363

RELIABILITY BASED CALIBRATION OF FOUNDATION STRENGTH FACTOR USING FULL-SCALE TEST DATA

A GUIDE FOR DESIGN ENGINEERS

Working Group
B2.07

December 2008

cigré

WG B2.07

Reliability based calibration of foundation strength factor using full-scale test data
A guide for design engineers

Prepared by Asim Haldar, on behalf of the WG

WG B2.07 members

Neil Cuer, *WG Convenor* (UK) and Eddie O'Connor, *Secretary* (Ireland)

Regular Members
Mike Vanner (UK), Anthony Digioia (USA), Branko Zadnik (Slovenia), Jean-Pierrre Chouteau (Canada), Asim Haldar (Canada), Eugene Dembicki (Poland), Aureo Ruffier (Brazil), Sylvain Muyard (France), Mike Buckley (Ireland), Maurizio Leva (Italy), Michael Pietschke (Germany), John P. Sivertsen (Norway), Boud Boumecid (UK), Toshiyuki Ozaki(Japan)

Corresponding Members
Kari Sahla (Finland), Graeme Paterson (Australia), N Sabri (Switzerland), Willem Combrinck (South Africa), Patrick Eychenne (France)

ISBN: 978- 2- 85873-050-6

TABLE OF CONTENTS

ABSTRACT

This technical brochure presents a reliability based calibration methodology of foundation strength factor using full-scale test data; it is shown that the simple model developed by Buckley (1994), is a subset of the general linear regression model provided the intercept is zero and the residual has a non constant variance. A simple statistical test is presented to check for zero intercept condition and a graphical method to test for variance trend of residuals. A flow diagram provides the necessary steps for the calibration process. A foundation design problem is also presented as an example to demonstrate the methodology

SOMMAIRE

Ce rapport présente une méthode de calibration du facteur de résistance des fondations basée sur une approche fiabiliste en utilisant des données d'essais en vraie grandeur. On montre que le modèle simple proposé par Buckley (1994) est un cas particulier du modèle de régression linéaire général, lorsque la résistance à l'origine est nulle et que la variance des résidus est variable. Un test statistique simple permet de déterminer la condition à l'origine (nul ou non nul) tandis que la tendance de la variance des résidus est vérifiée par méthode graphique. Le processus de calibration est présenté sous forme de réseau de décision qui détaille les principales étapes. Finalement, la méthode est illustrée à l'aide d'un exemple de conception type.

Key Words:

Reliability based design, probabilistic design, foundation strength factor, undercut foundation, uplift capacity, calibration, linear regression model, extrapolation, correlation, exclusion limit, characteristic capacity, frustum cone angle, shear model, confidence interval, prediction interval.

Mots clé:

Conception basée sur la fiabilité, conception probabiliste, facteur de résistance de fondation, fondation en redan, résistance à l'arrachement, calibration, modèle de régression linéaire, extrapolation, corrélation, limite d'exclusion, résistance caractéristique, angle de pyramide inversée, méthode du cisaillement vertical, intervalle de confiance, intervalle de prédiction.

ACKNOWLEDGEMENTS

The author would like to acknowledge the contributions of the following individuals during the preparation of this report. The author would like to thank Electricity Supply Board (ESB), Ireland for providing the uplift test data on undercut pad and chimney foundation. The author would also like to thank Dr. Jan Rogier from Belgium and Dr. Ruy Menezes from Brazil for reviewing the report and providing valuable comments. Finally, the author sincerely appreciates the long term support provided by the Senior Leadership Team of Newfoundland and Labrador Hydro.

WG 07 members	**Non members**
Mr. Jean-Pierre Chouteau (CA)	Professor A.H. S. Ang (USA)
Dr. Mike Buckley (IR)	Professor Gary Sneddon (CA)
Mr. Mike Vanner (UK)	Dr. Enrique Bazans (USA)
Mr. Neil Cuer (UK)	Ms. Cathy Murphy (CA)
Dr. Tony Digioia (USA)	Mr. Kyle Tucker (CA)
Mr. Eddie O'Connor (IR)	Ms. Connie Hilliard (CA)
Mr. Boud Boumecid (UK)	Mr. Kyle Tucker (CA)

EXECUTIVE SUMMARY

1.0 Introduction

Overhead line support structures are subjected to the effects of meteorological loads as well as to the longitudinal loads due to broken conductor and unbalanced ice shedding. Forces transmitted at the foundation level are often uplift (pullout) forces with small transverse shear forces for tangent lattice towers or large overturning moments with corresponding shear forces for single pole, angle or dead end structures. Foundation types are normally spread footings and steel grillages for lattice towers and drilled shafts for single poles, angle or dead end structures. Figure 1 depicts a typical undercut pad and chimney foundation under uplift load.

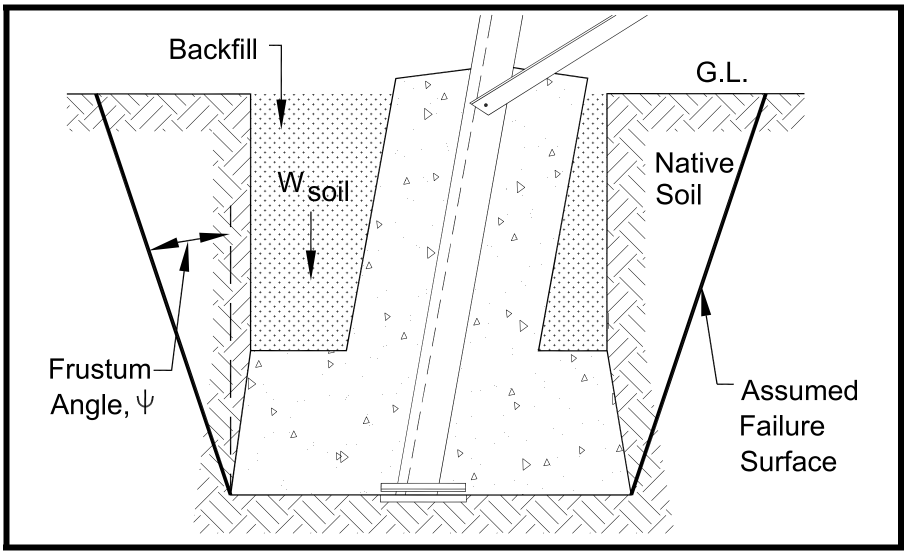


Figure 1 - A typical foundation under uplift load

Since the meteorological loads are probabilistic in nature, the transmitted forces acting on the foundation are also probabilistic and are subjected to a high level of uncertainties. In addition, spatial variation of soil strength parameters also contributes a high level of uncertainty. Therefore, the foundation design must consider all these uncertainties quantitatively.

2.0 Objective

The primary objective of this study is to present method(s) of undertaking the calibration of the theoretical geotechnical design model and hence determine the characteristic capacity of the foundation, based on full-scale test data. To develop this calibration methodology, a framework to determine the prediction interval of the foundation test capacity based on a limited number of full-scale test data is developed using a linear regression model. The prediction interval provides a confidence on a new test capacity value predicted given a single geotechnical design capacity value, R_n. The lower bound of the prediction interval is directly linked to the foundation strength factor, ϕ_F which can be used in reliability based design equation for a single foundation component. The foundation strength factor with an exclusion limit is defined as the ratio of the lower limit of the prediction interval (predicted test capacity) with a certain confidence level (analogous to characteristic capacity) and the corresponding geotechnical design capacity. A 10% exclusion limit implies that the confidence level on the prediction is 90%.

3.0 Methodology

To develop a reliability based calibration methodology for foundations, the four steps identified here are followed:

- Reliability based design equation (IEC 60826, 2003)
- Assessment of foundation geotechnical design capacity
- Assessment of foundation test capacity from a field test programme
- Data analysis and calibration based on $\overline{m}$ - model (Buckley, 1994) and linear regression model

An example problem is presented to illustrate the methodology which is followed by summary and conclusions.

3.1 Reliability Based Design Equation (IEC 60826, 2003)

IEC 60826 provides a semi probabilistic design equation in terms of the limit load acting on a component and the strength of the component. The design equation (1) relates the characteristic capacity, R_C to the appropriate load effects, Q_T having a specified return period of T-years with a global strength factor, ϕ_R.

$$\phi_R R_C \geq Q_T \qquad (1)$$

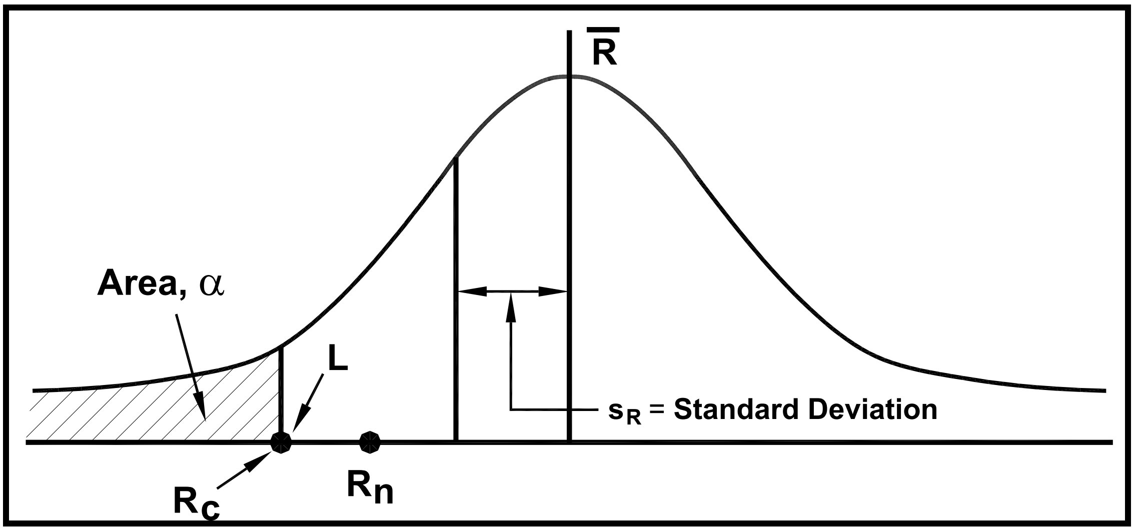


Figure 2 - Characteristic capacity of the test data

The global strength factor ϕ_R depends on strength coordination factor ϕ_S, number of components subjected to the maximum load intensity ϕ_N and quality of installation ϕ_Q; it also includes the characteristic strength factor ϕ_C, which takes into account the variation between the characteristic capacity for an exclusion limit of e% and the capacity R_C with an exclusion limit of 10%. For this study, the calibration is undertaken for a typical foundation only (one component) and therefore, the global strength factor, ϕ_R is not used explicitly.

Figure 2 depicts the characteristic capacity based on e% exclusion limit for a normal distribution. For a 10% exclusion limit the characteristic capacity, $R_{C,10\%}$ is

$$R_{C,10\%} = \overline{R}\ (1 - k_\alpha\ v_R) \tag{2}$$

where $\overline{R}$ and v_R are the mean and the coefficient of variation (ratio of the standard deviation, s_R and the mean value, $\overline{R}$) of the test capacity values for a finite sample size, n. The term k_α is a measure of how far the point "L" is from the mean to obtain the 10% exclusion limit (k_α =1.28). For a finite sample size, n < 30, k_α should be corrected based on the Student t-distribution. The exclusion limit 10% implies that the characteristic capacity is for the 90% confidence level and that there is a 10% probability that the capacity could fall below this value.

Within the framework of IEC 60826 (2003), the characteristic capacity R_C should be related to the geotechnical design capacity R_n and this could be done through calibration. If the full-scale test data is available, the appropriate foundation strength factor ϕ_F

$$\phi_F = \frac{R_C}{R_n} \tag{3}$$

can be used to relate the geotechnical design capacity R_n to the characteristic capacity R_C with a defined exclusion limit (5% for ASCE and 10% for IEC 60826). The geotechnical design capacity R_n in figure 2 is derived from analytical and/or semi empirical relationship or numerical models. Depending on the calibration assumptions, the relationship could be linear or non linear.

3.2 Assessment of foundation geotechnical design capacity

The geotechnical design capacity R_n of an undercut pad and chimney foundation as shown in figure 1 is obtained from the frustum cone model. In this particular model, it is assumed that an inverted frustum starts at the base of the foundation and slopes at an angle, ψ to the vertical to the surface. The weight of the soils within this frustum and the foundation weight itself resist the uplift force.

3.3 Assessment of foundation test capacity

Once the geotechnical design capacity R_n has been determined, the foundation designer is given guidance on how to undertake a full-scale foundation test programme. Several steps need to be considered before a full-scale foundation test programme can be commenced. These include: (a) selection of the test site(s), (b) selecting the applicable foundation geotechnical design model(s) to determine the foundation's geotechnical design capacity, (3) subsurface investigation to determine the various geotechnical strength parameters for the foundation design model(s), (4) sample size requirement for the test programme and finally the test results in terms of a typical load versus displacement curve (figure 3).

Subsequently, a reference is made to IEC 61773 which outlines a number of methodologies to determine the foundation's test capacity in terms of damage limit as well as the ultimate limit capacity based on the test result. A review of the information presented shows that the estimated ultimate capacity could vary considerably depending on the method selected. However, the foundation designer can also choose a test capacity (damage limit capacity, IEC 60826, 2003) based on a prescribed displacement (subjective). For a pad and chimney foundation used in the example, load at a 10 mm displacement is defined as the test capacity (Buckley, 1994).

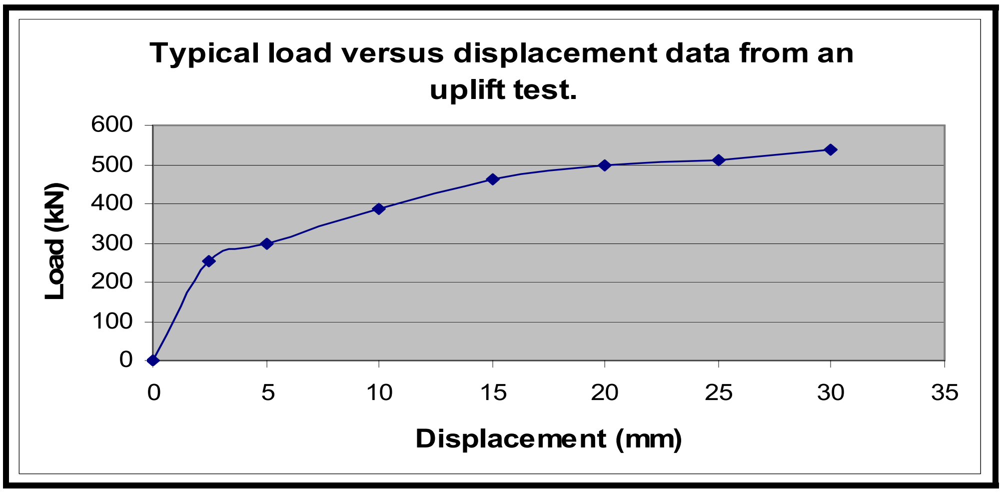


Figure 3 - Typical load versus displacement plot

3.4 Data analysis and calibration based on $\overline{m}$ - model and linear regression model

3.4.1 General

The uncertainty in the prediction of the characteristic capacity R_C is introduced through two factors: (i) model error and (ii) data error due to a finite sample size n. However, there is a need to assess both the quality of the data set (correlation coefficient, r) and its influence on the prediction interval in order to develop a general methodology.

The correlation coefficient, r of the dataset is determined from

$$r = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{(n-1)s_x s_y} \quad (4)$$

where $\overline{x}$ and $\overline{y}$ are the mean values and s_x and s_y are the standard deviations of the geotechnical design capacity values x_i and the test capacity values y_i respectively. A low correlation value (r^2) would imply that there is no linear relationship between the foundation test capacity and the geotechnical design capacity values. Table 1 provides some guidance on the correlation value r^2 and the linear association between x and y. Figures 4a and 4b present two datasets with very different correlation properties.

Table 1 – Suggested ranges of correlation value

Correlation Value, r^2	**Linear association between x and y**
Low , $r^2 \le 0.3$	weak
Moderate $(0.3 \le r^2 \le 0.8)$	moderate
High, $r^2 > 0.8$	strong

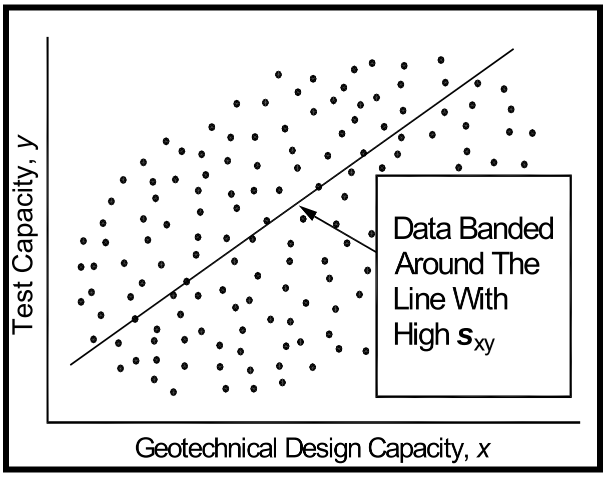


Figure 4a - Low correlation

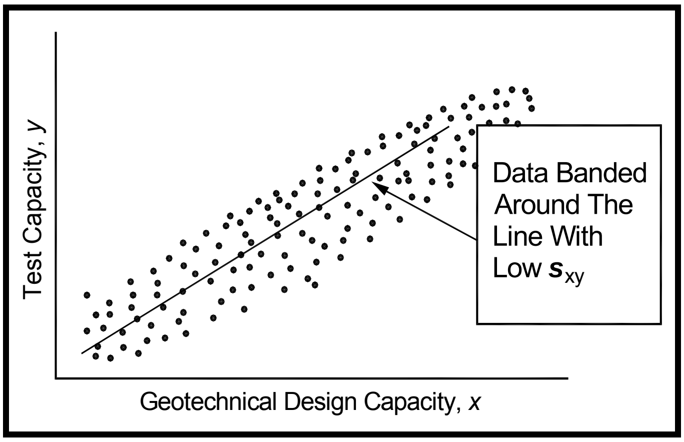


Figure 4b - High correlation

3.4.2 $\overline{m}$ - model - no correlation (repeated test data at one specific test site)

Since $x_i = \overline{x}$, for the repeated test data at one specific site, the correlation coefficient r=0.0 (equation 4) indicates that the data is unique to a specific test site. In this case, the foundation strength factor ϕ_F can be obtained directly from equation (2) dividing R_c by the model predicted geotechnical design capacity R_n (figure 2). The foundation strength factor cannot be extrapolated to other sites, unless the designer can ensure that the sites have very similar soil characteristics and the foundation has a similar aspect $(\frac{Depth}{Base\ Width})$ ratio.

3.4.3 $\overline{m}$ - model - with correlation (scattered test data along a line route)

Once the user has determined the test and geotechnical design capacities for the various test sites (s) and foundation sizes, the user can choose a simple calibration model based on $m = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity}$ ratio(s) following (Buckley, 1994). Using this approach, it can be shown that the foundation strength factor, ϕ_F for a large sample size (n >30) is

$$\phi_F = \frac{R_C}{R_n} = \overline{m}\,(1- k_\alpha * v_m) \qquad (5)$$

where

$\overline{m}$ = mean value of the m-values for a finite number of tests and

v_m = coefficient of variation of the m-values= $\frac{s_m}{\overline{m}}$ (s_m = standard deviation)

However, to use this model, the user needs to ensure that the correlation of the data is high (Table 1) and the coefficient of variation of the model error, v_m is low. Provided these two conditions are met, it can be shown that the $\overline{m}$ -model is a subset of the general linear regression model (figure 5). The specific conditions are: (1) zero intercept and (2) non constant variance of residuals, where the residual error values vary linearly with geotechnical design capacity values (x-values). The residual error value ε defines the difference between the actual test capacity and the estimated mean test capacity value from the linear regression model. For other situations where this is not the case, the foundation designer should undertake a more detailed analysis based on a linear regression model; either to ensure that the intercept effect can be neglected or alternatively to improve the theoretical design model. If the intercept effect cannot be neglected, then an alternate methodology based on a linear regression model is also presented illustrating how to include this in the analysis and to estimate the predicted capacity with the required confidence interval.

