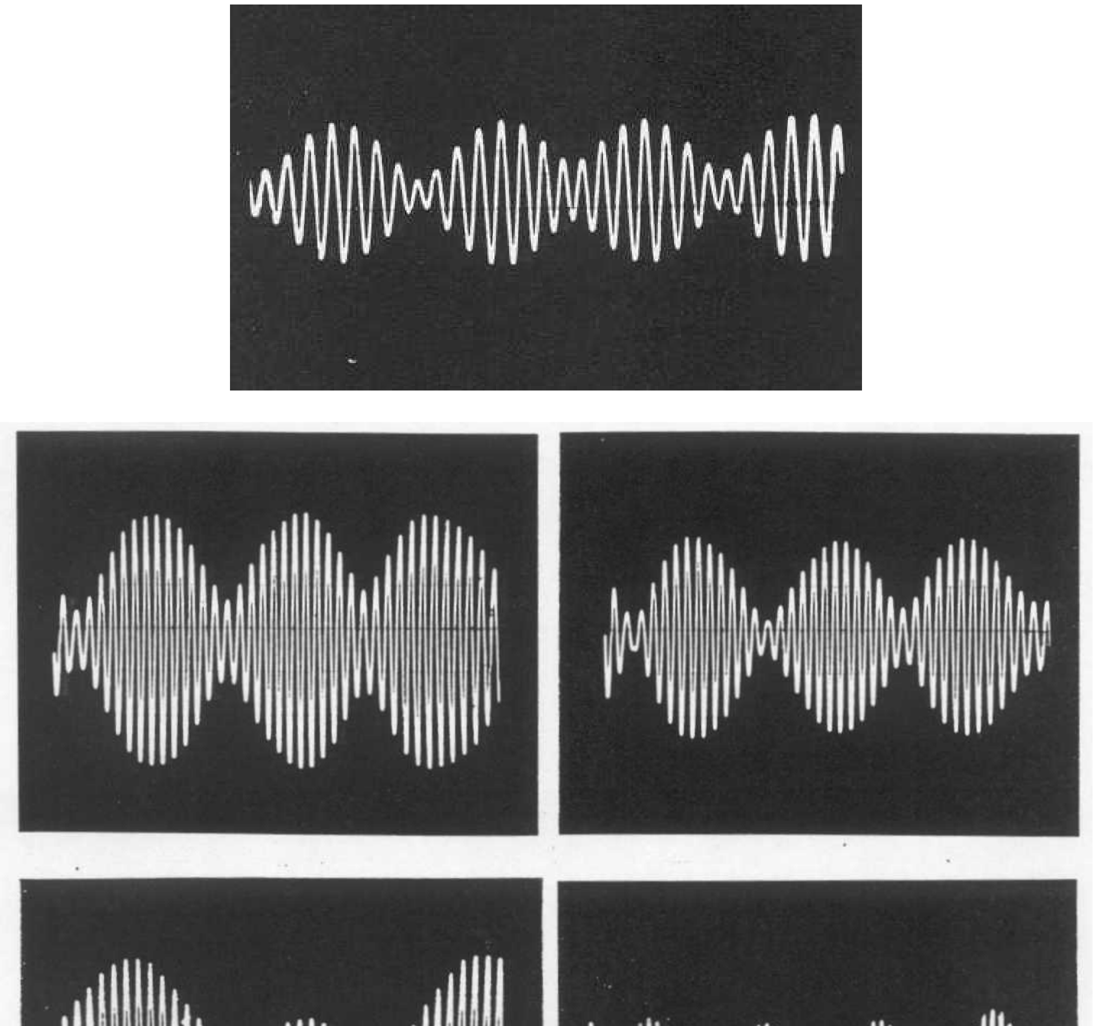

**Figure 12**: Records of natural Aeolian vibrations.

## 6.3 Vibration Assessment

In overhead conductors, fatigue failure of strands is the most common form of damage resulting from Aeolian vibrations (Figure 13). Conductor fatigue may also result from galloping and subspan oscillation, but is not the main problem associated with those motions.

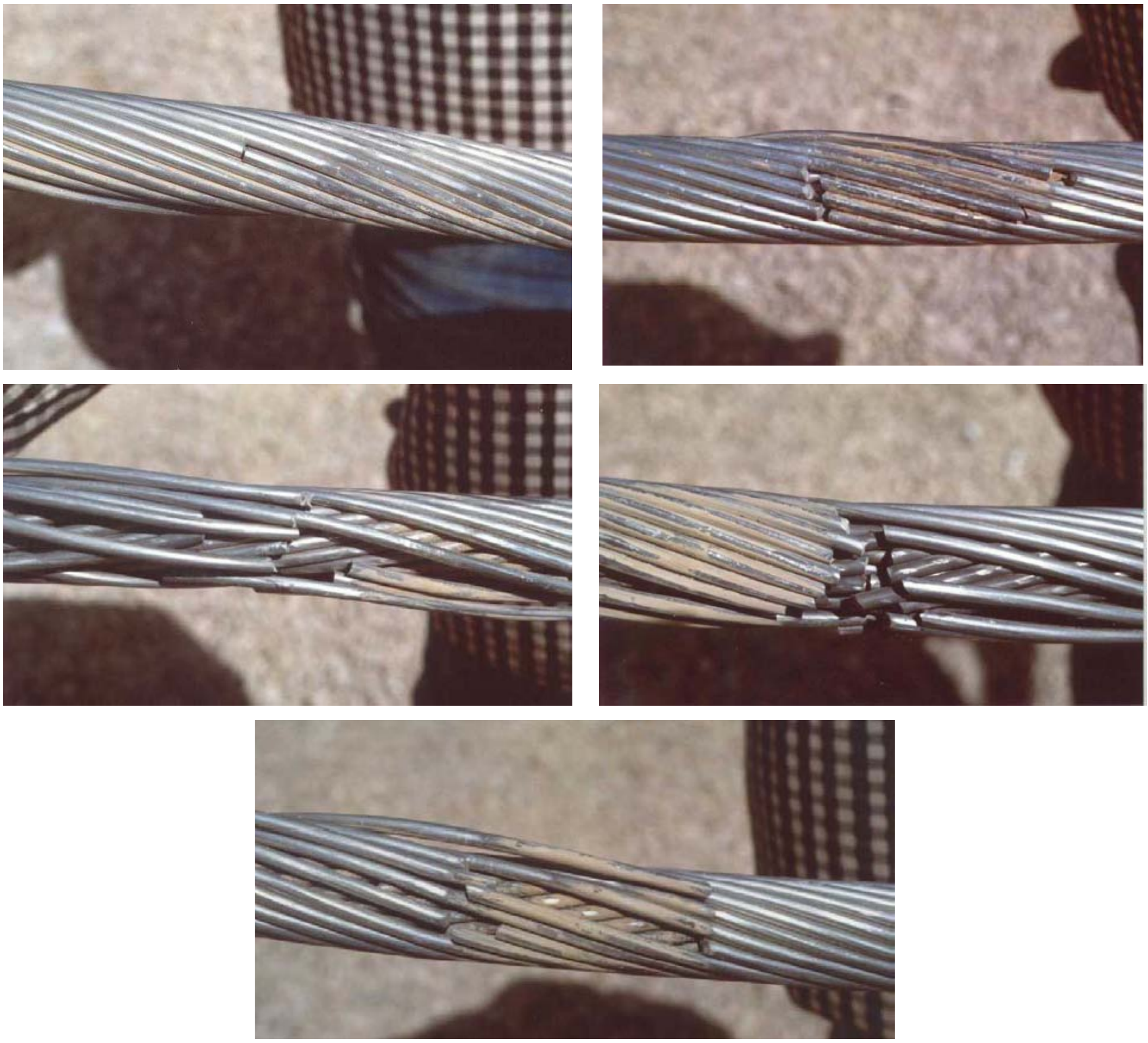

**Figure 13**: Strand failures due to Aeolian vibrations.

Fatigue of conductor strands, of any type, is caused by the alternating stresses produced by vibration at points where conductor motion is constrained, e.g. where the conductor is secured to fittings. Thus, typical locations are: suspension clamps, dead-end clamps, splices, clamps of spacers and dampers, warning devices and anti-galloping devices. Among these locations, the most critical is at the suspension clamp because of its rigidity in the direction of Aeolian vibrations (mainly vertical) and the cumulative static stress due to conductor curvature, tensile load and clamping effect. All the other fittings show a certain degree of mobility, but poorly designed units, especially spacers and dampers, may produce strand failures at their location or may fail themselves under vibrations (Figure 14).