In the linear regression model, the designer determines the slope, intercept and the variance of residuals. Figure 5 depicts this information. Figure 5 also presents the confidence interval on the estimated mean test capacity value, as well as, the prediction interval on a new test capacity value given a geotechnical design capacity value. This figure is very similar to figure 2, except that in figure 2 the characteristic capacity is computed for the test data only; while, in figure 5 this is determined from the linear relationship between the test capacity and geotechnical design capacity values.

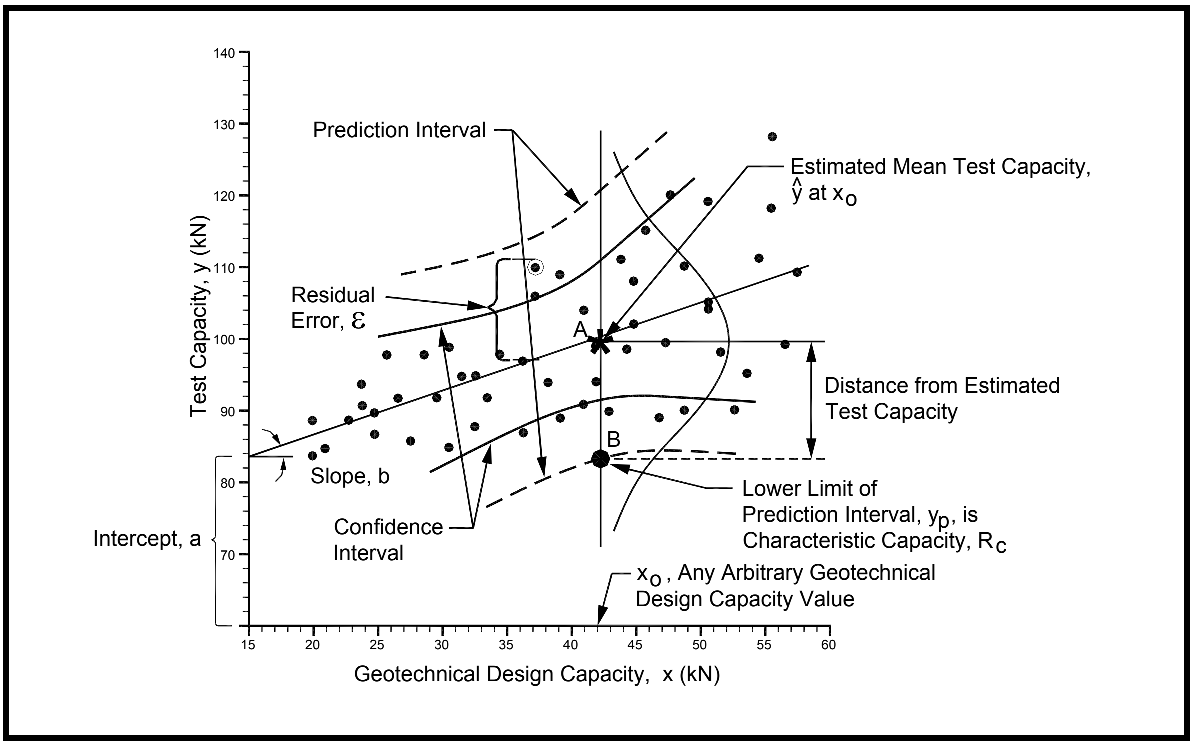


Figure 5 - Linear regression model

A simple statistical test is presented to check for zero intercept condition ("a" on figure 5) with a slope, $\overline{m}$ (b on figure 5) and a graphical method to test for variance trend of residuals (figure 6). For a non constant variance of residuals, an appropriate weight ($w_i = \frac{1}{x_i^2}$) can be used to make a correction to the original dataset (Ang and Tang, 1975). The non constant variance of residuals is defined as the residual error value, ε in figure 6b increasing with the geotechnical design capacity value. Moreover, the effect of constant or non constant variance of residuals is presented and its effect is emphasized in analyzing the data.

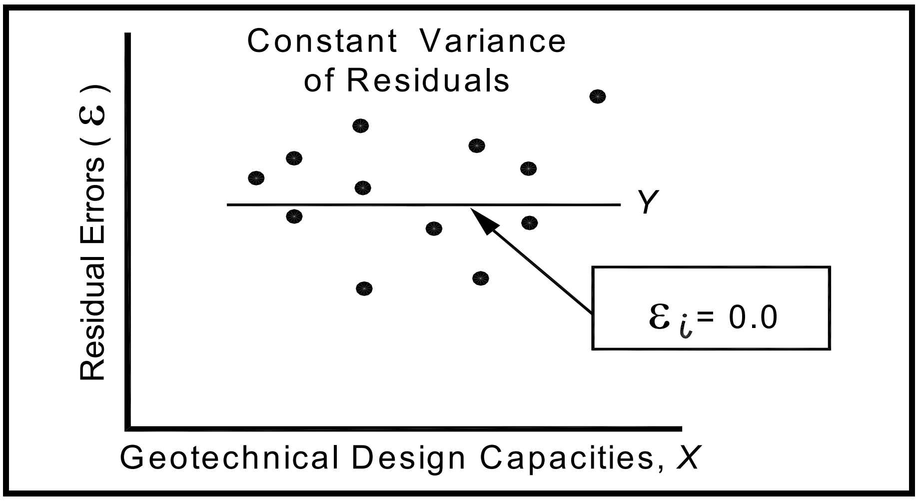


Figure 6a – Constant variance of residuals

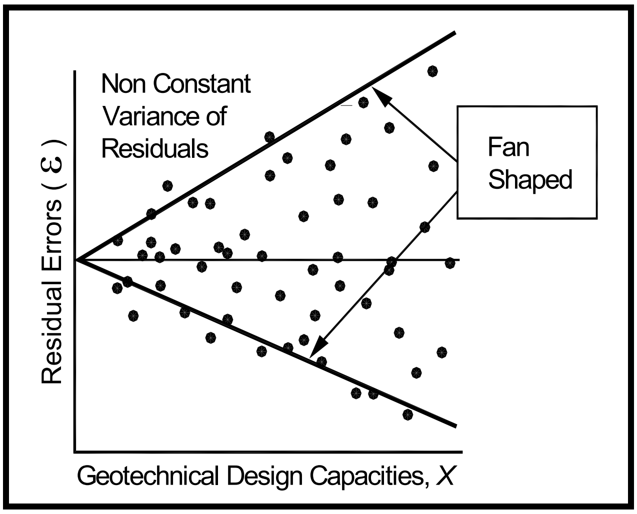


Figure 6b – Non constant variance of residuals

3.4.4 Foundation strength factor, ϕ_F under linear regression model

The foundation strength factor, ϕ_F under the linear regression model is defined as the ratio between the lower limit of the prediction interval, y_p (figure 5, predicted capacity) and the geotechnical design capacity value, x_0 used to determine this capacity.

$$\phi_F = \frac{y_p}{x_0} \tag{6}$$

Since there are two options (constant versus non constant variance of residuals) under each of these conditions (zero or non zero intercept condition), the user will encounter four possible cases for analyzing the data. Table 2 can be used as a guide for calibrating the foundation strength factor value for a specific foundation type. Each case in Table 2 will provide a lower limit on the predicted capacity. The equation for each case is given in the technical brochure. The test for zero intercept is definitive. However, the test for constant versus non constant variance of residuals is not quantitative and is rather subjective; because the designer needs to plot the residual error values against the geotechnical design capacity values and derive a trend line similar to figures 6. A flow diagram is developed to assist the foundation engineer in choosing a particular case.

Table 2- Analysis combinations

Intercept	**Variance of Residuals**	
	Constant	**Non constant**
Non Zero	Case A	Case B
Zero	Case C	Case D

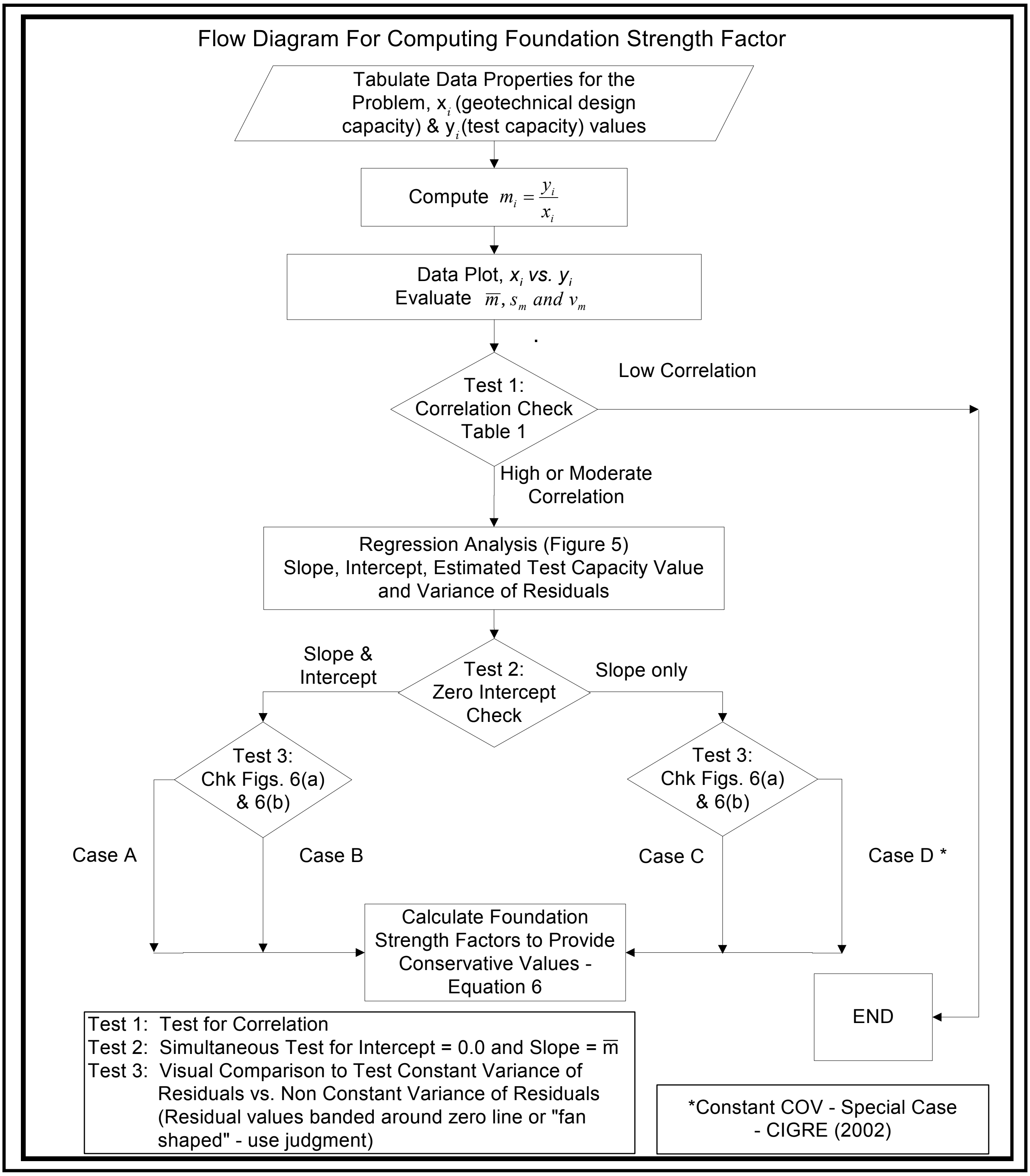


Figure 7 - Flow diagram

4.0 Example Problem

4.1 $\overline{m}$ -model

Electricity Supply Board (ESB), Ireland provided data for pad and chimney foundations tested under various soils conditions. The nine uplift foundation tests carried out in cohesionless soils are initially analyzed using the $\overline{m}$ -model. The mean values of the test and the geotechnical design capacities are 378 kN and 295 kN respectively, while the coefficients of variation are approximately 35%. The $\overline{m}$ and v_m values are 1.30 and 0.156 respectively.

4.2 Linear Regression Model

Figure 8 presents the data for regression analysis. The correlation value r^2 is 0.81 which is high (Table 1). The 30^0 frustum cone model in general, appears to be conservative because of large bias, i. e. the model in general underestimates the test capacity value by 30%. A linear regression model (CASE A) is run first following the flow diagram in figure 7 and Table 2. The statistical test for intercept indicated that the zero intercept assumption and the slope, $\overline{m}$ =1.30 ($\overline{m}$ -model) can be accepted. Therefore, the designer has to choose between Case C and Case D respectively (figure 7).

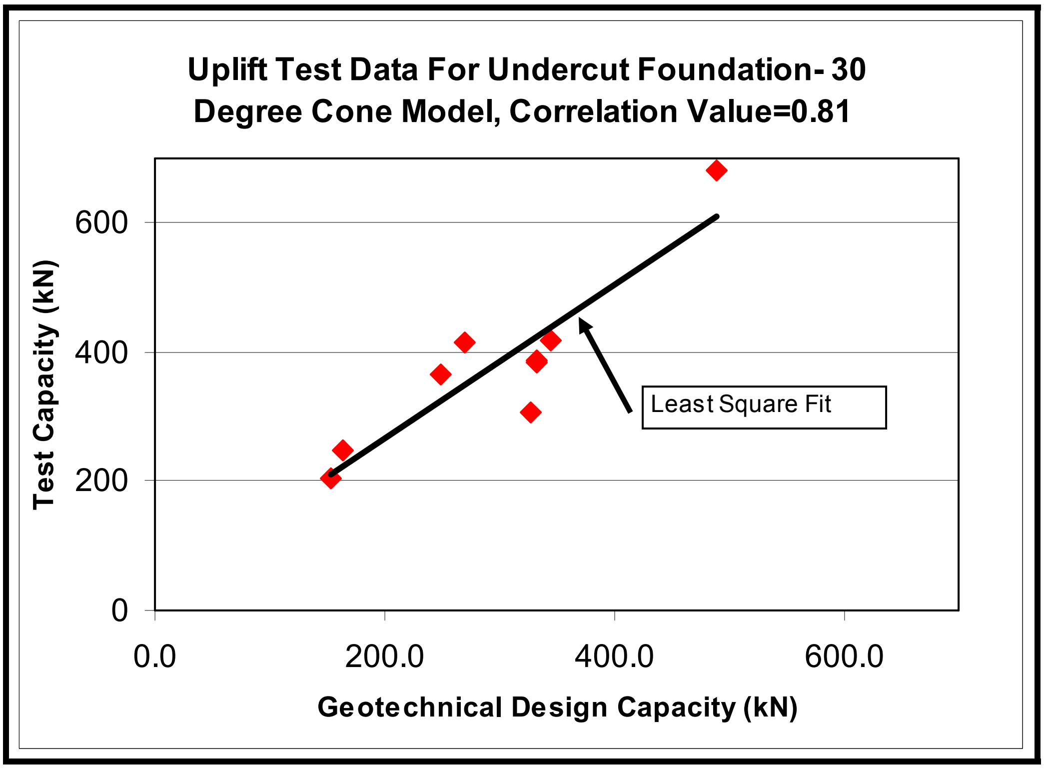


Figure 8 - ESB uplift data plot

Next, figure 9 presents the residual plot. Two lines are drawn to see a trend line that the variance of residuals is non constant (similar to figure 6b) and it is not fully definitive that this is the case. Therefore, the designer should try both cases C and D for determining the foundation strength factor values. Figure 10 depicts the comparison of foundation strength factor, ϕ_F based on CASE C and CASE D respectively; it is interesting to note that for CASE D the foundation strength factor is constant across all geotechnical design capacity values.

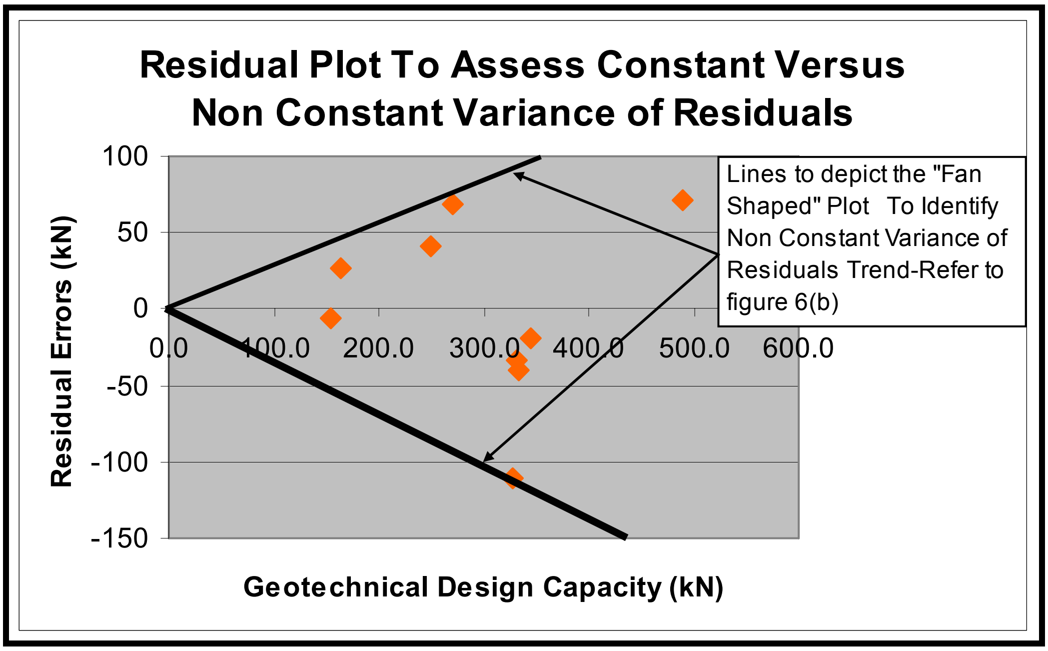


Figure 9 – Residual errors versus geotechnical design capacity values

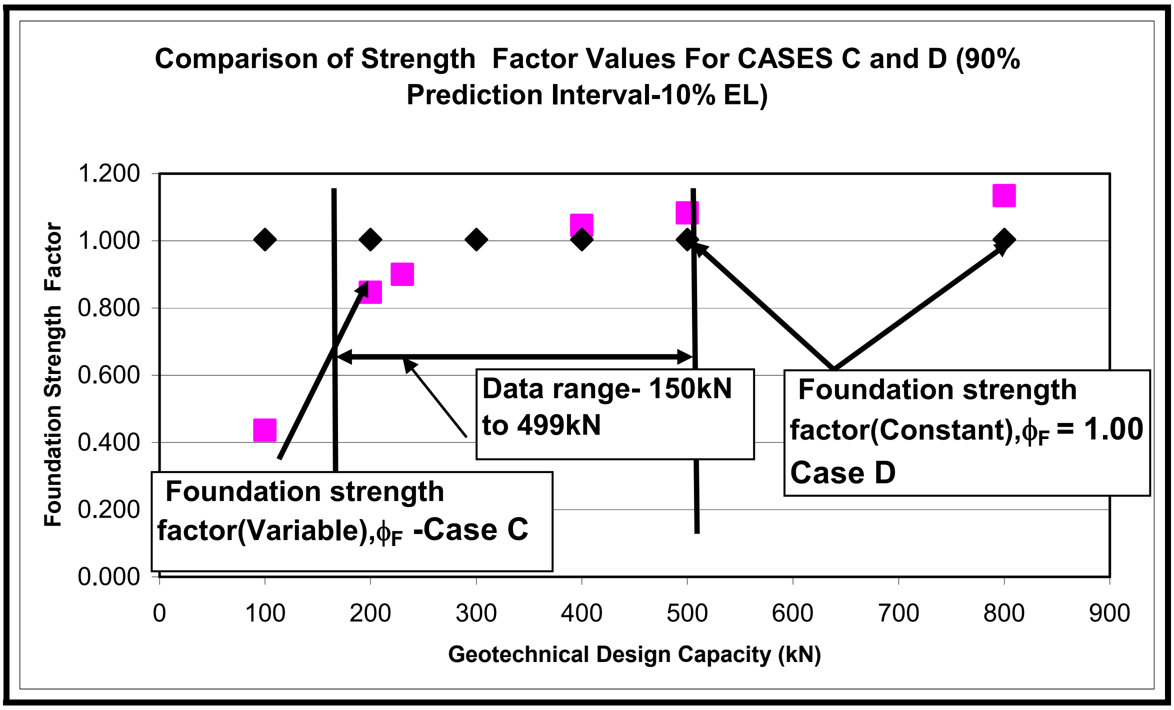


Figure 10 - Comparison of foundation strength factor values (sample size effect included)

However, if the foundation engineer uses CASE C, where the variation of residual errors with respect to x-values in figure 9 is interpreted as constant (figure 6a), the foundation strength factor becomes strongly dependent on the geotechnical design capacity value (x-value) as the predictor values are further away from the mean capacity value (295 kN). Therefore, in this situation, the foundation designer may want to consider the minimum of the two foundation strength factor values. A foundation design problem presented in the brochure shows that the difference in the design depth of foundation could be 9% or more depending on the assumption the foundation engineer chooses.