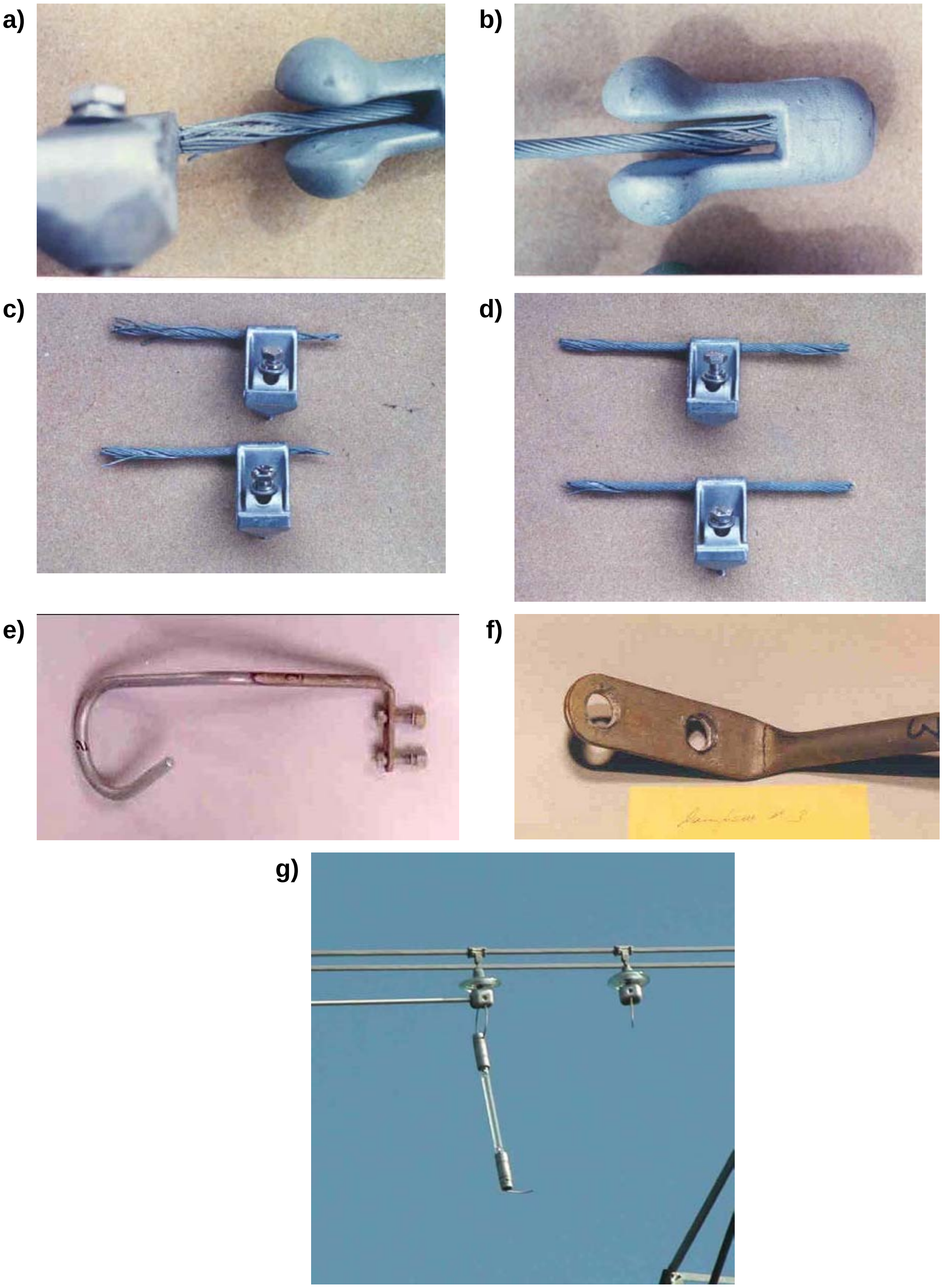

**Figure 14**: Damages to conductor fittings due to Aeolian vibration.

Some inspection procedures are available to assess strand failure or to estimate whether fatigue on strand may eventually occur during the economic life of the conductor. The most common are:

1. Visual inspection of conductor surface
2. Radiographic inspection
3. Conductor vibration measurement

The need to apply one of these procedures may be indicated by certain “early warnings”.

Generic information about excessive vibration levels on an overhead transmission line can be gathered by means of line-crew reports about conductor or structure vibrations or visible damage or looseness of hardware components (Figure 14e, f, g), cross-arm members and conductor fittings. Nevertheless, these reports should be carefully evaluated since they do not necessarily indicate danger to the line. Further investigation is necessary to clarify whether the damage is isolated to a single event or to a single component or it is the clue of a major design problem.

Strand failures of Stockbridge damper messenger cables and loss of damper weights are amongst the warning signs (Figure 14a, b, c, d). Still, this symptom should be carefully analysed as it can indicate a poor unit design or the result of galloping or the consequence of Aeolian vibration on conductor covered with hoarfrost or ice.

Failure of hardware components, having natural frequencies in the range of Aeolian vibrations, can occur for excessive conductor vibration although it can be produced by the damped residual vibrations that are easily endured by the conductor. Figure 14e shows an arcing horn failed, as evidenced in the detail of Figure 14f, under the effect of Aeolian vibrations. Figure 14g shows the same phenomenon on a night warning device.

Hardware components that show signs of chafing or rotation may provide evidence that vibration had occurred. Fretting at the inter-strand contacts produces black metal oxide powder. The appearance of this powder at the surface of the conductor indicates vigorous vibration activity.

Visual inspection is appropriate when there is strong or specific evidence that damage has occurred, but can not be performed systematically during periodic maintenance or line survey. In any case, strand failures may be difficult to detect, as they occur near the last point of contact between conductor and clamp. For example, failures at suspension clamps generally occur on the lower side of the conductor that is inside the clamp mouth. Reliable inspections require that the conductor be separated from the clamp. When armour rods or elastomer-lined clamps with helical rods are used, the search of strand failures requires the removal of these components.

In any case, this technique allows the detection of outer-layer damage only and thus may overlook evidence of excessive vibration severity.

Aluminium based conductors, having more than one layer of aluminium strands, may show the first strand failure either in the outer layer or in the layer below (Doocy et al., 1979). According to laboratory tests, the first failures are more likely to happen in the inner layers so that when outer layer strand failures are observed the extension of the damage may be already large and the corrective actions late.

Radiographic inspection can give some results, but it is not a common practice since it is costly and rather complex. Moreover, the interpretation of the radiographs is sometimes difficult and the failure detection may be not completely reliable.

Thermographic inspection is not suitable for the early detection of strand failure. Test conducted in Italy on an ACSR (54/7) conductor[†] showed that no difference in temperature can be detected for failure of one and two outer layer strands, while only one degree was measured for three strand failure.

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[†] Test performed with: current 500 A, wind speed 0.5 m/s, ambient temperature 20°C, solar radiation 500 W/$m^2$.

**Figure 15**: Wear and failure of conductor strands due to spacer clamp loosening.

Another problem related to conductor vibration is the clamp loosening of fittings such as spacers, dampers, warning devices, etc. Loosening of spacer clamps causes serious damages to the conductor. Initially, the movement of the loose clamp causes abrasion on the conductor surface (Figure 15 a, c, d), then increasing looseness allows hammering between the conductor and the clamp that leads to complete failure of the conductor (Figure 15b, e, f, g). Looseness of single clamp fittings determines the progressive slipping of the units along the conductor toward the centre of the span.

Considering the above and as stated by Doocy et al. (1979), "*attentiveness to early warnings and use of vibration recordings*" can be used for the early detection of conductor failure or risk of failure.

## 6.4 Vibration Measurements on Actual Lines

Measurements of Aeolian vibrations on actual lines can be made in different ways using a variety of instruments that can be classified into four groups:

1. Generic transducers
2. Vibration detectors
3. Optical vibration monitoring devices
4. Specific vibration recorders (bending amplitude recorders)

### 6.4.1 Generic Transducers

Generic transducers like accelerometers, velocity pick-up, contactless displacement transducers, anemometers, thermometers, connected to a ground sites data acquisition system are normally used in outdoor test stations. In the past, they have also been used on several transmission lines to assess the vibration severity or for research purposes (Falco et al., 1973), (Diana et al., 1982). With the advent of commercial bending amplitude recorders this practice has been limited to test stations.

### 6.4.2 Vibration Detectors

The first conductor vibration recorders, such as Zenith and Servis recorders and Jacquet counters, were simply vibration detectors able to provide a quite rough relative index of vibration activity. They have been replaced by modern vibration recorders.

However, the use of vibration detectors of low cost, light weight and easy installation can be still of interest. One of these devices, for example, has been proposed recently by the Rand Afrikaans University (Du Plessis and Pretorcus, 1995).

### 6.4.3 Optical Vibration Monitoring Devices

Optical devices are sometimes used to assess the vibration level on overhead line conductors. Two systems are known so far:

1. Opto-electronic recorder (Figure 16a)
2. Laser recorder

The companies Karl Pfisterer and Ribe have developed a similar opto-electronic vibration-monitoring device.

Both types are mobile non-contact equipment that consists of three parts:

- vibration monitoring unit
- wind measuring units
- computer based data acquisition system

An electro-optical camera equipped with a telephoto lens is usually directed at the vibrating object from the ground. The camera transforms the vibration images into electrical signals whose frequency spectrum and time history can be displayed on an oscilloscope and stored in the computer system.

A prototype of a laser based vibration recorder has been developed by ENEL, Italy, in 1984. The device consists of ground equipment emitting a low power laser beam directed to a "scotchlite" target installed on the conductor. The laser light reflected by the target returns to the instrument and is analysed by a computer controlled opto-electronic system. Vibrations of amplitude from 50 micron to 7 m in the frequency range 0 to 150 Hz can be measured and recorded. Thus, the recorder is suitable for any kind of conductor motion.

### 6.4.4 Bending Amplitude Recorders

The dynamic bending strain measured at the mouth of the suspension clamp is a parameter closely related to conductor fatigue

The first measurements of this quantity were performed applying strain gauges as near as possible to the points of maximum bending (Hard, 1958), (Steidel, 1959), (Buckner et al., 1968). However, this method, suitable for laboratory tests presented serious application problems on the field.

Edward and Boyd (1963) proposed the use of a parameter directly related to the bending strain at the mouth of the suspension clamp and more accessible to measurements, the bending amplitude $Y_b$. This practice had been used successfully by Ontario Hydro for some 25 years and the same authors presented the first live-line recorder suitable for these measurements, to be installed on the suspension clamp (Figure 16b).

Bending amplitude ($Y_b$) was defined as the vertical displacement peak to peak of the conductor, measured relative to the suspension clamp, at a point 89 mm (3.5 inches) from the last point of contact between the clamp and the conductor (Figure 17a). A linear correlation between bending amplitude and the strain measured on the surface of the conductor adjacent to the clamp was established.

In 1966, the IEEE Task Force on the Standardization of Conductor Vibration Measurements, recommended the bending amplitude method (IEEE, 1966) as a practical method of assessing the severity of fatigue exposure of overhead conductor in all conventional suspension clamps. A simple but approximate equation was suggested to convert the bending amplitude into bending strain, and an evaluation criteria based on the maximum allowable bending strain was proposed.

Later, Poffenberger and Swart (1965) formulated the dynamic deflection field of the conductor in the vicinity of a fixed clamp and provided relations to convert the bending amplitude into dynamic curvature and bending stress in the outer layer strands at the mouth of a conventional metal to metal suspension clamp.

An alternative method known as the “inverse bending amplitude” (Figure 17b) method was proposed in 1981(Hardy et al., 1981) together with a relevant measuring device, the Roctest TVM 90. The recorder was fixed onto the conductor where it sensed the motion directly above the last point of contact between the conductor and the clamp. In a number of cases, the measured inverse bending amplitude can be converted to either bending amplitude or bending stress in order to express the measurement results in accordance with the IEEE standardization. This recorder also paved the way for an important development in data reduction of conductor vibration motion. The results are analysed in situ and recorded in a frequency/amplitude matrix. The initial 4x4 matrix of the TVM 90 was rapidly increased in digital recorders that followed, with the advent of microprocessors that became more accessible for these applications.

The commercial bending amplitude recorders can thus be divided into two categories: analog and digital devices. The analog recorders are the oldest and can be self-contained (Ontario Hydro) or requiring ground instrumentation (Hilda and similar). They can provide the time history of the conductor vibration, information of great interest but generally requiring a time-consuming process to reduce it to relevant data.

Digital recorders are microprocessor based, battery powered devices that dispose of a built-in memory storage. They can be connected to a computer for the set-up of recorder parameters and functions, before the measurements and to read out, display and print of measured data after the test. The most common bending amplitude recorders are the following:

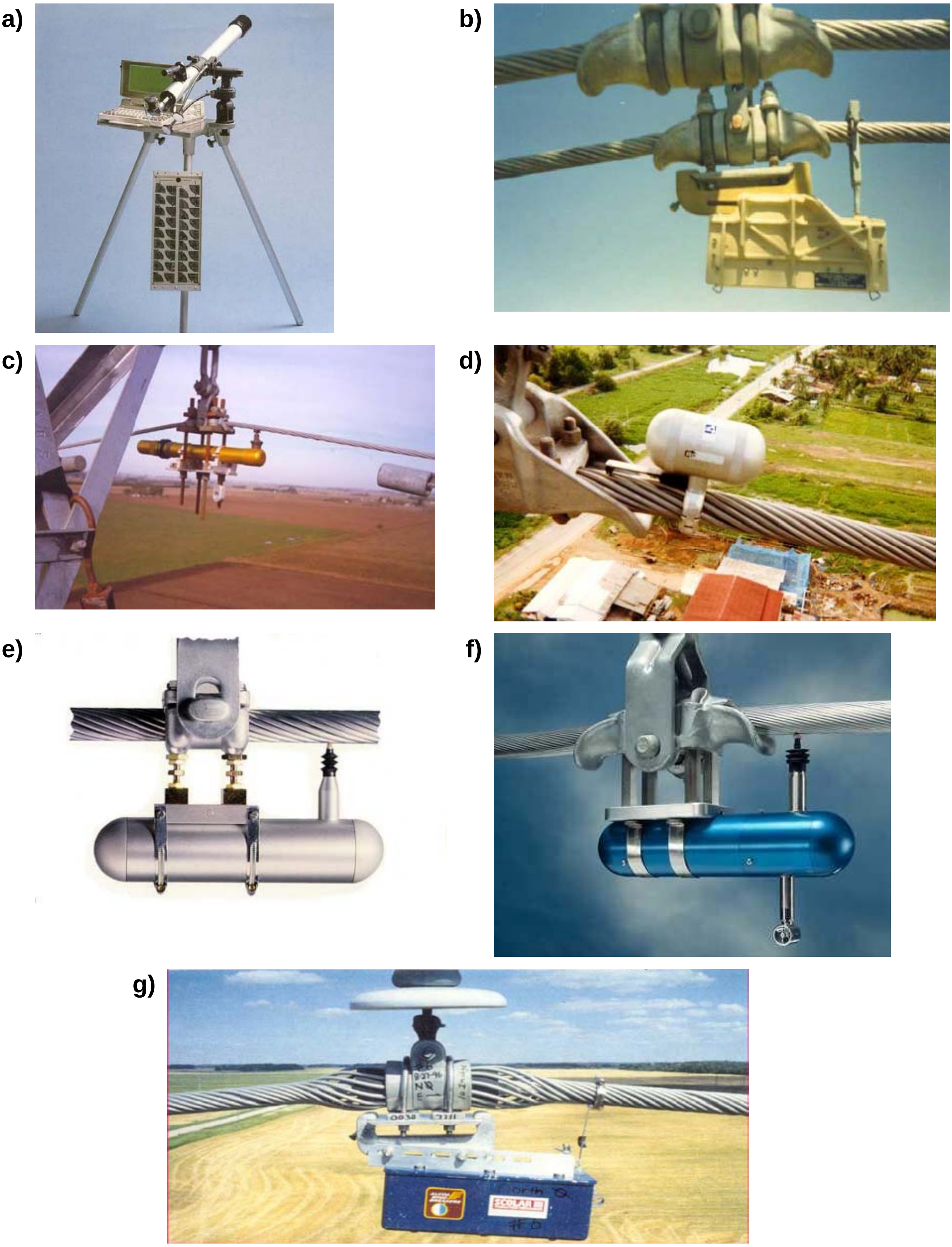
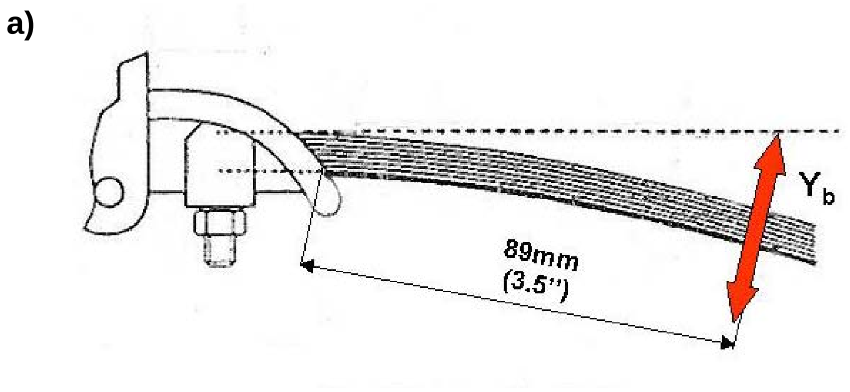
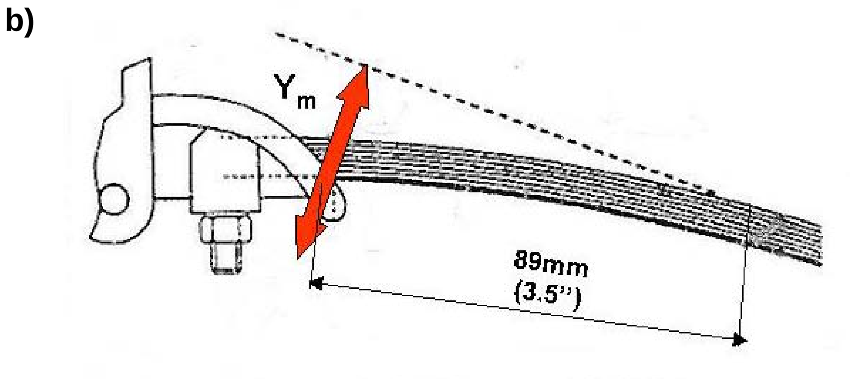