5.0 Summary and Conclusions

This study shows clearly that there are a number of steps one needs to consider before a foundation strength factor, for a particular foundation type, can be estimated based on a limited number of full-scale test capacity data and the theoretical foundation model predicted geotechnical design capacity. The foundation engineer also needs to understand that each dataset is different because of various test conditions, as well as model parameters imposed on them. The WG B2-07 recommends that a clear and consistent methodology based on accepted statistical theories, supported by valid assumptions, will provide a good basis for testing any database and its practical use for reliability based design of foundations. Simple tests, as proposed in this document include: correlation test, simultaneous test for zero intercept and slope, trend line for variance of residuals variation etc.

Based on these tests, the foundation designer can clearly decide whether to use the $\overline{m}$-model (CIGRE, 2002) or to use the linear regression model (alternative method), applicable to those databases where the results clearly show that the assumptions made under the $\overline{m}$-model may not be valid. A flow diagram is developed to assist the foundation engineer in choosing a particular option.

Some important conclusions are drawn from this study:

- A review of the current $\overline{m}$-model (Buckley, 1994, CIGRE, 2002) is presented; which has examined the underlying assumptions and their limitations in a systematic manner. The method is primarily based on the linear regression model and it is shown that it is valid under very specific conditions and assumptions, as detailed below.

- The Buckley method relies on two very specific conditions which imply that the intercept should be zero (figure 5) and the residual error values should vary with the geotechnical design capacity values in an increasing manner (figure 6b).

- To apply this CIGRE method to a wide range of reliability based foundation design calibrations with various degrees of correlation, the designer must be aware that unless these specific conditions are met, the prediction interval with a high degree of confidence (90% for IEC 60826 and 95% for ASCE) will produce results that may not be valid.

- A simple statistical test (simultaneous test) is proposed here to provide guidance to the foundation designer so that the assumption of "zero intercept" with a slope, $\overline{m}$, is validated properly.

Finally, it is shown that the general linear regression model with the unique test for "zero intercept" is quite applicable to the analysis of a wide range of foundation test data with various degree(s) of correlation, coefficient of variations and $\overline{m}$ values. The methodology presented here is quite robust and provides a consistent basis for analyzing a wide range of test data. The foundation designer is able to review the details of these calculations in order to understand the intricate relationships of various parameters that contribute to the estimation of the foundation strength factor, ϕ_F values. The foundation designer can make an informed decision to choose a particular option or options (Cases A to D) in determining the predicted foundation test capacity with a specified confidence level.

6.0 References

Ang, A. H. S and Tang, W.H. 1975 Probability Concepts in Engineering Planning and Design, Volume 1, John Wiley & Sons

Buckley, M 1994 Reliability Based Design of OHL Foundations, CIGRE Publication, SC 22-203, Overhead Lines, Paris Meeting, August 28-September 3,11p.

CIGRE 2002 The Design of Transmission Line Support Foundations- An Overview, WG 22.07, August, Technical Brochure No. 206

IEC 60826 2003 Design Criteria of Overhead Transmission Lines

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS

Symbol	Unit	Description	Equation Number/ Section Number(§)
A	m	Foundation depth (Fig. 3.1)	§3.1
a	kN	Intercept of the linear regression line	5.3
a*	kN	Arbitrary value of "a"	5.11
B	m	Foundation width at the bottom (Fig. 3.1)	§3.1
b	-	Slope of the linear regression line	5.2
C	m	Width of excavation (Fig. 3.1)	§3.1
CI	kN	Confidence interval for the mean test capacity	2.6
c	kN/m^2	Cohesion	3.4
D	m	Height of foundation pad (Fig. 3.1)	§3.1
D^0	m	Equivalent diameter of a rectangular foundation	§4.4.4
D_n	kN	Effects of dead load	2.14
E^*	kN	Error margin in estimating the confidence interval (to estimate the population mean)	2.12
E	m	Width of foundation chimney (Fig. 3.1)	§3.1
e	%	Exclusion limit	2.11
F_0	kN	Shear capacity (Frictional uplift resistance)	3.3
F^*	-	A value computed for "zero intercept" check	5.11
F	m	Height of foundation chimney above ground (Fig. 3.1)	§3.1
$F_{\alpha,2,\nu}$	-	A value obtained from F-distribution with significance level, α and degrees of freedom, ν & 2	5.11
G	m	Width of frustum at ground level	3.7
K	-	Coefficient of lateral earth pressure	3.4
k_α	-	Constant for determining exclusion limit for one sided interval (n> 30)	2.11
$k_{\alpha/2}$	-	Constant for determining exclusion limit for two sided interval (n> 30)	2.6
$\overline{m}$	-	Mean of m_i values	4.3
m_i	-	Ratio of test capacity, y_i and geotechnical design capacity, x_i, for sample test, i	4.2
N	-	SPT value to define soil strength parameters	§ 3.3
n	-	Sample size	2.1
PI	kN	Prediction interval for a new test capacity	2.8
p	-	Confidence level	2.5
Q_T	kN	Effect of a climatic load with a T-return period	2.16
$\overline{R}$	kN	Sample mean test capacity data (also noted as $\overline{y}$)	2.1
$R_{C,e\%}$	kN	Characteristic capacity defined as the capacity with a specified exclusion limit e %	2.11

Symbol	Unit	Description	Equation Number/ Section Number(§)
$R_{C,10\%}$	kN	Characteristic capacity defined as the capacity with a 10% exclusion limit (IEC 60826, R_C)	4.6
R_i	kN	Foundation test capacity for sample test, i (also noted as y_i)	2.1
R_n	kN	Geotechnical design capacity (also noted as x_i)	2.14
$\overline{R}_n$	kN	Mean geotechnical design capacity (also noted as $\overline{x}$)	5.3
R_{n+1}	kN	New foundation test capacity value for prediction	§ 2.3.2
r	-	Correlation coefficient	4.1
r^2	-	Correlation value	§ 4.3.1
S	m^2	Shear surface area	§3.2.2
s_{xy}	kN or non-dimensional	Standard deviation of the error in the linear regression analysis; for w_i =1.0, dimension is kN and for $w_i = \frac{1}{x_i^2}$, this is non dimensional	5.6
s_{xy}^2	(kN or non-dimensional)^2	Square of the standard deviation of the error in the linear regression analysis; Variance of residuals	5.6
s_x	kN	Standard deviation of x_i - data	2.2 & § 4.1
s_y	kN	Standard deviation of y_i - data	2.2& § 4.1
s_R	kN	Standard deviation of the foundation test capacity data	2.2
$t_{\alpha,\nu}$	-	Constant for determining exclusion limit for one sided interval (n< 30, Student t-distribution)	§ 2.4.2
$t_{\alpha/2,\nu}$	-	Constant for determining exclusion limit for two sided interval (n< 30, Student t-distribution)	2.7
U	kN	Uplift force	3.1
V_C	m^3	Total volume of the concrete	3.1
$V_{C,B}$	m^3	Volume of the concrete below the ground line	3.8
V_S	m^3	Volume of the soils within the frustum	3.5
$V_{S,F}$	m^3	Total volume within the frustum	3.6
v_m	-	Coefficient of variation of m-values	§4.2, 4.5
v_R	-	Coefficient of variation of the foundation test capacity data	2.3
W_C	kN	Weight of the concrete of the foundation	3.2
W_S	kN	Weight of the soil within the frustum	3.2
WI_n	kN	Effects of wind and ice loads on conductors and support	2.14

Symbol	Unit	Description	Equation Number/ Section Number(§)
w_i	-	Weight for sample test, i, in the linear regression analysis (w_i = 1.0 for constant variance of residuals and $w_i = \frac{1}{x_i^2}$ for non constant variance of residuals)	5.2
x_L	kN	Lower limit of the x_i data (predictor values)	§ 6.5
x_U	kN	Upper limit of the x_i data (predictor values)	§ 6.5
x_i	kN	Geotechnical design capacity for sample test, i	5.5
x_0	kN	Single geotechnical design capacity	5.10
$\bar{x}$	kN	Mean geotechnical design capacity data	4.1
y_i	kN	(Actual) test capacity for sample test, i	5.1
y_p	kN	Predicted test capacity for a given single geotechnical design capacity, x_0	5.10
$\hat{y}$	kN	Estimated mean test capacity from the linear regression analysis for a single geotechnical design capacity, x_0	5.17
$\hat{y}_i$	kN	Estimated mean test capacity from the linear regression analysis for sample test, i	5.5
$\bar{y}$	kN	Mean test capacity data	4.1
α	-	Significance level	2.5
β	-	Reliability Index	§ 2.6
γ_D	-	Load factor for the dead loads	2.19
γ_{WI}	-	Load factor for the combined wind and ice loads	2.19
ε_i	kN	Residual error value from the linear regression analysis (difference between the actual test capacity, y_i and estimated mean test capacity, $\hat{y}_i$)	5.1
η	-	Exponent of Mors (1965) equation	3.3
μ	kN	Population mean	§ 2.2
ν	-	Degrees of freedom (n-1 for one unknown; n-2 for two unknowns)	§ 2.3
ρ_C	kN/m^3	Unit weight of concrete	3.1
ρ_S	kN/m^3	Unit weight of soil within the frustum, soil density	3.1
σ	kN	Population standard deviation	§ 2.2
φ	-	Capacity reduction factor	2.19
ϕ	°	Angle of internal friction	3.4
ϕ_F	-	Foundation strength factor defined as the ratio of R_C and R_n	2.18
ϕ_C	-	Characteristic strength factor taking into account the variation between the actual exclusion limit	§ 2.6.2

Symbol	Unit	Description	Equation Number/ Section Number(§)
		and the supposed exclusion limit of 10% of the characteristic capacity R_C (IEC 60826)	
ϕ_N	-	Strength factor related to the number of components subjected to maximum load intensity	§ 2.6.2
ϕ_Q	-	Strength factor related to the quality of installation	§ 2.6.2
ϕ_R	-	Global strength factor	2.17
ϕ_S	-	Strength factor related to the coordination of strength	§ 2.6.2
ψ	°	Frustum angle for uplift capacity calculation	3.7
ψ^*	-	Resistance factor	2.15
τ	kN/m^2	Soil shear strength	3.3

GLOSSARY OF TERMS

Support - structure to carry overhead line conductors and ground wires.

Foundation - a structure below the ground to transfer support loads to the soils.

Sample mean - the arithmetic mean of a sample data of finite size.

Standard deviation - the dispersion of a random variable from its mean.

Population mean – mean of a normal distribution population.

Test capacity - load corresponding to a prescribed foundation movement.

Geotechnical design capacity - computed based on theoretical and/or semi empirical design model; for example, the frustum cone model is used in this study to determine the foundation geotechnical design capacity under uplift.

Coefficient of variation - the ratio of standard deviation to the mean value.

Correlation coefficient - a measure for the linear association of y with x.

Correlation value - square of the correlation coefficient.

Variance - square of the standard deviation.

Null hypothesis -a proposition which is believed to be true; this could be that the line passes through the origin while the alternative hypothesis could be the line does not pass through the origin.

Significance level - a probability measure to reject a null hypothesis even though the hypothesis is in fact true.

Exclusion limit - a variable defined in terms of percentage (%) to define the low side of the mean value from a probability distribution.

Confidence interval - provides a probability measure of how close the population mean is to the sample mean given a finite sample size; population mean is normally unknown and is an unobservable quantity.

Prediction interval - provides a probability measure that a new test value drawn randomly from a distribution will lie within a certain interval given a finite sample size.

Characteristic capacity - provides a capacity measure in probabilistic term with a specified exclusion limit; capacity with 5% or 10% exclusion limit means that the capacity will be higher than the specified value with 95% or 90% probability.

Normal distribution - a symmetric distribution commonly used in statistical analysis; normally valid for infinite sample size where the tail extends to infinity.

Log normal distribution - a probability distribution of any random variable whose logarithmic value is normally distributed.

Student t distribution - a distribution derived from normal distribution when the sample size is finite.

F-distribution -a joint probability distribution with two degrees of freedom.

Reliability index - a measure of structural reliability taking into account the variations in load and strength.

Undercut foundation – a type of foundation where the soil above the base of the foundation is undercut, such the foundation is in direct contact with the soil; very common in European countries.

Frustum angle - an angle used to define the failure surface in the frustum model for determining the uplift capacity.

Limit load - climatic load corresponding to a return period, T-years without imposing any load factor.

Elastic limit capacity - the load at which no significant (reversible) displacement is observed.

Damage limit (working limit) - the load at which some permanent (irreversible) displacement is observed and is similar to serviceability limit state in building codes.

Ultimate capacity - the load at which the foundation has undergone a significant movement and a complete failure will happen if this load is exceeded.

Residual - difference between the actual test capacity value and the estimated mean test capacity value from the linear regression line.

Constant variance of residuals - square of the residuals provide a constant value across all values along x-axis (geotechnical design capacity).

Non constant variance of residuals - square of the residuals vary with values along x-axis (geotechnical design capacity).

Linearity trend - provides a basis to predict y from x using linear relationship.

Foundation Strength factor - provides a measure of the relationship (linear or non linear) between the characteristic capacity and the geotechnical design capacity using full-scale test data.

Standard error of estimate (Variance of residuals) - a term in linear regression analysis to measure the dispersion from the estimated mean test capacity value (sum of the square of residuals).

Weighted residual - a term used in regression analysis to correct the data when the variance of residuals is non constant.

1.0 INTRODUCTION

1.1 General

Foundations for overhead line supports are frequently subjected to the effects of various types of meteorological loads, i.e. due to wind, ice and combined wind and ice loads on conductors and the support. Apart from these loads, supports are also subjected to broken conductor loads as well as longitudinal unbalanced loads due to ice shedding. The support foundation design must take into account the effects of all these forces.

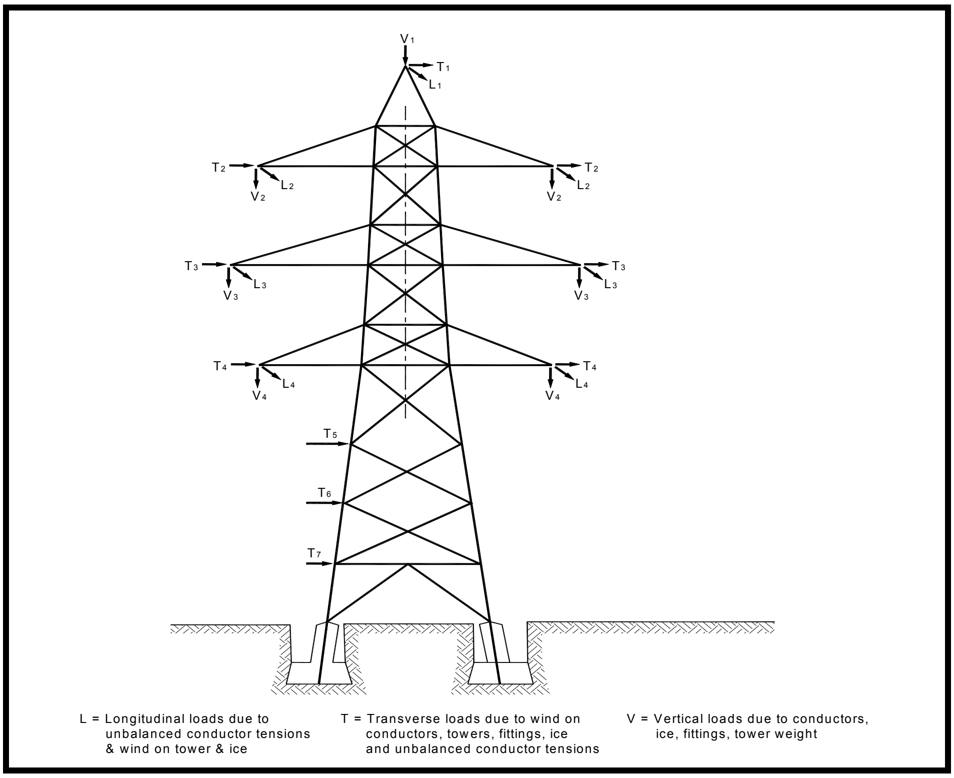


Figure 1.1 - Typical loads on an overhead line structure (Buckley, 2001)

Forces transmitted at the foundation level are often uplift (pullout) forces with small transverse shear forces for tangent lattice towers or large overturning moments with corresponding shear forces for single pole, angle or dead end structures. Foundation types are normally spread footings and steel grillages for lattice towers and drilled shafts for single poles, angle or dead end structures. Figure 1.1 depicts a typical double circuit tower with applied forces at the conductor attachment points. Figure 1.2 presents the resultant loads on a typical foundation, while figure 1.3 presents the various resistive forces acting on this foundation.

Since the variable forces acting on the conductor and tower are probabilistic in nature, the transmitted forces acting on the foundation are also probabilistic and are subjected to high levels of uncertainties. In addition, soil conditions and characteristics such as density, shear resistance, water level, vary also significantly from one site to the other and also contribute a high level of uncertainty. To resist the tower forces, the foundation must be designed based on the geotechnical design capacity which is normally obtained from a theoretical design model or an empirical method.

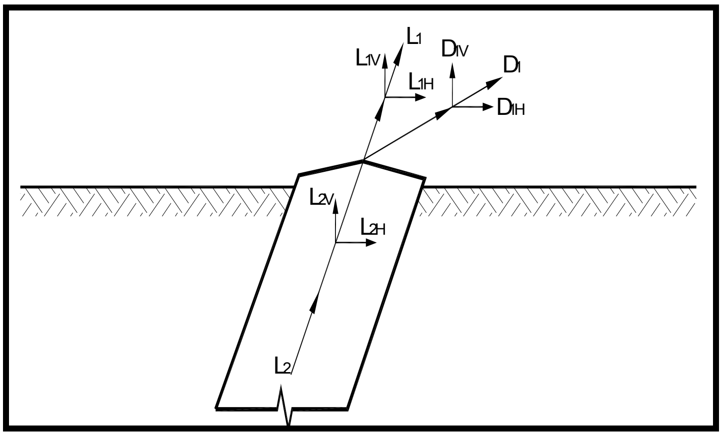


Figure 1.2 - Resultant forces acting on a typical foundation (Buckley, 2001)

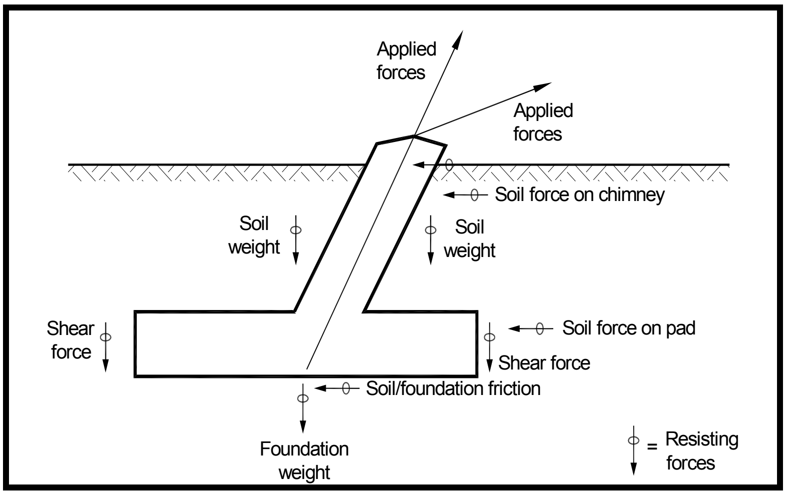


Figure 1.3 - Resistive forces on a typical foundation (Buckley, 2001)

Correspondingly, it is necessary to perform full-scale foundation tests to evaluate the actual capacity of the foundation compared to the theoretical design capacity.

1.2 Objective

The primary objective of this guide on reliability based calibration of foundation strength factor is to present method(s) of undertaking the calibration of the theoretical geotechnical design model and hence determine the characteristic capacity of the foundation, based on full-scale test data. To develop this calibration methodology, a framework to determine the prediction interval of the foundation test capacity based on a limited full-scale test data is developed based on a linear regression model. The prediction interval provides the upper and lower limits of a new test capacity predicted with a specified confidence level given the sample information. A direct relationship is established between the lower bound of the prediction interval and the foundation strength factor, ϕ_F which can be used in reliability based design equation of a single foundation component. The foundation strength factor with an exclusion limit is defined as the ratio of the predicted test capacity with a specified confidence level and the corresponding geotechnical design capacity. To assist readers, the report will present a step-by-step procedure to ensure that the foundation engineer is aware of all the assumptions made and the corresponding limitations in applying the calibration model(s). Specific recommendations are made to mitigate some of the limitations of the earlier model developed in CIGRE (2002).

1.3 Section Overview

For clarity, this report is divided in the nine sections

- Section 1 presents the introduction and outlines the objective of the study.

- Section 2 provides the information on confidence intervals, prediction intervals for a new test capacity, characteristic capacity (strength), sample size estimation and reviews the reliability based design with particular reference to allowable stress design (ASD), ASCE load and resistance factor design (ASCE- LRFD) and IEC 60826 (2003) design methodology.

- Section 3 reviews various theoretical models for computing uplift capacity of a pad and chimney foundation based on frustum angle and shear models. The necessary geotechnical parameters are identified for calculating the uplift capacity. The foundation designer is given guidance on how to undertake a full-scale foundation test programme. A number of steps are outlined how to carry out a field test programme and to obtain the full-scale test data. Two scenarios are considered. These are: (1) repeated test data for one specific site and (2) scattered test data at three sites along a transmission line route.

- Section 4 presents the determination of foundation strength factor based on the $\overline{m}$ -model developed by Buckley (1994) and presented in CIGRE (2002). Various assumptions of this model are presented. This model is applied to the above two test data. This is followed by an analysis which includes the actual test data provided by Electricity Supply Board (ESB) in Ireland.

- Section 5 presents an alternative methodology to calibrate the foundation strength factor based on a linear regression model; it is shown that the $\overline{m}$ -model is a subset of this generalized linear regression model. A flow diagram is included to provide guidance, to the foundation designer, for analyzing the data. A design problem based on ESB test data (section 4) is reanalyzed using the linear regression model.

- Section 6 discusses the general criteria for extrapolation and its limitation within the context of a linear regression model. A design example for foundation characteristic capacity (predicted) is presented to show the effects of interpolation versus extrapolation.

- Section 7 presents the summary and conclusions.

- Section 8 presents the references.

- Section 9 presents an APPENDIX with an example problem. A 3 point scattered test data is analyzed to show how the foundation engineer can calculate all the necessary parameters manually for determining the foundation strength factor.

2.0 RELIABILITY BASED DESIGN (RBD)-BACKGROUND

2.1 General

This section presents the background information on confidence interval and prediction interval on full-scale foundation test data only. The confidence interval on mean foundation test capacity is linked to an estimation of the sample size, while the prediction interval is linked to characteristic capacity with a defined exclusion limit. A reference is also made to the estimation procedure that can be used in the context of linear regression model when the test capacity data is related to model predicted geotechnical design capacity. This is followed by a brief review of the deterministic and semi-probabilistic methods used in foundation design such as allowable stress design (ASD), IEC 60826 (2003) and load and resistance factor design (ASCE-LRFD).

2.2 Sample Statistics

The mean, ($\overline{R}$), standard deviation, (s_R), and the coefficient of variation, (v_R), of a full-scale data on foundation test capacity of sample size, (n), are calculated based on the following formulae:

$$\overline{R} = (\sum_{1}^{n} R_i)/n \tag{2.1}$$

$$s_R{}^2 = \frac{1}{n-1}\sum_{1}^{n}(R_i - \overline{R})^2 \tag{2.2}$$

$$v_R = \frac{s_R}{\overline{R}} \tag{2.3}$$

where R_i, $i = 1,...n$ are the data on foundation test capacity.

Normally the test capacity value (R), is a random variable and is assumed to follow a distribution (normal, log normal etc.). The population mean resistance (μ), is defined as the estimated mean test capacity value of this random variable with a standard deviation (σ), which are normally unknown. The variance of residuals is defined as the square of the standard deviation and is a measure of the dispersion around the estimated mean test capacity value.

2.3 Estimation of Confidence Interval (CI) and Prediction Interval (PI)

2.3.1 Estimation of Confidence Interval (CI)

The mean of a sample drawn randomly from the population is a point estimate of the population mean (μ), and will vary for different samples. Given that the sample mean is unlikely to be equal to population mean, how reliable $\overline{R}$ is the measure of μ. The confidence interval provides this measure. In theory, as the number of samples approaches infinity ($n \to \infty$), there will be no difference between the sample statistics and population statistics. However, a large sample size is not practical for full-scale foundation testing and therefore, a confidence interval is often sought to provide an accuracy of the estimate on the mean test capacity value, ($\overline{R}$).

Figure 2.1 depicts a typical normal probability density function of R, as $f(R)$ and is described by the following equation:

$$f(R) = \frac{1}{s_R\sqrt{2\pi}} e^{-\frac{1}{2}[\frac{(R-\overline{R})}{s_R}]^2} \qquad (2.4)$$

Figure 2.1 also presents the concept of two sided confidence interval (CI), confidence level and the level of significance. The significance level is based on the area under the shaded region. The total area under the curve is 1.0.

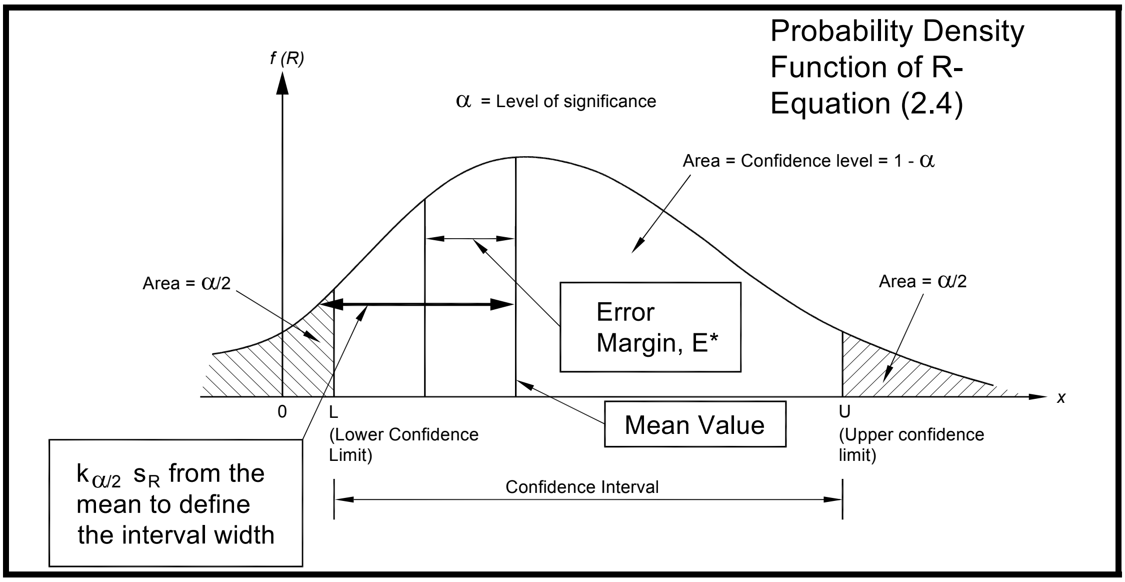


Figure 2.1 - Two sided confidence interval and significance level (Rao, 1982)

The confidence interval provides an estimate where the true value may lie within the upper and lower limits. For this case the confidence level is the area under the probability density curve between the upper confidence (U) and the lower confidence (L) limits marked on figure 2.1. Once the confidence level (p), is stated, the probability of true mean (population mean) lying in the confidence interval is p. Therefore, there is a probability of (1-p) % that the mean value will not lie within the confidence interval. This quantity is also defined as the significance level, α (Rao, 1982) and is given by equation (2.5)

$$\alpha = 1\text{-}p \qquad (2.5)$$

The significance level, α is very similar to exclusion limit (e %) which will be discussed in section 2.4.1, $k_{\alpha/2}s_R$ in figure 2.1 provides the required offset from the mean to define the confidence interval.

Table 2.1 presents $k_{\alpha/2}$-values for various confidence levels. For example $k_{\alpha/2}$ = 1.96 for 95% confidence level while $k_{\alpha/2}$ = 1.645 for 90% confidence level.

Table 2.1 - $k_{\alpha/2}$ & k_{α} - Values for various confidence levels (Benjamin & Cornell, 1970)

Confidence level in %	$k_{\alpha/2}$ in Figure 2.1 (Two Sided)	k_{α} in Figure 2.2 (One Sided)
99.9%	3.291	3.090
99%	2.576	2.326
95%	1.960	1.645
90%	1.645	1.282
80%	1.282	0.842

If the sample standard deviation, s_R is the same as the population standard deviation, σ (normally for large sample size, n > 30) and is known, the two sided confidence interval is given as

$$CI=\overline{R} \pm k_{\alpha/2}\frac{s_R}{\sqrt{n}} \qquad (2.6)$$

where $k_{\alpha/2}$ is obtained from Table 2.1.

The population standard deviation (σ), is not known with certainty when the sample size is less than 30 (n < 30). In this case, the two sided confidence interval (CI) is obtained from equation (2.7)

$$CI=\overline{R} \pm t_{\alpha/2,\nu}\frac{s_R}{\sqrt{n}} \qquad (2.7)$$

where $t_{\alpha/2,\nu}$ replaces $k_{\alpha/2}$ in equation (2.6) and is obtained from Student-t distribution, ν is the degrees of freedom and in this case, ν = n-1.

Table 2.2 provides some typical $t_{\alpha/2,\nu}$ values for two confidence levels, p= 90% and 95% respectively. This is normally the case when the sample size, n is less than 30.

2.3.1.1 Example Problem

If we assume that five full-scale-foundation tests (n = 5) have been undertaken for a specific foundation type and the test results are 200 kN, 250 kN, 275 kN, 280 kN, and 300 kN respectively. Following equations (2.1), (2.2) and (2.3), the mean, standard deviation and the coefficient of variation are $\overline{R}$ =261 kN, s_R=38.5 kN, and $v_R = \frac{s_R}{\overline{R}}$ =0.148, respectively.

The 90% confidence interval (CI) on the population mean, neglecting the effect of small sample size (n=5), is obtained from equation (2.6) as $261 \pm 1.645x\frac{38.5}{\sqrt{5}}$ = 261 $\pm$ 28kN. The term $k_{\alpha/2}$ in equation (2.6) is obtained from Table 2.1. This means that the populations mean, (μ), will lie within $\pm$ 28kN of the sample mean ($\overline{R}$), with a p= 90% probability. The engineer may state that the true mean value of the foundation uplift capacity lies between 233kN and 289kN with a 10% significance level. The two limits of the confidence level (CI) are 233kN (L) and 289kN (U). In figure 2.1, the value of E^* =$\pm$ 28 kN is the error margin and will vary depending on the confidence level sought p, sample size n, and the sample standard deviation s_R.

Table 2.2 - $t_{\alpha/2,\nu}$ & $t_{\alpha,\nu}$ Values for figures 2.1 and 2.2 (Benjamin & Cornell, 1970)

Degrees of Freedom, ν	$t_{\alpha/2,\nu}$		$t_{\alpha,\nu}$	
	90% ($\alpha/2 = 0.05$)	**95%** ($\alpha/2 = 0.025$)	**90%** ($\alpha = 0.1$)	**95%** ($\alpha = 0.05$)
1	6.32	12.7	3.08	6.32
2	2.92	4.30	1.89	2.92
3	2.35	3.18	1.64	2.35
4	2.13	2.78	1.53	2.13
8	1.86	2.31	1.40	1.86
10	1.81	2.23	1.37	1.81
11	1.80	2.20	1.36	1.80
30	1.70	2.04	1.31	1.70
∞	1.64	1.96	1.28	1.64

Considering the effect of sample size $n = 5$, equation (2.7) is used with $t_{\alpha/2,\nu}$ = 2.13 for 90% confidence level (table 2.2). The revised calculation shows that the lower and upper values are 224 kN and 298 kN respectively. ($261 \pm 2.13x\frac{38.5}{\sqrt{5}}$ = 261$\pm$ 37 kN); however, it is noted that the error margin ($\pm E^*$) has now increased by 33% (37 kN versus 28 kN) because of the sample size effect. For the case when the sample size is small, the sample size and the confidence interval width, $\pm E^*$ = $\pm t_{\alpha/2,\nu}\frac{s_R}{\sqrt{n}}$ are interrelated because $t_{\alpha/2,\nu}$ depends on ν which is also dependent on n (table 2.2). This will be discussed further in section 2.5 in determining the sample size for foundation tests.

2.3.2 Estimation of Prediction Interval (PI)

Similar to confidence interval (CI), the foundation designer can also consider an interval such that there is a p% probability that a new test capacity drawn randomly from the population will occur within the interval. This is called the prediction interval. The confidence interval is for a population parameter (unobservable quantity and unknown), while the prediction interval is for a single test capacity value (future observation) given the sample information. The prediction interval (PI) for a new test capacity value, (R_{n+1}), for a given sample size, (n) is

$$PI=\overline{R} \pm k_{\alpha/2} s_R [1+\frac{1}{n}]^{0.5} \quad (2.8)$$

The prediction interval (PI) for a new test capacity value, R_{n+1} for a small sample size (n less than 30) is

$$PI=\overline{R} \pm t_{\alpha/2,\nu} s_R [1+\frac{1}{n}]^{0.5} \quad (2.9)$$

The prediction interval (PI) is wider than the confidence interval (CI in equation 2.6) because R_{n+1} is a variable quantity while the sample mean, ($\overline{R}$), is a constant value. For large sample size n, equation (2.8) can be approximated as

$$PI=\overline{R} \pm k_{\alpha/2} s_R \quad (2.10)$$

This implies that a future observation will lie between ($\overline{R} + k_{\alpha/2} s_R$) and ($\overline{R} - k_{\alpha/2} s_R$) with a significance level, (α).

2.4 Comparison of Predicted Capacity and Characteristic Capacity

2.4.1 Predicted capacity

However, in certain applications, the prediction interval (PI) can be used to derive a guaranteed minimum test capacity with a specified confidence level. If the designer is only interested in the minimum value (lower limit of the prediction interval) then this becomes $\overline{R} - k_{\alpha} s_R$ (one sided). Equation (2.10) can be used to provide this capacity considering the lower limit only and figure 2.2 presents this case as a one sided interval and $k_{\alpha/2}$ is replaced by k_{α} (table 2.1).

2.4.2 Characteristic capacity (IEC 60826)

The guaranteed minimum test capacity is analogous to characteristic capacity, (R_c), an engineering term, defined as the low side of the estimated mean test capacity with a specified exclusion limit and is depicted in figure 2.3 (IEC 60826, Melchers, 1999). For one sided prediction interval, the significance level can also be interpreted as the lower exclusion limit. For a large sample size, the predicted capacity and the characteristic capacity will be almost identical. However, for a small sample size, the difference could be 3% as shown below.

For example, if we have foundation test data of a large sample size with mean, ($\overline{R}$), and a coefficient of variation, ($v_R = \frac{s_R}{\overline{R}}$), then the characteristic foundation capacity with an exclusion limit, e % (similar to significance level, α) can be estimated from equation (2.11)

$$R_{C,e\%} = \overline{R}\ (1 - k_\alpha\ v_R) \qquad (2.11)$$

where k_α is a measure from the mean value to the point L, on the x-axis where a specified exclusion limit is sought. The exclusion limit e=10% implies that the mean characteristic foundation capacity is guaranteed for 90% confidence level and there is a 10% probability that the mean capacity could fall below this minimum value. This implies that the area under the curve from left of point “L” is 0.10 and the area under the curve to the right of point “L” is 0.90. The geotechnical design capacity, R_n in figure 2.3 is calculated based on analytical and/or semi empirical relationship or numerical models (Buckley, 1994, Digioia et al, 1993).

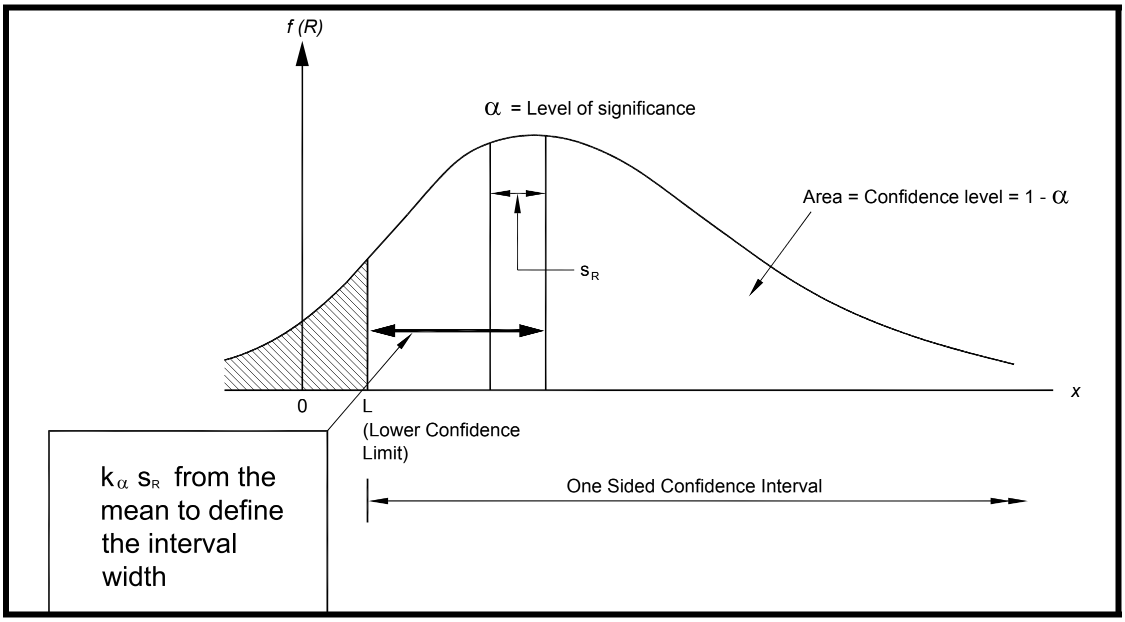


Figure 2.2 - One sided confidence interval, p and significance level α (Rao, 1982)

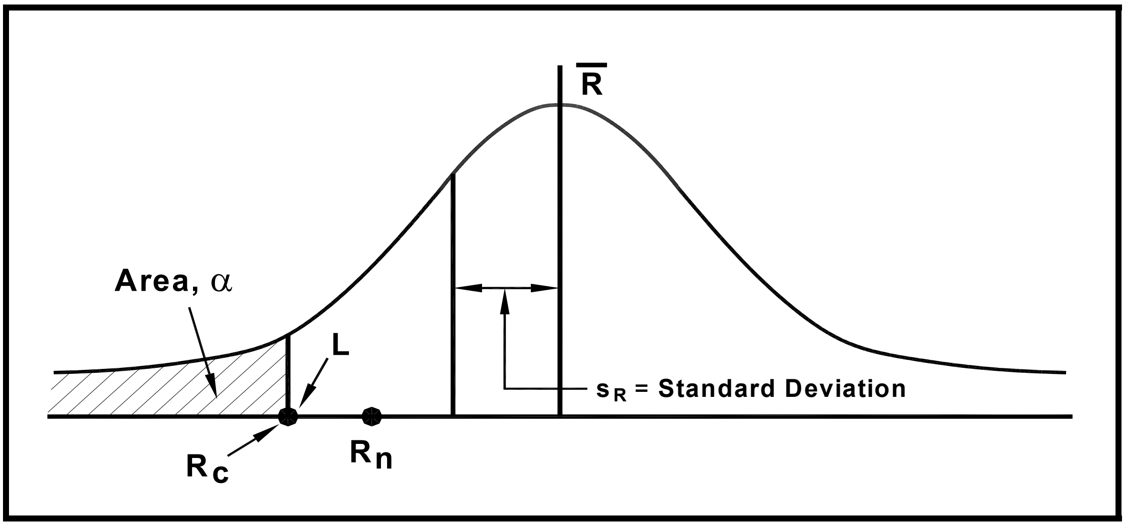


Figure 2.3 - Characteristic strength (R_C) of a foundation based on test data

For sample size n greater than 30, k_α value is 1.28 for 90% confidence level (10% exclusion limit) and can be obtained from Table 2.1. However, for a finite sample size, n < 30, $k_\alpha \ s_R$ should be corrected based on Student t-distribution (table 2.2).