**Figure 16**: Vibration recorders for overhead conductors.



Bending amplitude $Y_b$



Inverse bending amplitude $Y_m$

**Figure 17**: Bending amplitude and inverse bending amplitude.

#### 6.4.4.1 Most Common Bending Amplitude Recorders

- <u>Ontario Hydro Recorder</u>

This analog vibration recorder developed by Edward and Boyd (1963) is no longer available on the market. However, it has been widely used all over the world for many years. The recorder (Figure 16b) contains an internal clock and is timed to obtain one second recordings every 15 minutes. The recording system uses a tungsten carbide stylus that marks a trace on a clear 16 mm cellulose film. The trace is mechanically amplified five times.

The film is moved by a battery-powered mechanism during the one second recording at the speed of 6.4 mm/sec. The maximum countable frequency is 150 Hz and the maximum allowable amplitude is 50 mils. (1.27 mm).

The instrument autonomy is about three weeks in a temperature range –20 to +40°C.

The weight of the instrument is 4.5 kg (7.8 kg with standard fittings).

- <u>HILDA (High Line Data Acquisition System)</u>

The HILDA analog vibration recorder comprises a sensor/transmitter (Figure 16c) mounted on the suspension clamp and a ground site receiving and recording equipment.

The ground equipment consists of a weatherproof cabinet enclosing a high frequency receiver and recording devices such as magnetic recorder, paper recorder, etc.

The receiver collects the radio signals containing the vibration data sent by the transmitter, via a coaxially connected antenna installed on the tower.

Wind direction sensor and cup anemometer can be coupled to the vibration sensor to give simultaneous readings.

The maximum recordable bending amplitude is 2.54 mm peak to peak in a frequency range 1 to 100 Hz.

The autonomy of the transmitter is more than 200 days.

The weight of the line unit is 0.64 kg.

This device also is no more available commercially. However a number of units may be still in operation.

- Sistemel Recorder

Designed in Argentina, this analog vibration recorder has been used for many years but only inside the Country. The basic design principles and the performance characteristics are similar to the ones of HILDA. The wind velocity and direction transducers and temperature sensor are standard accessories.

- Pavica Recorder

The digital vibration recorder Pavica (Figure 16d) is the last version of a series of Roctest vibration recorders designed to be installed directly on the conductor (Hardy and Brunelle, 1991). Its light weight (0.5 kg) enables the recorder to perform measurements on location other than suspension clamps (Figure 18).

**Figure 18**: Vibration recorder Pavica installed near a spacer damper clamp.

The vibration sensor is a blade equipped with strain gauges.

A built-in serial interface (RS-232C) allows direct connection to a PC. A utility program Pavicom, supplied with the recorder, is used to set-up recorder parameters, read out, display and print of measured data. Moreover, it allows additional graphical presentations and conductor lifetime estimation in accordance with the CIGRE WG 22-04 method, for some conductor-clamp configuration.

The maximum measurable bending amplitude is 1.3 mm peak to peak in a frequency range 1 to 255 Hz (127 Hz in the latest version). Temperature recordings are also available.

A version with a split vibration sensor is available for measurements on small conductors.

The autonomy of the recorder at the standard sampling rate is between one and three months, depending from the battery type and the environmental temperature.

Automatic start/stop function is implemented.

- Vibrec400

The digital vibration recorder Vibrec 400 (Figure 16f), made in Switzerland by the company Pfisterer-Sefag can be used to measure the vibrations of transmission line conductors as well as the wind velocity perpendicular to the line and the ambient temperature. Previous versions still in operation are the Vibrec 100 and Vibrec 300.

A special version with vibration sensor split from the main body is available for use on shield wire and small conductors (Figure 19).

A built-in serial interface (RS232) allows direct connection to a PC. A utility program Life400, supplied with the recorder, allows to set-up recorder parameters, read out, display and print of measured data. Moreover, it allows additional graphical presentations and conductor lifetime estimation. Time histories of the recorded bending amplitude signal are also available. A tri-dimensional matrix shows recording of amplitude/frequency data associated with the relevant wind speed.

The maximum measurable bending amplitude is 2 mm peak to peak in a frequency range 0.2 to 200 Hz.

Automatic start/stop function is implemented.

The autonomy of the recorder is about six months. The weight of the instrument is 1.7 kg.

**Figure 19**: Vibration recorder Vibrec 400 with split displacement transducer.

- Ribe LVR Vibration Recorder

This recorder, made in Germany, (Figure 16e) has basic design principles and performance characteristics similar to the ones of Vibrec 400. The vibration sensor is an opto-electronic type. No wind and temperature measurements are provided. A built-in serial interface (RS232 C) allows direct connection to a PC. A utility program, supplied with the recorder, allows to set-up recorder parameters, read out, display and print of measured data.

Automatic start/stop function is implemented.

- Scolar III

The Scolar III digital vibration recorder (Figure 16g), made by the company ALCOA, uses a rotary encoder as vibration sensor. Recorded data can be read with a standard audiocassette through a plug-in connector built into the unit and processed by a computer compatible with the data format. Read out time is 65 seconds. The recorder is equipped with a liquid crystal display (eight digits, one inch high) that can be read from the ground by means of binoculars or telescope. The display shows in sequence the content of each memory cell.

The maximum measurable bending amplitude is 2.54 mm peak to peak in a frequency range 1 to 100 Hz.

The autonomy of the recorder is about three months.

The weight of the instrument is 3.1 kg (6.1 kg with standard fittings).

- Eskom Vibration Recorder

Eskom has developed Eskom 2002) a new bending amplitude recorder using an accelerometer as vibration sensor. The performance of this recorder is not known in details yet; only a photo is available but it does not clarify how the relative displacement of the conductor with respect to the suspension clamp is measured.

The standardization of conductor vibration measurements provided the industry with the possibility to compare results obtained from different operating conditions.

The CIGRE SC22 WG04 (1979) recommended a method to determine the lifetime of conductors under the effect of Aeolian vibrations. The method makes use of the bending amplitude measurements and based on the Miner’s rule proposes calculation for the estimation of the lifetime of a conductor subjected to complex bending strain spectra.

Another CIGRE document (CIGRE SC22 WG11, 1995) was published to provide a comprehensive guide to vibration measurements on overhead conductors performed by means of bending amplitude recorders.