The characteristic capacity with a 10% exclusion limit for the five tests presented earlier is obtained from equation (2.11).

$R_{C,10\%}$ = $\overline{R}$ (1- $k_\alpha \ v_R$) = 261 (1-1.28* 0.148) = 212kN

This capacity is based on a k_α value of 1.28 when the sample size effect is not included. Since the number of test is less than 30 (n < 30), $t_{\alpha,\nu}$ value is used to correct for the small sample size effect. Using $t_{\alpha,\nu}$ value = 1.53 from Table 2.2 (for α = 0.10 and ν = n-1 = 4), the revised estimate of the characteristic capacity is:

$R_{C,10\%}$ = $\overline{R}$ (1- $t_{\nu,\alpha} \ v_R$) = 261(1-1.53*0.148) = 202kN

This shows that the characteristic capacity is overestimated in equation (2.11) by 5% when the effect of small sample size is not taken into account. The minimum guaranteed capacity based on the prediction interval concept would be 196 kN (3% less) compared to 202 kN. The characteristic capacity is applicable to single variable data (test capacity only), while the concept of prediction interval can be used for the problem where more than one variable is involved, such as the calibration methodology involving the test capacity versus geotechnical design capacity.

2.5 Estimating Sample Size, n

Quite often, the question is asked "what sample size do I need to estimate the sample mean test capacity value which will be accurate to within $\pm$ E^* of the population mean value". $\pm$ E^* defines the error margin in Figure 2.1.

2.5.1 Estimation of sample size when σ is known (n >30)

In order to match the accuracy between the sample mean and the population mean by, $\pm$ E^* the required sample size is:

$$n = \left(\frac{k_{\alpha/2} s_R}{E^*}\right)^2 \tag{2.12}$$

However, if the sample size, n becomes less than 30 then a correction needs to be made based on Student t-distribution. This will be discussed in the next section.

Therefore, to estimate the sample size, n we need to know the confidence level (Figure 2.1), error margin and the standard deviation. Often we need to estimate the standard deviation from prior knowledge or from the sample data.

For example, the engineer wants to estimate how many tests need to be done in order to get an accuracy of E^* = $\pm$28 kN with a 90% confidence level (p=0.9) when the standard deviation of the test data, s_R is known to be 38.5 kN. In this case, $\frac{E^*}{s_R}$ $= \frac{28}{38.5}$ = 0.73. Therefore, the sample size is estimated as:

$n = (\frac{k_{\alpha/2} s_R}{E^*})^2$ = $(\frac{1.64x38.5}{28})^2$ = 4.98 $\cong$ 5. This was the sample size given in Section 2.2.

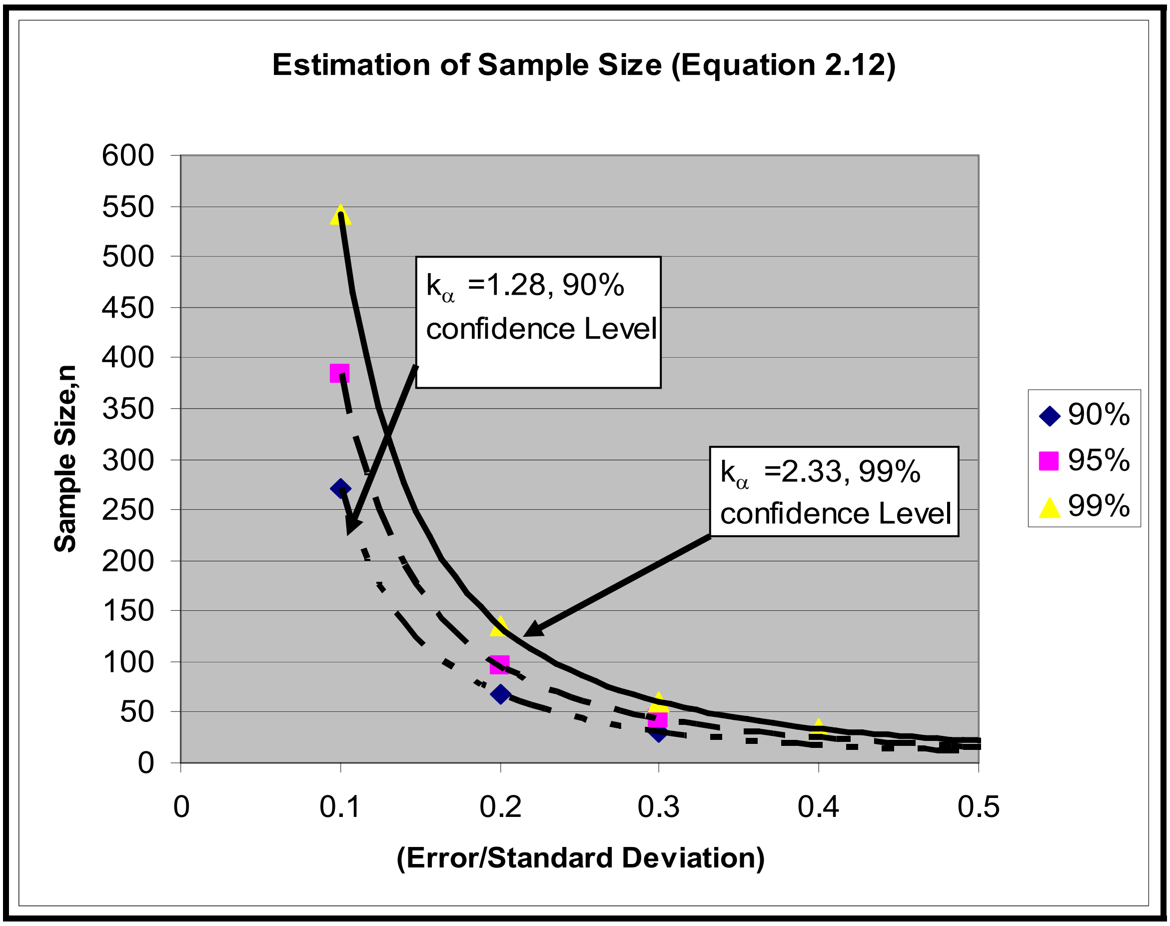


Figure 2.4 - Sample size versus ($\frac{E^*}{s_R}$) for various confidence levels

However, if the engineer decreases the accuracy of the prediction by increasing the E^* value to E^* = ± 40kN, the estimated sample size becomes:

$n = (\frac{k_{\alpha/2} s_R}{E^*})^2$ = $(\frac{1.645x38}{40})^2$ = 2.44 $\cong$ 3

Therefore, the sample size, n starts decreasing as we increase the error margin.

2.5.2 Estimation of sample size when σ is unknown (n < 30)

In this case, we need to replace $k_{\alpha/2}$ in equation (2.12) by $t_{\alpha/2,\nu}$ (Table 2.2) and the sample size becomes:

$$n = (\frac{t_{\alpha/2,\nu} s_R}{E^*})^2 \quad (2.13)$$

The problem arises in solving this equation (2.13) for n because $t_{\alpha/2,\nu}$ is not known a-priori; since $t_{\alpha/2,\nu}$ is dependent on sample size through ν because ν = n-1. However, a trial and error procedure can be used in solving equation (2.13).

Let us assume that $t_{\alpha/2,\nu}$ initially is equal to $k_{\alpha/2}$ which is 1.645 for 90% confidence level. Equation (2.13) provides a value of n=5 which gives $t_{\alpha/2,\nu}$ = 2.13 from Table 2.2. Using this value in equation (2.12), we find n=8.57 $\cong$ 9. Therefore, the revised $t_{\alpha/2,\nu}$ = 1.86 and we obtain n=8.54 in the second iteration. Therefore we need at least n=9 tests to maintain the same level of error margin of $\pm$ 28 kN.

Figure 2.4 presents this when the standard deviation is known (n > 30) and the sample size estimate is based on equation (2.12). The foundation engineer can also estimate sample size for (n < 30) following the iterative scheme presented before.

2.6 Reliability Based Design Methodologies

2.6.1 Allowable stress design (ASD)

Current practices for design of overhead line support foundations are primarily based on deterministic assumptions where the load effects on the foundation and the foundation capacity are both defined in terms of single values. The load effects and the capacity could be either defined in terms of the geotechnical design capacity or the ultimate capacity with appropriate factors of safety. In simple form, this is expressed as:

$$\frac{R_n}{FS} \geq (D_n + WI_n) \quad (2.14)$$

or

$$\psi^* R_n \geq (D_n + WI_n) \quad (2.15)$$

where:

R_n = geotechnical design capacity,
FS = factor of safety,
D_n = the effects of dead load such as tower weight, conductor weight etc;
WI_n = the effects of wind and ice loads on conductor and tower and
ψ^* = resistance factor =1/FS.

If ice is not present WI_n can be replaced by wind load effect only as, W_n and R_n could be calculated based on semi empirical formulae or numerical models.

2.6.2 IEC 60826 (2003)

IEC 60826 (2003) provides a basic framework where the semi probabilistic design equation is given in terms of the climatic load acting on a component and the strength of the component. The basic design equation relates the characteristic strength (capacity) value to the appropriate load effects having a specified return period of T years. In simple form, this can be expressed as:

$$R_C \geq Q_T \tag{2.16}$$

where R_C is the characteristic strength (capacity) of the foundation with e% exclusion limit and Q_T is the load effect with a T-year return period.

The design equation is based on the assumption that the distribution of load is Gumbel type 1 and the strength follows a normal distribution. The design equation is valid with a constant annual failure probability between (1/T to 1/2T) provided the coefficient of variation of strength is $0.05 \leq v_R \leq 0.20$ and of the load effect is $0.20 \leq v_Q \leq 0.50$ respectively.

Since IEC 60826 recommends the use of annual failure probability of (1/T to 1/2T), it is important that the coefficient of variation of strength be kept under 0.3 to meet the above criteria. For foundation, this may not be always possible. The capacity is normally based on the available test data with a typical exclusion limit of 10% with a 50 year return period normally assumed for the load. IEC 60826 also recommends larger return periods for higher reliability levels.

In designing individual line components based on a system approach, IEC 60826 also recommends to apply a global strength factor, ϕ_R in equation (2.16)

$$\phi_R \; R_C \geq Q_T \tag{2.17}$$

where ϕ_R is a global strength factor which depends on strength coordination ϕ_S, number of components subjected to maximum load intensity ϕ_N and quality of installation ϕ_Q; it also includes the characteristic strength factor ϕ_C which takes into account the variation between the characteristic capacity for an exclusion limit of e% and the capacity R_C with an exclusion limit of 10%

Within the framework of IEC 60826, the characteristic capacity should be related to the model predicted geotechnical design capacity and this could be done through calibration. If the full-scale test data are available, the appropriate foundation strength factor ϕ_F can be used to relate the geotechnical design capacity to the full-scale test capacity with a defined exclusion limit (5% for ASCE, 10% for IEC 60826).

A multiplicative form of ϕ_F has been presented by Buckley (1994). The characteristic capacity, R_C in equation (2.16) for a single foundation can then be modified as:

$$\phi_F R_n = Q_T \qquad (2.18)$$

where ϕ_F is a function of the statistical properties of the test and the geotechnical design capacities. The details of computing the calibration factor, ϕ_F based on $\overline{m}$ - model and linear regression model will be discussed in sections 4.0 and 5.0 respectively.

2.6.3 Load and Resistance Factor Design (ASCE- LRFD)

Although the ASD approach has served the utility industry reasonably well, increasingly utilities' design practices are moving towards a reliability based design methodology, in order to quantify the foundation reliability in a predictable manner. A simple reliability based design format is the use of load and resistance factor design (LRFD) approach where the geotechnical design capacity and the load effects are factored appropriately to account for a specified target reliability index β. The reliability index β provides a measure of reliability and is directly related to the failure probability which can be determined from normal tables (Melchers, 1999).

The LRFD format can be expressed as follows;

$$\varphi\, R_n \geq \gamma_D D_n + \gamma_{WI} WI_n \qquad (2.19)$$

where

φ = the capacity reduction factor or resistance factor and is always less than 1.0,

γ_D = load factor for dead load D_n,

γ_{WI} = load factor for the combined wind and ice loads WI_n.

If ice is not present, γ_{WI} can be replaced as γ_W, to represent the load factor for wind load effect, W_n. These factors are calibrated with respect to existing design practices, which are often deterministic in nature or against a design format prescribed, by the standard or code. For a simple log normal distribution of load effects and strength values, these factors can be computed analytically (Melchers, 1999).

2.7 Summary

This section provided a basis for determining (1) the confidence interval on mean value and (2) the prediction interval on a future test value. The lower limit of the prediction interval can be used to determine the guaranteed minimum foundation capacity, with a specified confidence level; while, the characteristic capacity is defined as the low side of the mean foundation capacity with a specified exclusion limit. For a large sample size, both will provide almost an identical value. The characteristic capacity is applicable to single variable data (test capacity only); while, the concept of prediction interval based on a linear regression model can also be applied to the problem where more than one variable is involved, (such as the test capacity versus geotechnical design capacity). Since the method here is applied to a set of test data only, it will be shown in section 5.0 that the same methodology can be used also, to provide the confidence interval for the estimated mean test capacity as well as the prediction interval on a new test capacity, for a given geotechnical design capacity value. It will be also shown in section 5.0 that the foundation strength factor, ϕ_F can be related directly to the lower limit on the prediction interval.

3.0 FOUNDATION CAPACITY ASSESSMENT

3.1 General

This section of the report presents a brief review of various design models in determining the geotechnical design capacity of a pad and chimney foundation under uplift condition. The soil strength parameters are identified for capacity calculation followed by the field data collection programme. A short description of the field test programme is presented which outlines the various steps that need to be followed in collecting the test data as well as the data interpretation and analysis using the methodology described in IEC 61773.

The fundamental premise of the reliability based design is to determine the uncertainty in the design process and to quantify this appropriately. To account for this uncertainty in assessing both the test capacity as well as geotechnical design capacity, the designer needs to test a number of foundations in various soil types (strength parameters), to ensure that the data represents a wide range of foundation conditions along the transmission line route. Figure 3.1 depicts a typical line route, which covers a wide range of soil conditions usually encountered in a project.

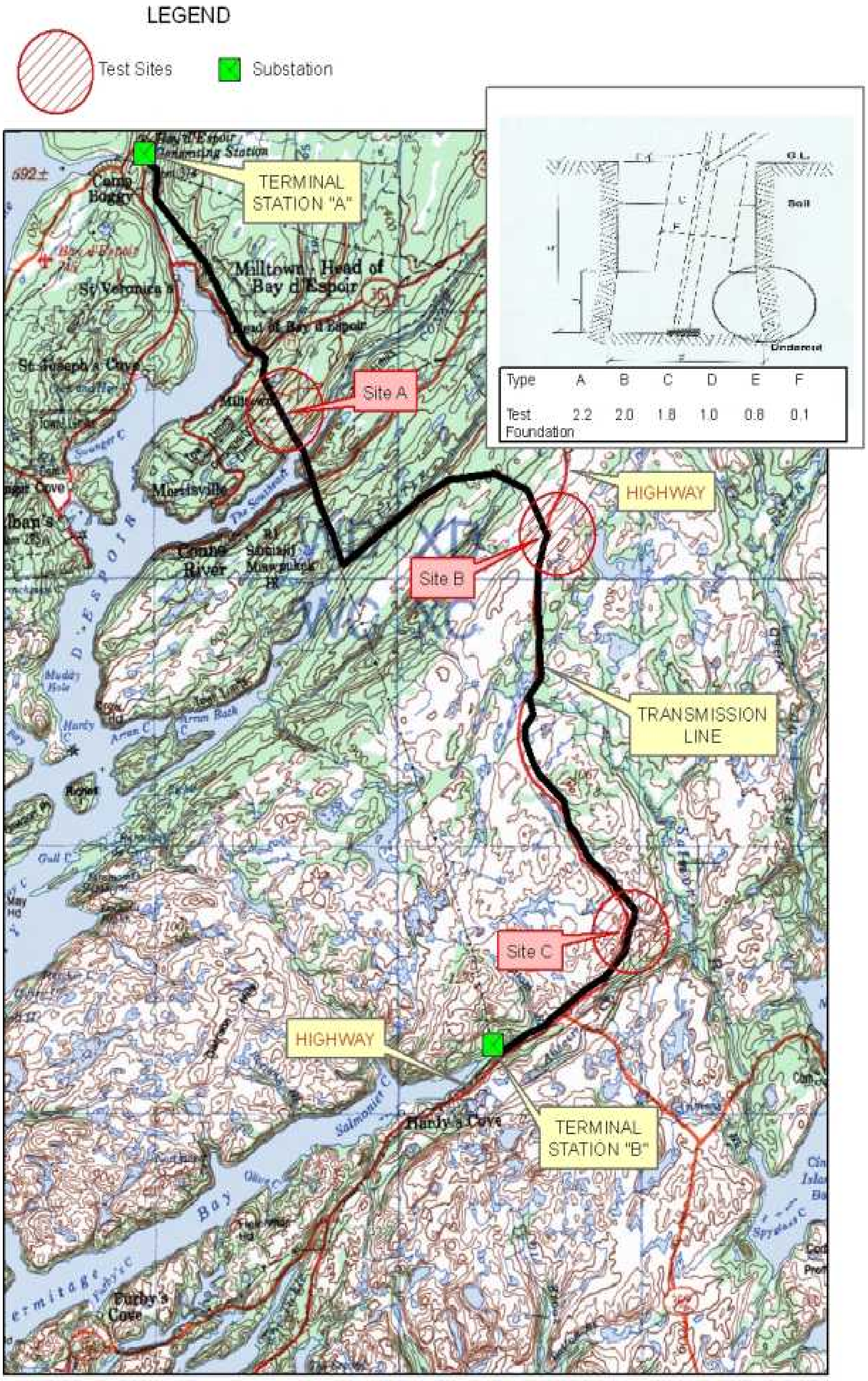
The transmission line engineer can do these tests in three different ways. The engineer can carry out a number of tests with a foundation of fixed dimensions, in homogeneous soils, with fixed installation technique to ensure that all tests will produce similar results in soils of similar categories. This would provide a good coefficient of variation of a specific foundation type and can be compared with the coefficient of variations of other line components such as the support, conductor, and insulator to coordinate the strength as per IEC 60826.

Alternatively, the engineer can vary the dimensions and carry out these tests in similar soils to assess the effect of the geometry and can estimate the coefficient of variation of the capacity.

Finally, the engineer can test a typical foundation of fixed dimensions, in various soil types with repeated tests, thereby obtaining a wide range of coefficient of variations, which will be more representative of a typical transmission line route. Of course, the last two approaches will cost the utility more compared to the test carried out in one soil type.

The overall foundation capacity is primarily dependent on two basic parameters. These are (1) geometrical parameters which define the foundation dimensions and (2) soil strength parameters such as shear strength, cohesion, density, overburden pressure, effective friction angle, ground water table, etc. Although the geometrical parameters can be defined deterministically for a specific foundation type, there is normally a considerable amount of scatter in soil strength data. Therefore, the overall variation of the foundation capacity is a direct function of the variation of soil strength parameters.

If we assume that the engineer has designed a typical foundation and wants to determine the actual or characteristic capacity with certain confidence limits, then it is necessary to test the foundation for a wide range of soil conditions to provide a good level of confidence in the predicted capacity. In testing a specific foundation type, the engineer needs to ask a few questions and these are listed as follows.



Notes: Geotechnical properties at the test sites are given in table 3.5

Figure 3.1 - Typical line route showing the test sites (Topographic Map)

- Is there any data available with respect to this foundation type?
- What design model(s) do I use?
- Is there a standard design model that I can use?
- What subsurface information do I need to collect?
- What sample size do I need to get a reasonable prediction?
- Do I need to collect additional data?
- What criteria should I use for capacity?

Assuming that the specific foundation type is an undercut pad and chimney foundation, and the foundation is to be tested under uplift load, the first thing the designer needs to ask is “what design model is appropriate for predicting the geotechnical design capacity of this type of foundation”. Figure 3.2 depicts a typical undercut pad and chimney foundation.

An undercut foundation is where the base of the foundation is slightly wider than the width at the neck level. The foundation is excavated by machine to the width ‘C’ in figure 3.2. The bottom of the excavation is widened to the width ‘B’ in figure 3.2 using a handheld shovel. The undercut is usually 0.1 m on each side of the foundation. Table 3.1 presents the dimension of a typical pad and chimney foundation.

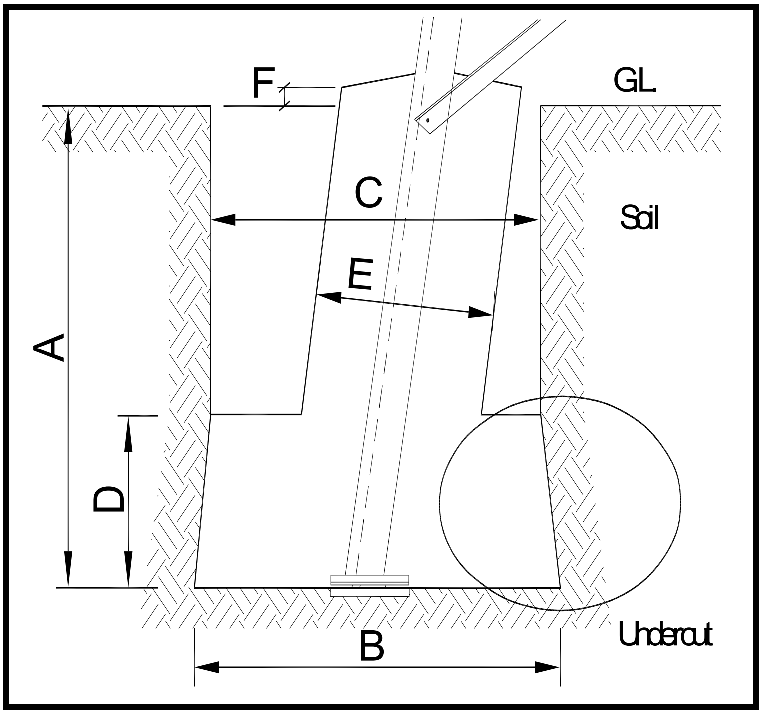


Figure 3.2 - Typical undercut pad and chimney foundation

Table 3.1- Typical dimensions (m) of an undercut pad and chimney foundation

Type	A	B	C	D	E	F
Square Based	2.2	2.0	1.8	1.0	0.8	0.1

CIGRE (2002) presents a review of a number of theoretical design models that can be used to determine the geotechnical design capacity for this type of foundation. Two models are discussed here. These are (1) frustum angle model (inverted truncated cone) in Section 3.2.1 and (2) shear model in Section 3.2.2.

3.2 Uplift Failure Mode

The failure mode will often depend on the type of the installation used. For example, if the foundation excavation is done in such a way that the excavated hole is sized sufficiently to place the foundation vertically or cast in place vertically with the surrounding space filled by compacted backfill, then the failure zone would most likely be covered by both native as well as the backfill soils. Figure 3.3 depicts a situation where the failure zone is in the native soils. However if the excavation is "open pit type" and is sufficiently large then it is possible to have the failure only within the backfill zone (Figure 3.4). In this case, the backfill soil properties would determine the geotechnical design capacity. This is more appropriate for sites with boulder, cobbles etc. where often the excavation cannot be done narrowly or where the walls of the excavation cannot stand up sufficiently safely.

3.2.1 Frustum model (inverted truncated cone)

In this particular model, it is assumed that an inverted frustum starts at the base of the undercut foundation and slopes at an angle, ψ to the vertical to the surface (refer to figures 3.3 and 3.4) and the weight of the soils within this frustum and the foundation weight itself resist the uplift force. Mathematically, this is described as follows:

$$U = V_S \rho_S + V_C \rho_C \tag{3.1}$$

where V_S and V_C are the volume of the soils and the concrete within the frustum while ρ_S and ρ_C are the unit weight of the soils and the concrete respectively. U is the uplift force.

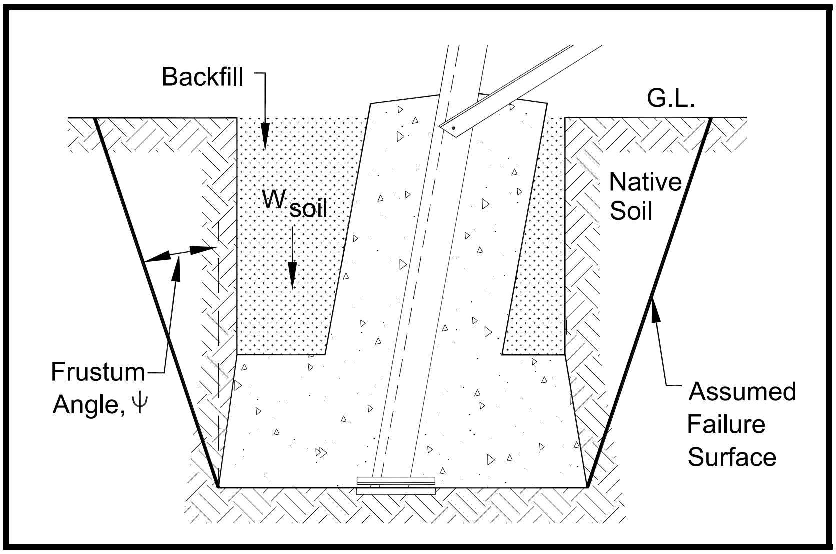


Figure 3.3 - Failure in native soils

IEEE (1985) provides information on the frustum angle, ψ based on soil type, overburden pressure, construction method and the groundwater condition. The submerged weight of the soil should be used where the portion of the foundation is below the ground water line.

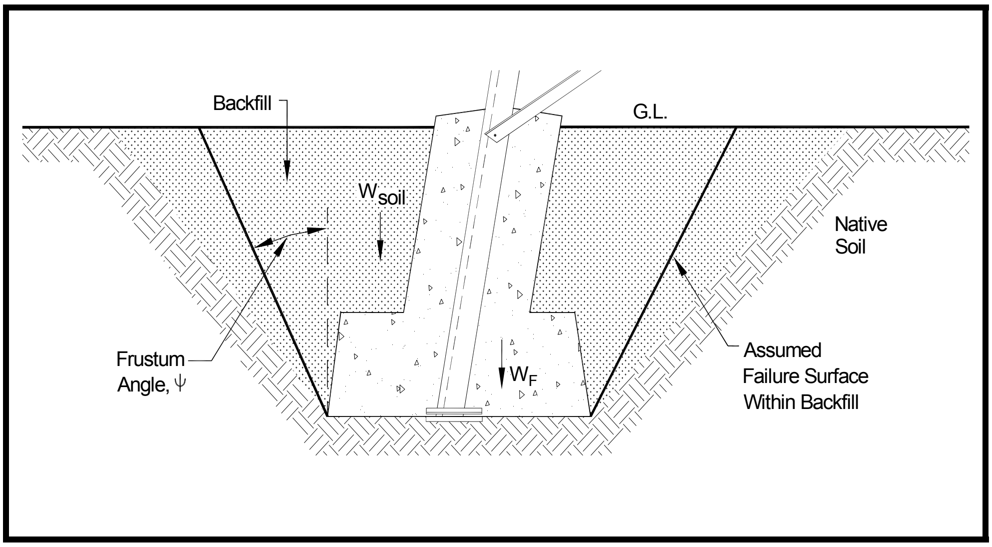


Figure 3.4 - Failure in backfill soils

3.2.2 Shear model

In this model, it is assumed that the uplift force is resisted by shear force acting along the vertical shear surface circumscribing the base of the foundation, the weight of the concrete foundation, W_C and the weight of the soil above the concrete base within the frustum, W_S . The uplift force is determined as:

$$U = S\tau + W_S + W_C \qquad (3.2)$$

where S is the shear surface area, τ is the shear strength.

Although a constant shear strength value of 19.6 kN/m^2 was used by Killer (1953), Buckley (1994) recommended varying shear strength values with depth to ensure that the predicted ultimate capacity is more realistic and comparable with the test capacity. Mors (1965) presented an empirical expression for the frictional resistance (shear capacity):

$$F_0 = S\tau A^{\eta} \qquad (3.3)$$

where η is a constant (1.5 to 2.0), A is the depth of embedment, S is the shear surface area and τ is the shear strength which should be adjusted to 50% of the value if there is groundwater. IEEE (1985) also provides the frictional resistance value for a square foundation :

F_0 = 4 cBA + 2K ρ_S B A^2 $\tan\phi$ (3.4)

where c = cohesion, K= coefficient of lateral earth pressure and ϕ = angle of internal friction. B and A are the width and depth of the foundation respectively.

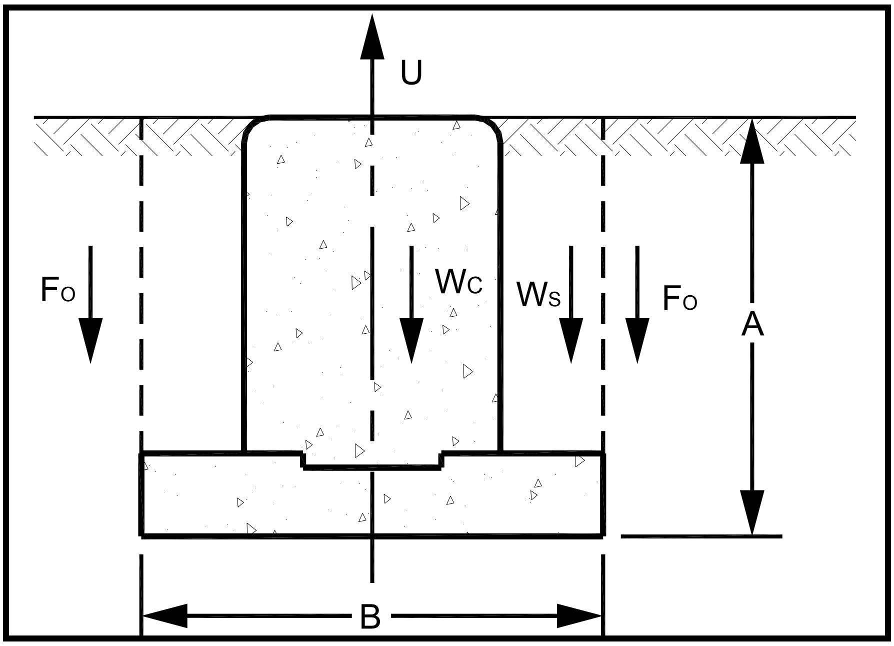


Figure 3.5 - Shear model (IEEE, 1985)

CIGRE (2002) presents a comprehensive summary of various methods available in the literature for determining the uplift capacity of a spread footing.

3.3 Determination of Strength Parameters

The strength parameters required for an undercut pad and chimney foundation under uplift load are :(1) soil density, (ρ_S), (2) concrete unit weight, (ρ_C), and (3) shear strength, (τ), along the depth depending on the model used.

Once the number of sites along the line route (Figure 3.1) has been chosen, soil densities can be obtained by using a sand cone test in the test pit(s) near the installation site. Alternatively, a standard penetration test (SPT) can be carried out in undisturbed soils, to measure the penetration resistance in terms of blow counts (N-value); which can be related to soil characteristics and strength parameters. The sand cone test can be done at various depths for easy profiling; while, the SPT test provides a boring log which can be further interpreted for soil density and angle of internal friction (Brink et al, 1982). The shear strength values, τ can be estimated once the soil classification and the angle of internal friction, ϕ has been determined. For more detail evaluation, sample needs to be tested in laboratory under triaxial load condition. If the excavated hole is entirely backfilled, the soil properties of the backfill should be obtained from historical data or specific test in the backfill soil.

3.4 Example Problem (Calculation of Geotechnical Design Capacity)

Fig. 3.6 depicts a subsurface profile for a specific test site for a pad and chimney foundation shown in Figure 3.2. The N-values from SPT test for native soils in two layers are shown as 20 and 30, respectively; it is also assumed that the native soils will be used as backfill materials with the compaction level better than medium dense soils (N = 30).

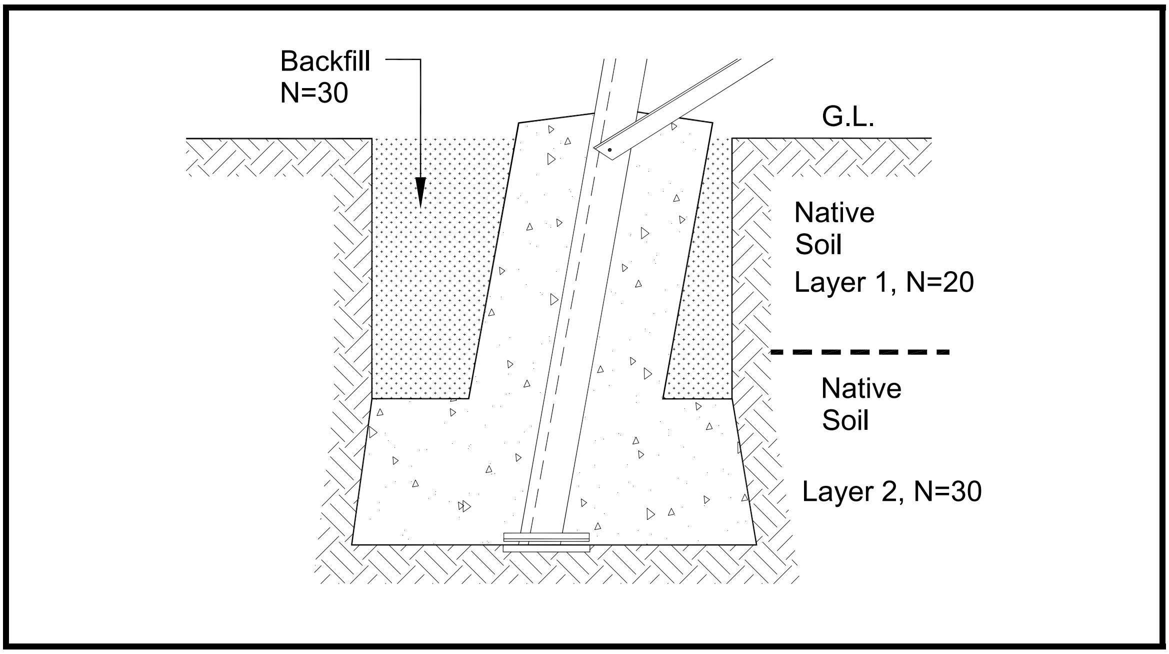


Figure 3.6 - Typical subsurface profile of a specific test site

Based on the above information, the following properties (IEEE, 1985) are chosen in figure 3.7 for the native soils and the backfill soils.

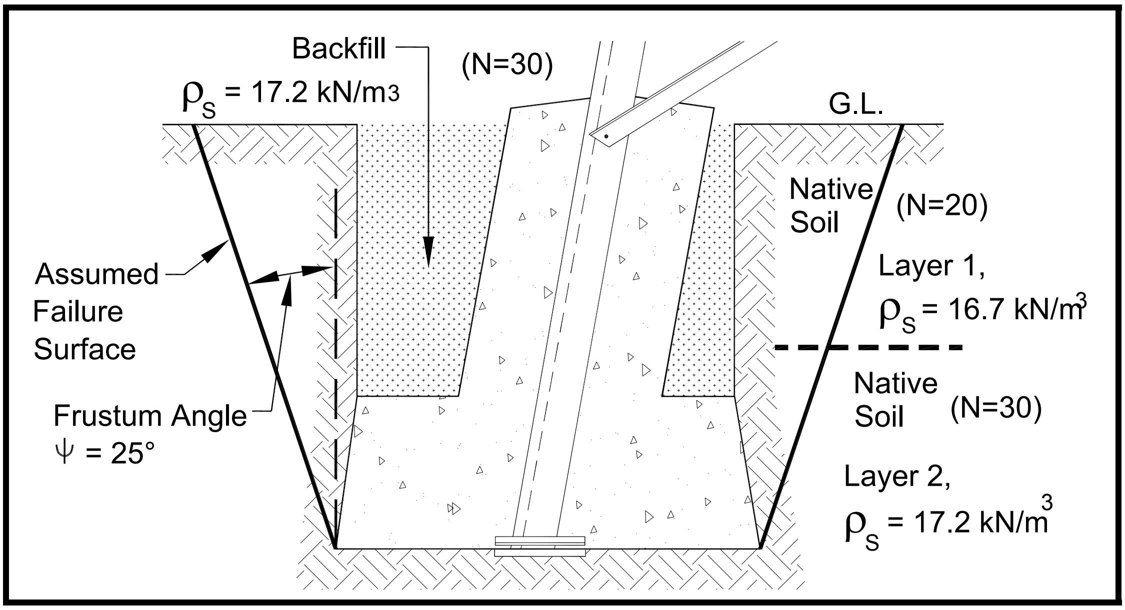


Figure 3.7 - Subsurface soil profiles with assumed failure surface

Since the backfill soil is well compacted granular materials and assuming that the site is well drained, we select the frustum angle, ψ = 25^0 (IEEE, 1985). Fig. 3.7 also depicts the failure diagram based on the frustum model with appropriate unit weights labeled. In the calculation, soil density was assumed as ρ_S =16.7kN/ m^3 (minimum value in figure 3.7) while the concrete unit weight is assumed as ρ_C = 22.6 kN/ m^3. The calculated uplift capacity is based on equation (3.1) where V_S and V_C are computed based on equations (3.5) and (3.9) respectively. The input data are taken from Table 3.1 and figure 3.2 for a square base foundation.

$$V_S = V_{S,F} - V_{C,B} \qquad (3.5)$$

where

$$V_{S,F} = \frac{A}{3}[G^2 + B^2 + (B*G)] \qquad (3.6)$$

$$G = B + 2\,(A*\tan\psi) \qquad (3.7)$$

and

$$V_{C,B} = \frac{D}{3}[B^2 + C^2 + (B*C)] + ((A-D)*(E^2)) \qquad (3.8)$$

where $V_{C,B}$ represents the volume of concrete below the ground line and $V_{S,F}$ is the volume of the frustum. The total concrete volume is given by equation (3.9).

$$V_C = \frac{D}{3}[B^2 + C^2 + (B*C)] + ((A-D)*(E^2)) + (E^2*F) \qquad (3.9)$$

The above formulae are approximate and are based on a square base foundation. If the base is rectangular, the appropriate adjustment should be made to equations (3.6) to (3.9). The calculated geotechnical design capacity for the foundation type shown in figure 3.2 is 375 kN. A sensitivity study was done by varying the failure angle and the result is presented in Table 3.2.