IEEE will publish an updated “Guide for Aeolian Vibration Field Measurements of Overhead Conductors”. The draft 22.0 of this Guide, dated June 2005, was made available for the purposes of this report.

#### 6.4.4.2 Data Sampling and Reduction

It has been common practice, since early application of the bending amplitude method, to perform measurements of a few seconds at regular intervals. The first analog recorders were timed to obtain one second recording every 15 minutes. The Hilda recorder allows a choice of a one and three seconds recording every 15 minutes, while the Scolar III can be set up for 1 to 4 seconds every 10 minutes. Other digital recorders allow the setting up of different measuring and waiting periods. The most used intervals are: ten seconds every fifteen minutes.

The digital recorders perform an on-line data reduction. The analog signal is sampled and reduced in digital form. The frequency and the amplitude of the vibration cycles are measured by suitable algorithms and stored in a memory matrix according to the procedure suggested by IEEE (1966). The matrix contains a number of frequency classes and amplitude classes (typically 10 x 21, 32 x 32 or 64 x 64) forming “cells” in which, each amplitude/frequency combination is stored as a single event.

Data relevant to temperature and wind speed perpendicular to the conductor, where available, are stored in separate arrays.

### 6.4.4.3 Recorder Positioning

Bending amplitude recorders are generally installed on the suspension clamp. The only exception is the Pavica recorder that is designed for direct installation on the conductor. The “lever arm” i.e. the distance between the sensor tip position (or the recorder position, in case of Pavica) and the last point of contact between conductor and the suspension clamp is preferably maintained at the standard position of 89 mm. This may not be possible, for example, on rubber lined clamps with helical rod attachment and on long suspension clamps for crossing spans.

When rods are used forming a cage around the clamp the sensor is located outside the cage area. Applications of the Pavica recorder along the helical rods and at their extremities as well as near spacer damper clamps (Figure 18) have been reported.

In the event that the lever arm is set up at a distance other than 89 mm, the measured amplitudes can be converted to the corresponding bending amplitude or bending strains values using the Poffenberger and Swart theory and considering the actual distance of the sensor tip from the suspension clamp.

Ontario Hydro and Alcoa provide correction curves for these cases. Other digital recorders can do these conversions during data elaboration by means of the relevant utility software.

IEEE suggest that the effect of the shift in lever arm distance may be approximately corrected by multiplying all recorded amplitudes by $(89/x_b)^2$ where $x_b$ is the actual lever arm distance.

It should be considered that the distance of 89 mm has been selected to get a measurable displacement while maintaining the sensor tip in a conductor zone which shape, during vibration, is governed by the stiffness effect alone and not by the inertial forces acting in the vibration loops.

Recommendation should be provided by the recorder manufacturers for an easy evaluation of the maximum allowable distance between the sensor tip and the suspension clamp for each type of conductor and relevant tensile loads.

### 6.4.4.4 Installation of the Vibration Recorders

The installation of the vibration recorders is a delicate operation. It must be performed or witnessed by an engineer with suitable experience in this field. Generally, these engineers do not install directly the recorder, unless the conductor under test can be reached with a suitable bucket truck. In most cases, the engineer instructs linemen to do it. The best solution is to arrange a conductor/clamp assembly for installation training at ground level. Each lineman should be invited to install the recorder on the assembly, during which the correct sequence of operations can be carefully explained.

Generally, the installation and removal of the live-line recorder are made during an outage of few hours. In some cases, the installation has been made on energized lines using hot sticks or the bare hand technique (Figure 20).

**Figure 20**: Installation of a vibration recorder on an energized line using the bare hand technique.

When available, the use of the automatic start/stop function is advantageous since it prevents the recording of the conductor movements during the linemen's operations and does not require the manual switching of the recorder after the installation and before the removal.

### 6.4.4.5 Measurements at Clamps Other Than Conventional Suspension Clamps

The bending amplitude method has been established for conventional metal to metal clamps. It proved to be reliable for clamps with mouth radii ranging form 0.4 mm to 152 mm (IEEE, 1966). Elastomer lined clamps and clamps with helical rod attachments do not behave like metallic clamps, and for them the relationship between bending amplitude and bending strains should be determined by laboratory vibration tests. The clamp manufacturers should provide recommendations regarding the optimum positioning of the vibration sensor and for the interpretation of the measurements.

For clamps incorporating elastomeric inserts the CIGRE guide (CIGRE SC22 WG11, 1995) suggests to use the PS formula, considering the center line of the suspension as the last point of contact between the conductor and the clamp *pending a more specific analysis of these supports*.

Dangerous dynamic bending strains can occur, also, at the clamps of other fitting such as dampers, spacers, warning devices and so on. Measurements of dynamic bending amplitudes at these clamps are not as simple as measurements at a suspension clamp. They require light recorders or a different measurement approach. Moreover the fatigue endurance limits of these specific conductor/clamp combinations as well as the relationship between bending amplitude and bending strain, if required, have to be determined by laboratory tests.

### 6.4.4.6 Measurement Inaccuracies

In bending amplitude measurements, there are several possible sources of measurement inaccuracies that should be duly considered and possibly reduced to a tolerable value.

Measurements errors can arise from the instrument performance e.g. calibration inaccuracy, linearity deviation, electrical noise including Corona, magnetic field interference, temperature effect on electronic components and so on. The recorder supplier should provide evidences of the good performance of each unit together with the individual calibration certificate and be available for maintenance and periodic re-calibration services.

Recorder attachments to suspension clamps can be the cause of measurement errors. These holders are specially designed for each type and size of clamp and should be as light as possible but rigid. Cases of resonance of the recorder holders at frequencies within the measurement range have been reported (Cigada and Manenti, 1995). It is advisable to perform laboratory vibration test on any type of recorder-holder assembly prior to the installation at site.

Severe imprecision can be determined by the incorrect positioning of the recorder. The distance of the vibration sensor from the clamp must be measured accurately. Errors in evaluating this parameter translate into errors in the resulting bending amplitude. Some recorders are provided with gauges for the correct positioning of the sensor.

Distortion of bending amplitude measurements can be determined by loss of mechanical contact between the sensor tip and the conductor or from an excessive reduction of the measurement range due to an incorrect adjustment of the sensor rest position.

The mass and moment of inertia of the recorder and relevant holders may influence the bending amplitude measurements. This effect has been analysed theoretically as well as tested in laboratory and in the field (Krispin, 1992, 1993), (Heics and Havard, 1993). The influence of recorder-holders mass depends on the vibration frequency and is smaller at low frequency. The phenomenon seems to be more pronounced with small conductors and large additional inertia. For these reasons, the recorders should be as light and compact as possible.

The data obtained by the bending amplitude recorders should be analysed considering the operating characteristics of the equipment and the variety of conductor motions, which occurred during the test period.

The memory matrix of the digital bending amplitude recorders shows quite often entries at frequencies well below the minimum Aeolian vibration frequency calculated using the Strouhal formula.

The data stored under the lower frequency intervals (0.2 to 3 Hz) may show high amplitudes but a limited number of cycles. These entries are generally due to:

- unsustained oscillations of the cable that can occur under the effect of high-speed wind gusts. In these cases, the cable is subjected to variable drag forces inducing transversal oscillations and its vertical component is detected by the vibration sensor of the recorder. Moreover, the effect of the transversal oscillations increases when the axis of the displacement transducer is deviate from its vertically position;
- movements of the linemen, when the recorder is switched on manually, after the installation on the line and switched off manually, before the removal;
- the presence of amplitude filters built in into the data reduction algorithm to avoid a great number of entries because of signal noises. In this case, vibrations with amplitudes below a specific value are ignored, but this may cause entries with higher amplitude and lower frequency in respect to the actual vibration parameters, as explained in (Sefag, 2003).

Such low frequency entries have generally no influence on the calculation of conductor lifetime, for their limited number of cycles. They are not considered, if they exceed the maximum allowable bending amplitude, when this criterion is used for the evaluation of vibration severity.

### 6.4.4.7 Test Locations

Vibration measurements are generally performed on few spans of a transmission line. When, in some locations of the line, there is evidence of conductor strand or fitting failures or doubt of possible damages the measurements are taken on these points.