Table 3.2 - Foundation geotechnical design capacity (kN) under various frustum angles (ψ) assumptions

Frustum angle	15-degree	20-degree	25-degree	30-degree
Geotechnical design capacity (kN)	277	322	375	439

3.5 Full-scale Test - Field Program

The field testing program normally consists of the following steps

- Selection of sample size

- Selection of the test sites
- Subsurface investigation
- Foundation installation
- Instrumentation and data collection
- Loading scheme
- Loading sequence
- Data reduction
- Test results

Since this report describes the uplift test for a pad and chimney type foundation we will describe the steps required to carry out this type of test only. Other foundation tests such as drilled shaft under moment may require additional information and can be obtained from the test report (EPRI 2197, 1982). However, the overall steps would be very similar to those described here.

3.5.1 Selection of sample size

Section 2.5 has provided the required background information on how to estimate the sample size for cases where the standard deviation of foundation test capacity data is either known or unknown. The user can use this information as a guide to estimate the sample size for specified confidence level on the estimated mean test capacity value of foundation test capacity data (figure 2.4). For sample size less than 30, a correction needs to be made based on Student–t distribution.

3.5.2 Selection of test sites

As mentioned earlier, the user needs to know whether the tests will be done at one specific site or at several sites along the transmission line route (Figure 3.1). The key parameter for selecting a test site could be the access as well as its typical soil characteristics and distributions with respect to line routing.

3.5.3 Subsurface investigation

Depending on the testing program, the subsurface investigation could vary between simple to very complex. For example, the uplift test program requires the determination of a number of soil strength parameters. These are: soil types (cohesive versus. cohesionless), relative densities, unit weight and the $c-\phi$ properties. This information can be obtained from a sand cone test, SPT test or any other suitable test program (laboratory or in-situ).

3.5.4 Foundation installation

The foundation is excavated by machine to the width ‘C’ in Figure 3.2. The bottom of the excavation is widened to the width ‘B’ using a handheld shovel. Once the foundation is in place, the surrounding hole is backfilled with well compacted granular materials. The strength parameters of the compacted backfill soil at each layer (300mm to 450mm) can be determined from a nuclear densiometer test, sand cone test, etc. The detailed installation procedure for various foundation types can be found in CIGRE (2006) technical brochure.

3.5.5 Loading scheme and sequence

Figure 3.8 presents a loading test arrangement of a grillage foundation under inclined uplift load. The test was arranged to determine the anchor capacity of a grillage foundation in cohesionless soils for a 230 kV suspension guyed V-tower. The excavation was an “open pit type” and the capacity was primarily based on the backfill soil properties (Prasad and Haldar, 2001).

Figure 3.8 - Grillage foundation under uplift load (Prasad and Haldar, 2001)

The loading sequence can be simply one load cycle where the load is applied until the foundation fails or it could be cyclic where it follows a loading-unloading sequence. The rate of loading can also be varied. In this study, a cyclic loading was used.

3.5.6 Instrumentation and data collection

For typical uplift test, the user needs to record the load and displacement of the foundation. The load can be measured either using a dynamometer or electronically through a load cell. Similarly, the displacement can be measured using a survey transit, dial gages or using LVDT. The engineer can use any one of these or all depending on the accuracy and the verification desired in the measurement process.

3.5.7 Data reduction

This will include taking out any offsets or any other transformations required in the load and displacement measurements.

3.5.8 Test results

Once the data has been reduced, the final test result can be plotted in terms of load versus displacement as shown in Figure 3.9. In the next section, the user will be presented with the methodology to determine the ultimate capacity of the foundation. The capacity is performance based and the method follows IEC 61773.

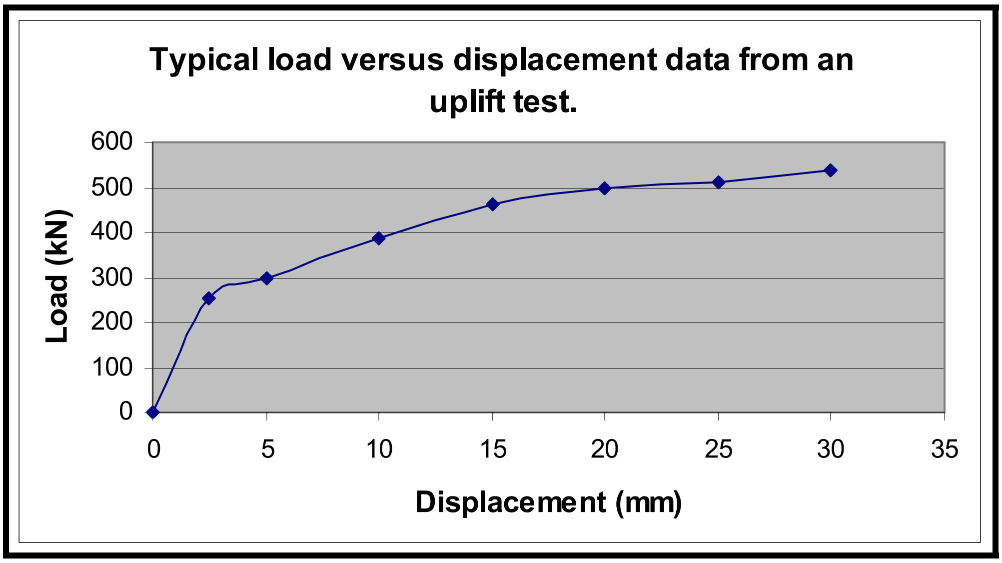


Figure 3.9 - Typical load displacement plot for a pad and chimney foundation

3.6 Interpretation of Test Data

The performance criterion of a foundation is normally based on three limit capacities. These are: (1) elastic limit capacity, (2) working (damage) limit capacity and (3) ultimate limit capacity. The ultimate limit capacity of a foundation is defined when it has reached its full plastic limit, such that little or no additional load is sufficient to produce considerable deflection (Davidson, 1981, EPRI EL 2197). However, the ultimate limit capacity may not be realized in many full-scale tests because of the various constraints imposed by the test set-up and the in-situ soil conditions. Therefore, some practical methods are required to determine the ultimate capacity.

For a given load displacement data set, the utility engineer can determine the elastic limit capacity and the ultimate limit capacity following the procedures described in the IEC 61773 entitled “Testing of Structure Foundations for Overhead Lines”. However, the definition of working limit (damage limit) capacity is more subjective.

Buckley (1994) carried out a number of foundation tests in various soil conditions in Ireland. The elastic limit load was defined as the load when the foundation movement upon unloading was insignificant. The working limit load was defined as the load associated with a 10 mm foundation movement. Details on the selection of 10 mm displacement are given in section 4.4.2. This is similar to “Damage load” defined in IEC 60826. The ultimate load was defined as the final pull out of the foundation. Since one can get a large movement under ultimate condition, it was reported that this load is not practical to use because the tower may not be able to sustain such a large movement. Buckley (1994) has recommended a 10 mm displacement as the determination of the working limit capacity for spread footing. For directly embedded single pole structure, the capacity associated with a 2^0 rotation can be used to define the damage limit (IEC 60826, 2003).

To illustrate the various methodologies for determining the test capacities, an example is presented. Figure 3.9 depicts the typical load displacement curve obtained from a full-scale test carried out in Ireland (ESB report). The elastic limit capacity at 2.5 mm displacement is 253 kN while the working limit capacity at 10 mm displacement is 388 kN. However, the ultimate limit capacities are determined based on the various methodologies described in the IEC 61773 document, which can vary considerably depending on the specific method used.

The same is also true if the user chooses to use different displacement criteria for the working limit capacity or damage limit as per IEC 60826 (2003). IEC Standard provides various methods to determine the limit capacities. Table 3.3 presents the comparison of ultimate capacities based on various methods described in IEC 61773.

Table 3.3 - Comparison of ultimate capacity of an undercut foundation (IEC 61773)

Methods	Tangent Intersection (TI)	90% Criteria	Parabolic	Hyperbolic	Slope Tangent (ST)	
					4 mm	10 mm
Estimated Ultimate Capacity (kN)	323	542	620	625	362	450

In a study conducted by EPRI (EPRI EL 2870), a 4 mm displacement was used in determining the ultimate capacity. The estimated mean test capacity based on a 4 mm displacement is 362kN (figure 3.9) while that using a 10 mm displacement is 450kN.

In the above sections, a specific example (figure 3.9) is presented to determine the various test capacities of an undercut pad and chimney foundation tested by ESB in Ireland. However, depending on the method used, the estimated ultimate capacity of this foundation can vary considerably; it could range from 323kN to 625kN. The mean value is 487kN with a coefficient of variation of 26.5%.

The calculated geotechnical design capacity can also vary significantly depending on the specific failure model used. As stated earlier, the overall variation of the foundation capacity is a direct function of the variation of soil strength parameters. The uncertainty in the model predicted geotechnical design capacity, R_n can be accounted theoretically only through the joint density function of all these parameters in a probabilistic sense. However, in practice this is not possible because the joint density function of all these parameters is often complex and is unknown. Obviously, the designer can obtain different ratios of $m = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity}$ depending on what methods are used in determining these limit capacities.

3.7 Test Data Collection-Field Programme, an Example

Let us assume that the design engineer wants to test a pad and chimney foundation (figure 3.2) to determine the characteristic capacity with a defined exclusion limit. The engineer has a number of choices. The engineer can test the foundation either in one specific site (Site A in Figure 3.1), or can arrange a testing programme along the line route at different site locations. The test can be repeated at each site to obtain a dispersion of the test capacity (Sites A, B and C in Figure 3.1), which can then be used in determining the confidence level. Also, by testing at various locations along the line route, the engineer will obtain the limit capacities, which are more representative of the entire line route.

3.7.1 Site specific repeat test data for single geotechnical design capacity (R_n)

The geotechnical characteristics of SITE A are obtained by SPT tests and Table 3.4 presents the data. The dimensions of this test foundation are presented in Table 3.1. Using equations (3.1) and (3.5 to 3.8), the calculated geotechnical design capacity based on a ψ =30^0 frustum angle (Section 3.2.1) is 439kN. To calculate this capacity, the minimum value of the soil density used is 16.7kN/m^3 (figure 3.7). The concrete unit weight, ρ_C is assumed to be 22.6kN/m^3. The engineer performed three repeated tests at this site (Site A in figure 3.1) and obtained working limit capacities at 10 mm displacement as 500kN, 540kN and 600kN respectively. These values can be obtained from actual tests similar to the plot presented in Figure 3.9. Figure 3.10 depicts the plot, which shows the scatter in the test capacity with respect to one fixed geotechnical design capacity value.

Table 3.4 - Geotechnical properties at Site A (Figure 3.1, IEEE, 1985)

Density	N-Values (SPT)	Relative Density, D_r in Percent	Angle of Internal Friction, ϕ	Unit Weight (kN/m^3)
Medium	10-30	35-65	35^0 - 40^0	16.7-19.6

Since the test capacities are representative for one single geotechnical design capacity value, there is no correlation between the test capacity and the geotechnical design capacity. A correlation defines the degree of measure of the linear relationship between the test capacity y and the geotechnical design capacity x. In this case, it relates the linearity between the test capacity and the geotechnical design capacity. In statistical data analysis, the degree of predictability will often depend on the quality of the dataset (mutual dependence between test capacity and geotechnical design capacity) and this is often measured by the statistical correlation coefficient r. This will be discussed in detail later in sections 4.0 and 5.0.

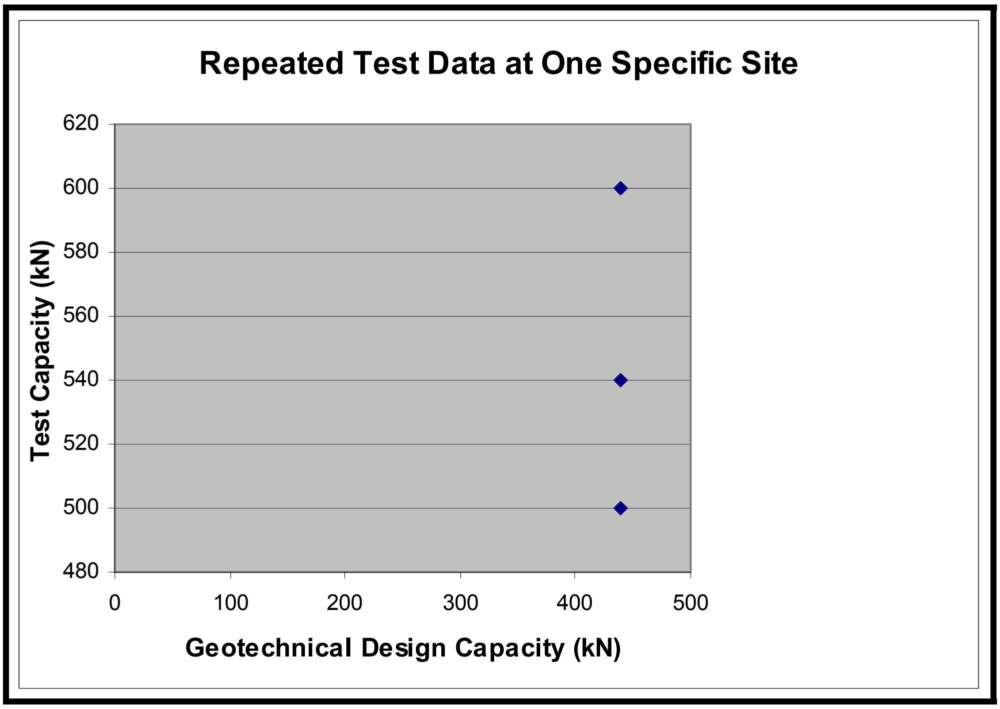


Figure 3.10 - Repeated test data for an undercut pad and chimney foundation-project specific

3.7.2 Site specific test data along the line route

Alternatively, the engineer decides to test this foundation type at three different locations along the line route considering that the capacities will be more representative of the soil conditions along the line route depicted, in Figure 3.1. These sites are marked in figure 3.1 as Site A, Site B and Site C; respectively. Based on SPT test data, Table 3.5 provides the soil characteristics and the estimated soil densities.

Table 3.5 - Geotechnical properties at three sites (Figure 3.1-IEEE, 1985)

Site Locations	Density	N-Values (SPT)	Relative Density, D_r in Percent	Angle of Internal Friction, ϕ	Unit Weight (kN/m^3)
Site A	Loose	4-10	15-35	30^0 - 35^0	13.7-16.7
Site B	Medium	10-30	35-65	35^0 - 40^0	16.7-19.7
Site C	Dense	30-50	65-85	40^0 - 43^0	19.7-21.6

The model foundation shown in Figure 3.2 is again used to calculate the geotechnical design capacity for the three different soil types. Using the frustum angle model, the calculated geotechnical design capacities are 379kN, 439kN and 498kN respectively for loose, medium and dense soil conditions. The damage capacities at 10 mm displacement are assumed to be 450kN, 540kN and 680kN respectively. Figure 3.11

depicts the plot for these tests. Table 3.6 presents the data with m_i values calculated for these three data points where

$$m_i = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity} \quad (3.10)$$

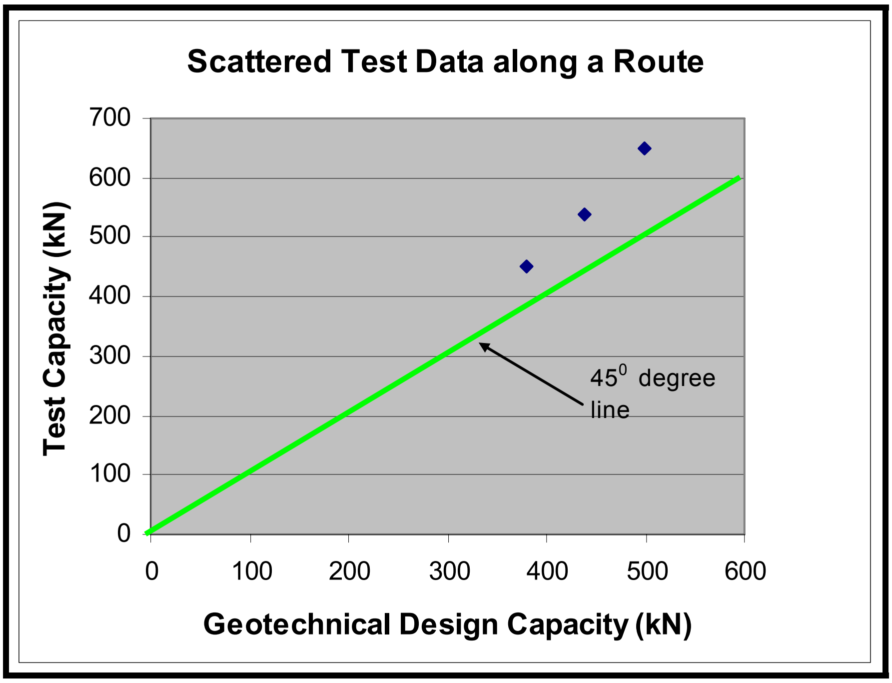


Figure 3.11 - Uplift test -scattered test data along the line route (damage limit capacity at 10mm displacement

Table 3.6- Test data versus geotechnical design capacities

Foundation Capacity (kN)	Site A	Site B	Site C
Geotechnical design capacity	379	439	498
Test Capacity (at 10mm displacement)	450	540	680
$m_i = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity}$	1.19	1.23	1.37

3.8 Summary

This section presented a review of the various steps involved in the assessment of foundation capacities from test data as well as from theoretical model predicted design capacities. The latter is defined as geotechnical design capacity value. This includes: (a) determination of the geotechnical design capacity based on specific model(s), (b) field test programme to determine the test capacity and associated strength parameters for the model, (c) sample size requirement for the test programme, and (d) testing programme and interpretation of data according to IEC

61773. Finally two typical databases, which the designer may encounter, are described. In the first case, three repeated tests are done at one specific site for a single geotechnical design capacity value while in the second case the tests are carried out at three different sites along the transmission line route (scattered test data). The next two sections will outline the calibration methodology and the appendix will use the scattered test data to demonstrate the methodology of computing the foundation strength factor based on a linear regression model.

4.0 CALIBRATION OF FOUNDATION STRENGTH FACTOR ($\overline{m}$ - MODEL OF BUCKLEY)

4.1 General

This section of the report presents a framework to calibrate the foundation strength factor based on a limited number of full-scale test data. The methodology is based on a simple mathematical model where the statistical properties of m_i values (Table 3.6) are used following Buckley (1994). Later, various assumptions of this model are examined and a linear regression model is proposed for general application. The correlation effect is discussed and its importance in the data analysis is emphasized.

The uncertainty in the prediction is primarily introduced through two factors: (i) model error and (ii) data error due to finite sample size. However, to develop a general methodology for the calibration of the foundation test data against the model predicted geotechnical design capacity (geotechnical foundation design model), the foundation designer needs to assess the quality of the dataset (correlation) and its influence on the predicted test capacity. The predicted test capacity is analogous to the characteristic capacity described in section 2.4.

In statistical data analysis, the degree of predictability will often depend on the quality of the dataset (mutual dependence between test capacity and geotechnical design capacity) and this is often measured by the statistical correlation coefficient, r.

The correlation coefficient of the dataset is determined from equation (4.1):

$$r = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{(n-1)\, s_x s_y} \qquad (4.1)$$

where s_x, s_y are the standard deviations of the geotechnical design capacity values x_i and the test capacity values y_i ; $\overline{x}$ and $\overline{y}$ are the mean values of the geotechnical design and test capacities respectively. The last two symbols are also analogous to $\overline{R}_n$ (mean geotechnical design capacity value) and $\overline{R}$ (mean test capacity value). The variables $\overline{x}$ (or $\overline{y}$) and s_x (or s_y) are computed from equation (2.1) and (2.2) replacing R_i and $\overline{R}$ by x_i (or y_i) and $\overline{x}$ (or $\overline{y}$) respectively. For convenience, x and y will represent geotechnical design capacity and test capacity values. For example, figures 4.1 (a) and (b) present two datasets with very different correlation properties.

Two scenarios are considered. In the first scenario, it is assumed that the tests have been carried out only at specific site with one type of foundation (repeated test data) while in the second scenario the data is available for multiple test sites along the transmission line route. This is identified as the scattered test data (refer to section 3.7).