As final acceptance test of the conductor damping system, the vibration recording is generally performed on one or two span of the line, in which the greatest exposure to the vibration-inducing wind can be anticipated. Those are, generally, the longest suspension spans, with the highest supports, which are stretched in flat desert areas or in open and plain lands, particularly near water, with low and sparse obstacle (trees, buildings, etc.), in areas where the presence of winds perpendicular to the conductors and in a wide range of speeds are likely to occur. Very long spans crossing rivers, channels and valleys, designed with structural characteristics and parameters different in respect to the rest of the line, are tested separately.

#### 6.4.4.8 Test Period

The test period is chosen in accordance with the purpose of the measurements.

If the purpose of the test is to measure the maximum bending amplitude, the IEEE standardization (IEEE, 1966) suggests a period of minimum two weeks.

For final acceptance test of the conductor damping system at the end of the line construction, most of the utilities' specifications require a minimum period of one month.

For comparative analysis of different damping systems the test period is not important providing to test simultaneously the units under examination.

To obtain results that are statistically meaningful, a minimum of three months period is deemed necessary(CIGRE SC22 WG11, 1995).

In areas where seasonal conditions change significantly e.g. different wind characteristics, different ambient temperature range, changing in ground roughness due to agriculture and snow, measurements shall include these different conditions. In alternative, the test period shall be established considering the yearly distribution of wind and ambient temperature in order to get, during the tests, the most severe meteorological conditions.

In any case, the vibration measurements should be associated with wind velocity measurements to verify that during the test period the whole range of wind velocities required to excite significant Aeolian vibrations was present. If not the test should be repeated.

#### 6.4.4.9 Interpretation of Recorded Data

The bending amplitude measurement is, with no doubts, the easiest way to investigate the causes of damages already found or to resolve doubts determined by "early warnings". However, regarding the reliability of these measurements in determining the risk of future fatigue damage, the following must be considered.

The inherent concept of these measurements is to take for a few weeks, on one or few spans, samples of the conductor vibrations. In general, the measurements are performed for about ten seconds every fifteen minutes for a period of one month. This means that information is collected for about 1 % of the time elapsed in a month, which covers about the 0.002% of the transmission line life.

Such information is supposed to establish:

1. If the conductors under tests, in the spans where measurements have been made, will face or not fatigue risks during his expected service life (30 to 50 years).
2. If the conclusion drawn for those conductors and spans can be extended to all the other spans of the line.

It must be pointed out that to achieve reliable conclusions, it is necessary that the vibration samples taken do really represent, at least, the predominant conditions that will exist on that line during its service life. Therefore, the correct choise of the test locations and the definition of the test period and duration are of primary importance.

#### 6.4.4.10 Evaluation Criteria

The following criteria are commonly used to assess the vibration severity on transmission line conductors:

- IEEE maximum allowable bending strain
- EPRI endurance limits
- CIGRE WG 22-04 method

The IEEE Task Force (IEEE, 1966), suggested, with the bending amplitude method, a general evaluation criterion based on a maximum allowable bending strain: *"The maximum bending strain that can be tolerated in ACSR conductors without eventually inducing fatigue damage can not yet be stated precisely……. It is speculated that the value of 150 $\mu$inch/inch (microstrains) peak to peak, which is given here only as a guide, is somewhat conservative and the strains of the order of 200 $\mu$inch/inch (peak to peak) may well prove to be safe".* This criterion proved to be rather conservative. However, many utilities, in many different countries, still require this procedure for the assessment of vibration severity as acceptance tests of the damping systems of new lines.

The EPRI book (Rawlins, 1979) provides, for various types of conductor, the values of the bending amplitude or bending stress/strain corresponding to "endurance limits". They are valid for conductors supported by rigid metallic clamps with smooth internal profile. The list of conductors includes mainly ACSR conductors, but also some AAAC, steel and copper conductors are considered.

The EPRI book also suggest that a general endurance limit for multi-layer ACSR conductor at the bending amplitude value of 9 mils (0.23 mm) could probably be used, as well as a bending stress value of 8.5 MPa. These limits can be applied to homogeneous aluminium conductors of 1350 and 5005 alloy also, while for 6201 and similar alloys a lower limit of 5.7 MPa is suggested.

Endurance limits for other conductors and for clamps other than metallic suspension clamps are not available in literature. Bending amplitude measurements on combinations of these conductors and clamps can be evaluated only when the actual endurance limits have been defined by means of laboratory tests.

The evaluation of the conductor fatigue danger based strictly on the maximum allowable bending amplitude or bending strain corresponding to the "endurance limit" may be considered excessively cautious. In fact, these limits can be exceeded up to a certain level and for a limited number of times with no practical effect on the conductor integrity. To reduce the severity of the method, some concessions are granted. For example, the following empirical limits are proposed in the revised version of the IEEE Guide (draft 22.0, June 2005) as "a widely used set of criteria":

- The measured bending amplitude may exceed the endurance limit for no more than 5% of the total cycles.
- No more that 1% of the cycles may exceed 1.5 times the endurance limit.
- No cycles may exceed 2 times the endurance limit.

The CIGRE WG 22-04 method for the evaluation of the lifetime of aluminium based conductors considers the cumulative effect of all the recorded vibration cycles. For this scope, the bending amplitude data, stored in the recorder memory matrix, are converted in bending stresses, then extrapolated to one year. Finally, the data are presented as an "accumulated stress curve" showing for each stress level $\sigma_i$, the number of cycles, $n_i(\sigma_i)$, to be expected in one year. Using the Miner's Rule, the above curve is compared with a fatigue curve worked out by CIGRE WG 22-04 known as "safe border line".

The conductor lifetime estimation can be useful for expressing vibration severity and in comparing alternative damping systems. However, considering the remarkable scatter in conductor fatigue characteristics, conductor life expectancy should be considered to have only qualitative significance. The following chapter presents a broad discussion to illustrate these issues.

### 6.4.4.11 Survey on the Evaluation Criteria

A survey on the evaluation criteria adopted by the industry for the assessment of vibration severity on transmission line conductors has been performed reviewing 80 Technical Specifications issued by the main Utilities worldwide in the past 20 years.

The survey shows that, for the evaluation of the vibration severity:

- 59% of the Specifications adopt the bending strain as endurance limits
- 16% adopt the bending amplitude endurance limits proposed by EPRI
- 6% adopt the CIGRE method for the evaluation of the lifetime
- 19% do not specify any criterion

Among the Utilities adopting the bending strain endurance limits:

- 27% prescribe 150 microstrains peak to peak
- 18% prescribe 200 microstrains peak to peak
- 4% prescribe 247 microstrains peak to peak (corresponding to 8.5 Mpa)
- 51% prescribe 300 microstrains peak to peak

It was evident during the survey that, in the industry, evaluation criteria of vibration severity are frequently prescribed with no consideration of whether the relevant reference limits available in literature are applicable or not to a specific conductor clamp combination. For example, endurance limits for aluminium based conductors in metallic clamps have been adopted for steel shield wires or OPGW or for measurements taken at the spacer clamps.

Laboratory tests are required when no specifications are available. It is expensive and time consuming to determine the actual endurance limits of a conductor/clamp system.

It is evident that many utilities are not aware of the recent development and, what is more, of the inherent limitation in the assessment of conductor vibration severity.

CIGRE SCB2 WG11 (previously SC22) and IEEE WG on Conductor Dynamics are committed to provide the industry with detailed and comprehensive guides on the subject.

# 7. Evaluation of Conductor Residual Life

Prediction of the remaining life of a conductor that has been exposed to damaging fatigue stresses is of interest to guide decisions on whether repair or replacement of conductors, or upgrading vibration protection, will be necessary and, if so, when. Cumulative damage theory is used for the estimation of residual life in fatigue.

Use of cumulative damage theory in connection with overhead conductors was proposed by Steidel (1960), and has since received intensive study within CIGRE SC22, leading to specific recommendations for its application (CIGRE SC22 WG04, 1979), as mentioned previously. In addition, some laboratory fatigue tests have been performed to test the precision of the leading cumulative damage model, Miner's Rule, in predicting remaining life in overhead conductors (Brunair et al., 1988), (Goudreau et al., 2003).

Miner's Rule derives from the Palmgren-Miner Law which considers that a material will fail in fatigue when the accumulated damage from all stress cycles reaches a certain value. The amount of damage each cycle causes depends on the amplitude of its stress. Under Miner's Rule, the amount of damage a cycle of stress $\sigma_i$ causes is equal to $1/N_i$, where $N_i$ is fatigue life when $\sigma_i$ is the sole fatigue-inducing stress. The Rule states that the total damage under exposure to $m$ different stress amplitudes is,

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \qquad (1)$$

Where:

$n_i$ is the number of cycles at $\sigma_i$ in the exposure.

Failure is expected to occur when the damage parameter $D$ exceeds 1. Equation (1) is the linear form of Miner's Rule, and is the most widely used.

Application of Miner's Rule requires two elements of information:

1. The fatigue characteristics of the conductor/clamp system where fatigue may be expected, that is, some form of the function $\sigma(N)$ or $N(\sigma)$ representing the fatigue or Wöhler curve.
2. Data on the exposure of that system to fatigue-inducing stresses in the field, that is, the distribution $n_i(\sigma_i)$ of the accumulated cycles at each of the stress levels experienced in the field. Such data are usually expressed as cycles, or megacycles, per year.

In the case of overhead conductors, the fatigue-inducing stresses at the interstrand contacts are not accessible to direct measurement. Instead, they are estimated from some closely related parameter that can be measured. For overhead conductors that parameter is almost universally the bending amplitude $Y_b$ (see Section 6.4).

## 7.1 Endurance Capability

Chapter 4 reviews the laboratory determination of fatigue characteristics of overhead conductors, and Rawlins (1979) had described how several broad classes of conductor types could be represented in only a few fatigue curves. For example, all multi-layer ACSR conductors can reasonably be represented by a single such curve when the stress is based on bending amplitude.

However, different versions of the curve arise depending upon: the definition of failure; the data source it is based on; and the intended application. Figure 21 shows several curves applicable to non armoured multilayer ACSR where stress was determined from bending amplitude. Three different data sources are represented, as indicated by the references. The top three curves would be used to predict when actual fatigue may be expected. The first two show the effect of how failure is defined relative to the number of broken strands. The difference between those two curves and the "Log mean life" reflects the difference in fatigue test data bases.

The fourth and fifth curves of Figure 21 are intended to define some level of assurance about the integrity of the transmission line. The "Safe Boundary" was constructed to fall below the envelope of a number of sets of fatigue tests of various types relative to overhead conductors. The "Safe Limit" curve uses data source (Rawlins, 1979) to define the locus where there is 95% probability of survival without any strand failure in an individual test. Judgement is required in the choice of which of these curves, if any, to use in a particular investigation.

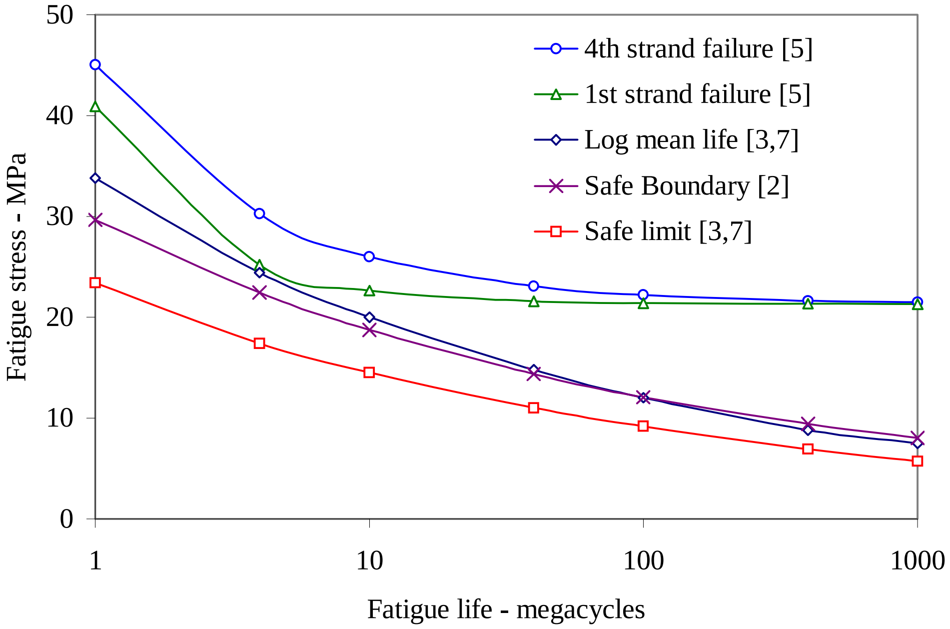


**Figure 21**: Fatigue characteristics of multilayer ACSR without armor rods (several sources). Symbols are curve identifiers, not data points.

## 7.2 Fatigue Exposure

Estimates of the exposure function $n_i(\sigma_i)$ can be based upon knowledge of the vibration amplitudes that have occurred up to the time the condition of a line has been brought into question (see Chapter 6). If the line under consideration has not yet been erected, the function may be based upon previous recordings from an existing similar line, or upon analytical methods such as the energy balance principle. In the latter case, reference should be made to (CIGRE SC22 WG11, 1998) relative to the accuracy to be expected in predicted vibration levels.

For purposes of calculating (1), it is common practice to express $n_i(\sigma_i)$ as the total numbers of stress cycles falling in successive increments of stress, as illustrated in Figure 22.

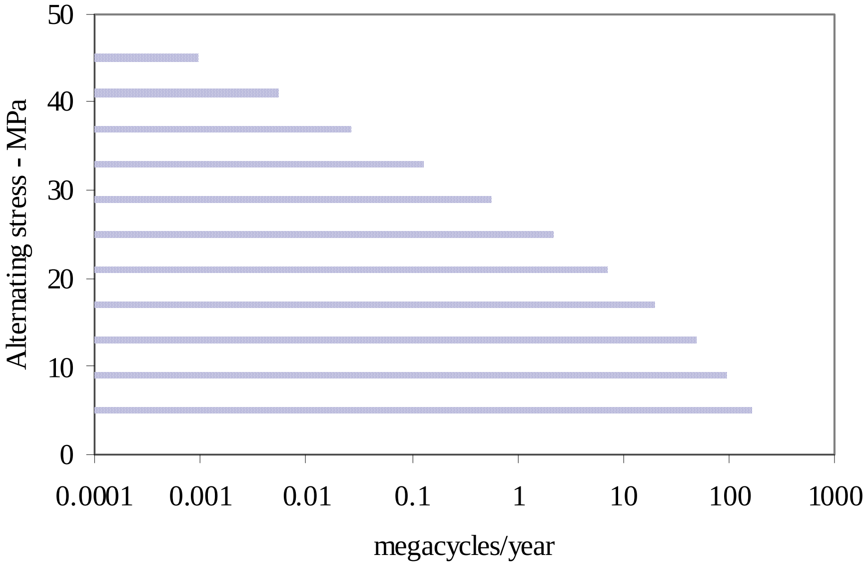


**Figure 22**: Illustration of fatigue exposure function.

## 7.3 Discussion

It should be remarked that exposure to stress cycles of various amplitudes occurs in various types of structure. A great deal of study and research has been devoted to the problem of assessing expected fatigue performance in such cases. For some structures such as aircraft components, well-supported technology has been developed for dealing with the problem. This technology generally employs laboratory fatigue tests in which stress waveforms typical of those that occur in actual service are applied to the structure. This procedure is used because damage accumulation is influenced not only by the statistical distribution of the stress amplitudes but also by the sequence in which they are applied. Applying the cycles in blocks of constant amplitude yields different results from applying them in random order, or in the order imposed during actual service.

Such technology has not been developed for conductor vibration, basically because of the difficulty and expense of conducting laboratory fatigue tests of conductor under the complex waveforms that occur in Aeolian vibration. This is a source of uncertainty when trying to apply cumulative damage theory to Aeolian vibration fatigue. There are other sources of uncertainty.

1. There is considerable scatter in results among laboratory fatigue tests of conductor that have been run under identical conditions at constant stress amplitude (Rawlins, 1979), (Cloutier et al., 1999). Fatigue life ranges of 2:1 are common in small groups of repeat tests, while 5:1 and greater ranges have been found in larger groups. Thus, fatigue life requires a statistical description even under the most-controlled conditions. That requirement was confronted in (Hardy and Leblond, 2001), resulting in the fifth curve of Figure 21.
2. Laboratory evaluation of the precision of Miner's Rule as applied to ACSR has shown average deviations from expected fatigue life by factors as large as 1.7 (Brunair et al., 1988). Deviations in some individual tests were obviously larger. The test program applied the stress cycles in blocks of constant amplitude, rather than in random sequence for the reason noted above. It has been argued (Cardou et al., 2002) that the experimental value of $D$ in Equation (1) should be considered a random variable, rather than assigned the constant value 1.
3. Where recordings on operating lines provide the basis for describing the past and predicting the future stress history, uncertainty arises from the need to extrapolate a relatively brief test period to cover the economic lifetime of an entire line. For example, Figure 23 shows results of a year-long study of vibration of a span in North Dakota (Poffenberger and Komenda, 1971). The average rate of strain cycle accumulation over the year (b) is contrasted with the rates estimated from the 2-week test periods showing the least (a) and the most (c) vibration activity. For this case, the most damaging strain levels would be those greater than 150 $\mu\varepsilon$. In that range, predicted accumulated damage for curve (a) would be only a small fraction of that for curve (c). It would therefore result in an estimated residual life many times that of (c).
4. Again, when estimates of fatigue damage are based on field recordings, data from only one or a few recording locations must be relied on to characterize the condition of an entire line. Some sense of the variability of fatigue exposure within a line can be obtained from Figure 24. The figure shows accumulated strain cycle curves from strain gages located at the three suspension clamps at a single tower. For two of the clamps, strain gages were located on both sides. The recorded data shows that there is a large dispersion in accumulated cycles per year, even among five locations which are nominally identical, and are at the same tower. At the 300 $\mu\xi$ (21 MPa) level, the dispersion covers a 10-to-1 range. The dispersion over the entire population of suspension clamps in the line would obviously be greater.
5. When analysis using the energy balance principle provides the basis, inaccuracies in background data on wind energy and conductor self damping, along with assumptions and approximations associated with the analytical procedure, introduce large uncertainties into estimates of expected stress amplitudes (CIGRE SC22 WG11, 1998).

Because of these uncertainties, estimates of absolute values for the residual life of conductors in years should be viewed as indicative approximations. However, estimated residual life can serve as a parameter for expressing vibration severity and in comparing alternative damping systems. This view reflects the cumulative effect of the foregoing and therefore, until progress is made, numerical evaluation of residual life must simply be viewed with caution (Rawlins, 2004).

There is a need for an integrated statistical approach to the residual life question. The need is dictated by the sources of uncertainty described above. The following variables are inherently random:

1. Fatigue life of a single specimen under constant fatigue stress, $N(\sigma)$;
2. The distribution of fatigue stresses upon each support in an actual line, $n(\sigma)$;

3. The distribution of the exposure functions $n(\sigma)$ over the population of supports in a line, due to differences in span length, terrain and other factors;
4. Possibly, the value of the damage parameter $D$ that corresponds to failure, over the population of supports.

Thus, the output of an assessment of residual life must be presented in statistical terms.

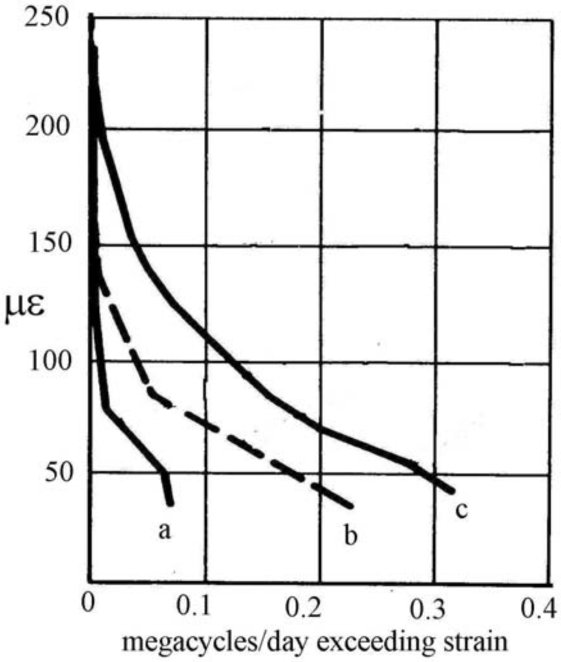


**Figure 23**: Megacycle accumulations in field recordings on a 230 kV line in North Dakota – megacycles/day exceeding given levels of microstrain (Poffenberger and Komenda, 1971).
a) Test period showing lowest activity; b) Average over a year; c) Test period showing greatest activity.

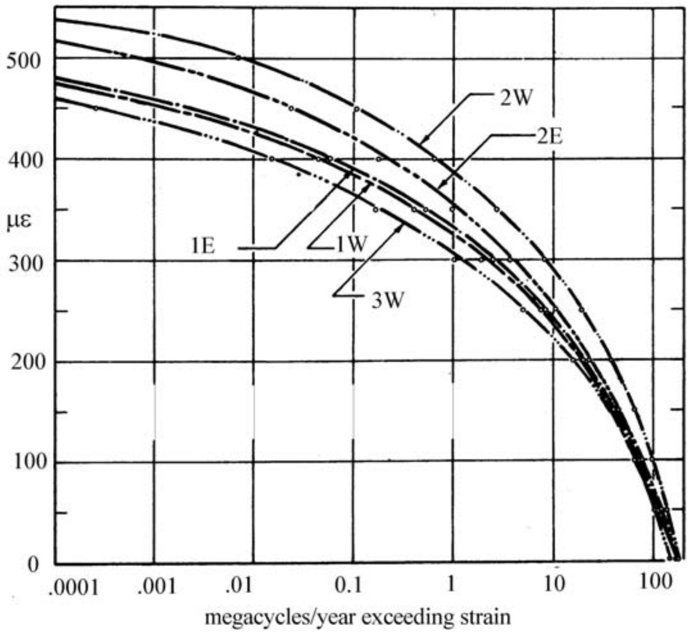


**Figure 24**: Megacycle accumulations in strain recordings at a tower in a 345 kV line in northwest United States. Chukar ACSR. 1W indicates west side of suspension clamp on Phase 1; 2E indicates east side of suspension clamp on Phase 2; etc.

# 8. Conclusion

This report concludes the update of present knowledge on the fatigue endurance capability of conductor/clamp systems.

The solutions retained until now to determine the fatigue endurance capability of conductor/clamp systems is a good example of the sound application of an engineering approach to solve a complex problem. An idealized stress related to vibration intensity, deduced from a simple model and that correlates well enough with conductor fatigue performance, was proposed to serve as a fatigue sensitivity index for a range of conductor strands and sizes of the same class. Future work encompassing notions of contact mechanics and of fracture mechanics should lead to the proposition of a model representing more closely the fretting fatigue mechanism actually causing the strand failures. A lot remains to be completed to model satisfactorily the fretting fatigue mechanism induced by conductor vibrations at supporting clamps.

The discussion in Chapter 7 exposes important limitations in our ability to define precisely the actual conditions prevailing in the field in order to correctly apply the present knowledge on conductor/clamp systems fatigue endurance capability. The challenge is to be able to present the output of residual life calculations in a way that pictures the variances and variations realistically, but in a manner that still is useful to utilities in making decisions on line inspection, assessment, repair and refurbishment.

It is important to still improve our knowledge of the conductor fatigue phenomena and the means to propose to the transmission line engineer to enable him to increase the reliability of overhead transmission lines.

However enough knowledge is now available for conducting in situ field recordings and to usefully contribute to the assessment of the condition of lines exposed to Aeolian vibrations. A Technical Brochure is under preparation to propose to the transmission line engineers *engineering guidelines* based on present knowledge. The document will also include a thorough review of the future work required to adequately comprehend and specify remedies to the fatigue endurance capability of conductor/clamp systems.

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# 10. Appendix – List of Formulas

Tensile stress in a wire:

$$\sigma_T = \frac{E_w \cos^2\beta_\ell}{\sum_w E_w A_w \cos^3\beta_\ell} T \tag{2}$$

Minimum bending stiffness of the conductor:

$$EI_{min} = \sum_w E_w \frac{\pi d_w^4}{64} \tag{3}$$

Maximum bending stiffness of the conductor:

$$EI_{max} = EI_{min} + \sum_w \frac{n_\ell}{2} E_w A_w r_\ell^2 \cos^3\beta_\ell \tag{4}$$

(Note: The cos β factors in the above equations, arises from the fact, that because of its helicity, the wire length is longer than the corresponding conductor axis length. These factors are for the “normal” lay angles (say up to 15°) close to 1 and are usually neglected)

Poffenberger-Swart formula:

$$\sigma_{PS} = \frac{E_a\ d_a\ p^2}{4\left(e^{-px_b} - 1 + px_b\right)}\ Y_b \tag{5}$$

with

$$p^2 = \frac{T}{EI_{min}} \tag{6}$$