4.2 $\overline{m}$ - Model – No Correlation (Repeated Test Data)

The $\overline{m}$ - model of Buckley is based on the ratio of the test capacity and the geotechnical design capacity.

$$m_i = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity} = \frac{R_i}{R_n} = \frac{y_i}{x_i} \quad (4.2)$$

$$\overline{m} = \frac{\sum_1^n (\frac{y_i}{x_i})}{n} \quad (4.3)$$

The standard deviation and the coefficient of variation of the m_i values are computed from equations (4.4) and (4.5)

$$s_m{}^2 = \frac{1}{n-1}\sum_1^n (m_i - \overline{m})^2 \quad (4.4)$$

$$v_m = \frac{s_m}{\overline{m}} \quad (4.5)$$

Since $x_i = \overline{x}$, for the repeated test data presented in Section 3.7.1, the correlation coefficient is zero ($r = 0.0$) from equation (4.1) indicating that the data is unique. Therefore, the exclusion limit for the repeated test data is computed from the test data only. In this case, the three data points are 500kN, 540kN and 600kN respectively (figure 3.10). The geotechnical design capacity for all three tests is 439kN (reference to section 3.7.1).

Using equations (2.1-2.3), the mean test capacity $\overline{R}$ = 556 kN and the coefficient of variation, $v_R = \frac{s_R}{\overline{R}} = \frac{115.9}{556} = 0.20$

Therefore, the characteristic capacity with a 10% exclusion limit (90% confidence level, one sided), is obtained from equation (2.11):

$R_{C,10\%} = \overline{R}\ (1 - k_\alpha\ v_R)$

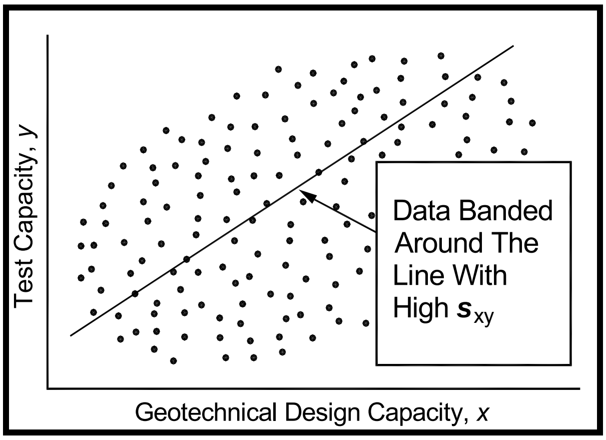


Figure 4.1 (a) - Low Correlation

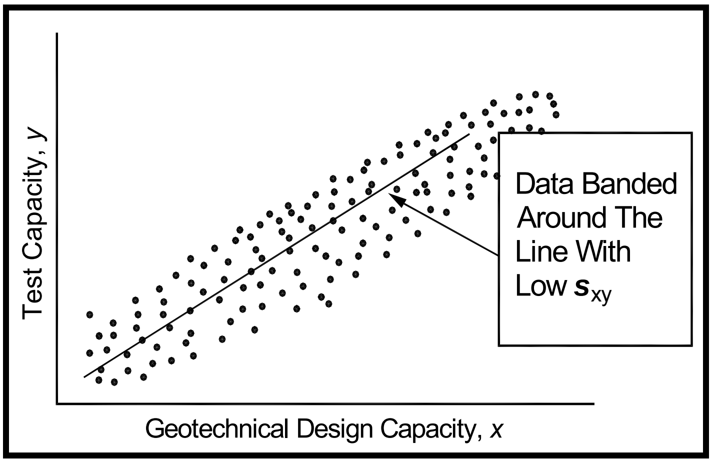


Figure 4.1 (b) - High Correlation

According to equation (2.18), the foundation strength factor, ϕ_F is defined here as the ratio between the characteristic capacity, R_C and the geotechnical design capacity, R_n. Based on the repeated test data, $\overline{R}$ = 556kN, R_n = 439 kN and k_α is 1.28 from Table 2.1. Therefore, $R_{C,10\%}$ = 0.744 $\overline{R}$, where $\overline{R}$ = 1.266 R_n and the foundation strength factor, ϕ_F = 0.744 x 1.266 = 0.942. This foundation strength factor value can be used directly in equation (2.18) following IEC 60826 (2003).

For a finite sample size n = 3, k_α in equation (2.11) should be replaced by $t_{\alpha,\nu}$ and

$$R_{C,10\%} = \overline{R}\ (1 - t_{\alpha,\nu}\ v_R) \tag{4.6}$$

where $t_{\alpha,\nu}$ = 1.90 from Table 2.2. Therefore, ϕ_F = 0.785 considering sample size effect; correspondingly the foundation strength factor is decreased by 20% when the sample size effect is considered.

Since the data for the repeated tests (reference to figure 3.10) are only valid for one specific test site and for one size of foundation, it has no correlation with respect to other test site locations along the line route unless the foundation designer can be sure of another location where the subsurface information is similar to the test location and the foundation has a similar aspect ratio (depth/width). For undercut foundation with rectangular base, an equivalent diameter D^0 may be considered as alternative for the width.

4.3 $\overline{m}$ - Model-With Correlation (Scattered Test Data)

In the second case when the data is scattered and the tests are undertaken at three different locations along the transmission line route (sites A, B and C in figure 3.1), the foundation designer can expect that the data from various test sites will have a degree of correlation. However, the degree of correlation will depend on how well the geotechnical design model can predict the foundation capacity at the various test sites. Using equation (4.1), the statistical correlation coefficient is r = 0.90 for the data presented in Table 3.6.

Based on figure 3.11 and the m_i data presented in table 3.6, it is obvious that the geotechnical design capacity based on a 30^0 frustum angle model is under-predicting the test capacity (working limit at 10 mm), since all three geotechnical design capacity values are below the 45^0 line passing through the origin. The mean, $\overline{m}$ (slope) and the coefficient of variation, v_m of these tests are obtained from equations (4.2 – 4.5) and are 1.26 and 0.085 respectively. Figure 4.2 presents the average slope, $\overline{m}$. This means that on average, the model predicted geotechnical design capacity underestimates the test capacity by 26%.

The foundation strength factor, ϕ_F is obtained from equation (4.3)

$$\phi_F = \frac{R_c}{R_n} = \overline{m}\,(1 - k_\alpha * v_m) \qquad (4.7)$$

The above equation was presented by (Buckley, 1994, $\overline{m}$ -model) and is recommended in CIGRE (2002).

The characteristic capacities for 5% (ASCE) and 10% (IEC 60826) exclusion limits can be obtained initially first from equation (4.7) neglecting the sample size (CIGRE 2002). The k_α values for 5% and 10% exclusion limits (95% and 90% confidence levels) are obtained from Table 2.2 and these values are 1.645 and 1.282 respectively.

$R_{C,5\%}$ =1.26(1 - 1.645*0.085) R_n = 1.0838 R_n ; ϕ_F =1.084

and

$R_{C,10\%}$ =1.26(1 - 1.282*0.085) R_n =1.1226 R_n; ϕ_F = 1.122

For finite sample size, (n less than 30, Section 2.1), k_α should be replaced by $t_{\alpha,\nu}$ in equation (4.7) and

$$\phi_F = \frac{R_c}{R_n} = \overline{m}\,(1 - t_{\alpha,\nu} * v_m) \qquad (4.8)$$

The $t_{\alpha,\nu}$ value is obtained from Table 2.2. Table 4.1 summarizes the foundation strength factor values with the plot shown in figure 4.2. Correspondingly, the foundation strength factor values are reduced by 6% and 15% when the sample size effect is considered $(\nu = n - 1 = 3 - 1 = 2)$.

$R_{C,5\%}$ =1.26(1 - 2.92*0.085) R_n = 0.947 R_n and ϕ_F =0.947

and

$R_{C,10\%}$ =1.26(1 - 1.90*0.085) R_n = 1.056 R_n and ϕ_F = 1.056

Table 4.1 - ϕ_F Values from equations (4.7 & 4.8)

Sample Size	$\phi_{F,5\%}$	$\phi_{F,10\%}$
n is large	1.084	1.122
n = 3, $\nu = 2$	0.947	1.056

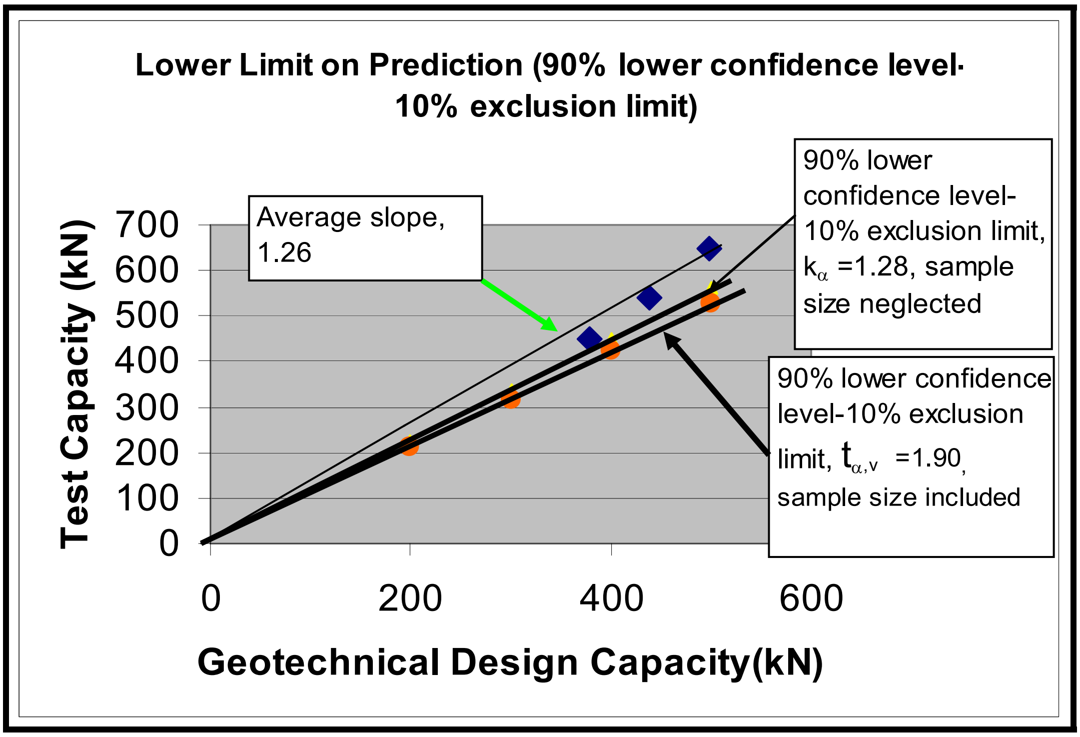


Figure 4.2 - Predicted test capacity based on $\overline{m}$ -model

4.3.1 Discussion

The simplicity of the above model (equation 4.3) is that once the relationship between the characteristic capacity, R_C and the geotechnical design capacity value, R_n is established through the foundation strength factor, ϕ_F , the geotechnical design capacity is easily determined from the characteristic capacity with a defined exclusion limit (e %). However, the method is based on the following five **key** assumptions.

- The relationship between the foundation test capacity versus model predicted geotechnical design capacity is derived from a linear regression model.
- The intercept is assumed to be zero, i.e., the regression line passes through the origin.
- The residual error, ε_i (the difference between the estimated mean test capacity and the actual test capacity), is non constant, i.e. it increases as the

model predicted geotechnical design capacity (R_n values along the x-axis) also increases. The determination of residual error is explained in Section 5.1.

- The sample size, n is large and the data follow a normal distribution.
- Correlation of the data is high and the coefficient of variation, v_m is small (< 0.10).

A low correlation value, r^2 would imply that there is no linear relationship between the foundation test capacity and the geotechnical design capacity. Table 4.2 presents some suggested correlation values and the degree of linear association between the test capacity, y and the model predicted geotechnical design capacity, x.

Table 4.2 – Suggested ranges of correlation value

Correlation Value, r^2	**Linear association between x and y**
Low, $r^2 \leq 0.3$	weak
Moderate $(0.3 \leq r^2 \leq 0.8)$	moderate
High, $r^2 > 0.8$	strong

Each of the above assumptions requires to be clearly understood. There are cases where these assumptions may not apply. For example, the assumption that the line passes through the origin needs to be validated. The assumption that the residual errors vary in a non-constant manner also needs to be validated. Without these validations, the lower limit on the prediction interval of the test capacity (analogous to characteristic capacity with exclusion limit, section 2.4) for a single geotechnical design capacity value, R_n and the extrapolation of the data across any geotechnical design capacity values (data along x-axis) could be **problematic and could lead to erroneous conclusions**. This will be discussed in detail in section 5 under "linear regression model" and in section 6 under "interpolation versus extrapolation".

4.4 Example Problem- ESB Data (Buckley, 1994)

4.4.1 Background

Electricity Supply Board (ESB), Ireland carried out a total of 68 full-scale foundation tests during the period 1984 to 1989. Fifty four (54) of these foundations were of the pad and chimney type and cast, where possible against undisturbed soil and undercut where feasible. These tests were carried out on foundations newly installed primarily for research purposes; while the remaining fourteen (14) old foundations were also pad and chimney type and made available for testing because of the removal of an existing 40 year old line. These were installed in the disturbed soils (c- ϕ). All of the fifty four (54) foundations tested were installed in various types of soils and using different installation techniques. Buckley (1994) has reported in detail the foundation test results and the next paragraph is a direct quote from his recent communication (Buckley, 2007).

“The tests were designed to provide generic test parameters for the vast majority of tower foundations on the ESB system. The vast majority of tower foundations in ESB system were set in consolidated glacial till (c-ϕ) soils and a substantial minority in cemented sand and gravel (ϕ) soils. There were also areas overlain, with peat and under laid with glacial till at shallow (1-2 meters) and occasionally deep (up to 6 meters) depth. The water levels in the glacial till were generally high and the cohesionless soils were usually dry. The selection of test sites was based on this spread of ground conditions and on the availability of the sites for extensive and disruptive foundation setting”.

Except for the existing 14 foundations, all the research foundations were installed in cohesive and/or cohesionless soils. Nine (9) of these installed foundations were in granular soils where the foundations were above the ground water table; while 22 of the research foundations were in soils partly below the ground water table and the soils were overconsolidated glacial tills (c-ϕ soils). The soil conditions at the existing fourteen foundation sites varied from cohesive to silty soils (c-ϕ). The majority of these foundations was pad and chimney type (figure 3.2) and is classified as undercut foundations.

All tests were completed to determine the ultimate capacity of the foundations under uplift condition. Uplift loads were applied using high capacity cranes, while the horizontal (shear) loads were applied via a tractor mounted winch. The typical testing period was 20 minutes and in some instances loads were cycled. The tests were carried out until the foundation failed, although in some instances the ultimate foundation capacity could not be reached due to limitations of the test equipment.

4.4.2 Defining foundation capacities

The ESB study defined the foundation failure capacity in terms of the foundation movement. These were: (i) elastic limit load, (ii) working limit load and (iii) ultimate limit load. The elastic limit load was defined as the load when the foundation movement upon unloading was insignificant. The working limit load was defined as the load associated with a 10 mm foundation movement (similar to “Damage load” defined in IEC 60826) and was based on the following criteria:

- The load was normally 60% -70% of the ultimate load determined based on the other criteria such as the tangent intersection, slope tangent etc (IEC 61773).

- A number of these foundations initially tested with the lower capacity crane equipment (50 tonnes) experienced movement more than 10 mm but did not fail. These were later tested again to failure with the larger equipment (200 tonnes) and showed the same results under the repeat tests.

- Earlier work by (Mors, 1964) indicated that the tower can sustain a 10 mm differential leg movement without any damage to the tower.

The ultimate load was defined as the final pull-out capacity of the foundation. Since usually the foundation displacement under ultimate condition is large, it was reported that is not practical to use this load because the tower would not be able to sustain such a large displacement. Therefore, in all subsequent data analyses, a working limit load at 10 mm is used as the ultimate capacity.

4.4.3 Design models

In calculating the geotechnical design capacity of each of the foundations tested, Buckley (1994) used three different geotechnical design models. These were (1) the frustum cone model, (2) German design model (DIN EN 50341-3-4, 2001) and (3) the shear model (Killer, 1953). Details of these models are given in this paper. According to Buckley (2007), none of these models accurately reflects the failure mechanisms of spread footings. Also, the assumed soil density and the concrete unit weight used to determine the geotechnical design capacities for various foundation types did not always reflect actual in-situ properties but are simple to use, provided the designer understands the limitations. The geotechnical design capacities determined for ESB data are based on a fixed soil density and a concrete unit weight. The progressive failure model introduced in 1994 is more realistic but requires more detailed soil knowledge which is seldom available; it provides the ultimate load. The foundations were arranged into various categories based on their actual dimensions and the soil types in which they were installed. A total of 10 categories were used to subdivide all foundations groups tested. The primary objective of the study was to compare foundation strength factor values for various foundation types based on the different models.

4.4.4 Foundation data for uplift test (Category 2-9 tests)

The nine data points for the "Category 2" foundation test are analyzed here using the $\overline{m}$ -model (equations 4.7 and 4.8). Table 4.2 provides the data, i.e. the geotechnical design capacity based on a 30^0 frustum angle model (section 3.1.1) and the measured test capacity at 10 mm displacement (working limit capacity) for each test. This data is taken directly from ESB report. The unit weights used are 22.56kN/m^3 for concrete and 15.7kN/m^3 for soil respectively. Table 4.3 also presents two dimensions for each foundation, the depth and the equivalent diameter at the base. The equivalent diameter was used since all of the tested foundations had a rectangular base (Buckley, 2001).

Table 4.3 - ESB Test Data (CiGRE, 2001)

Foundation Type	ML1A	ML1B	ML1C	ML1D	ML2A	ML2B	ML4B	ML8A	ML8B
Depth-A (m)	1.5	1.7	1.9	2.1	2.3	2.3	2.7	2.2	2.3
Equivalent Base Diameter-D^0 (m)	1.53	1.34	1.63	1.49	1.54	1.52	1.65	1.66	1.59
Aspect Ratio- ($\frac{A}{D^0}$)	0.98	1.27	1.65	1.41	1.49	1.51	1.63	1.32	1.45
Model Predicted –Geotechnical design capacity (kN)	154**	163	249	270	332	328	489	332	345
Test Capacity at 10 mm displacement (kN)	204*	248	364	416	389	306	680	383	418
$m_i = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity}$	1.32	1.52	1.46	1.54	1.17	0.93	1.39	1.15	1.21

* Numbers have been rounded to nearest digit

Table 4.4 presents the data statistics. The mean values of the test and the geotechnical design capacity are 378kN and 295kN respectively, while the coefficient of variation values are approximately 35%. The $\overline{m}$ and v_m values are 1.30 and

0.154, respectively. These values can be calculated from equations (4.2) to (4.5). Similarly the mean value and the coefficient of variation of the aspect ratio are also given in Table 4.4.The model in general, appears to be conservative because of large bias, i.e., the model generally underestimates the test capacity value by 30%.

Table 4.4 - Data Statistics

n	$\bar{x}$	s_x	$\bar{y}$	s_y	$\bar{m}$	v_m	Average Aspect Ratio	Coefficient of Variation of Aspect Ratio
9	296	102	378	135.4	1.3	0.154	1.36	0.15

Table 4.5 presents the foundation strength factor values for 5% and 10% exclusion limits (prediction with 95% and 90% confidence levels respectively).

Table 4.5 – Foundation Strength Factor Values for 9-point dataset (ESB Data)

Confidence Level	90%	95%
n	9	9
$\bar{m}$	1.30	1.30
v_m	0.154	0.154
k_α -values	1.28	1.64
ϕ_F for k_α -values (equation 4.7)	1.04	0.97
$t_{\alpha,\nu}$ where ν = n-1=8 (Table 2.2)	1.40	1.86
ϕ_F for $t_{\alpha,\nu}$ (equation 4.8)	1.02	0.93

4.5 Summary

This section of the report presented a simple mathematical model to compute the foundation strength factor following Buckley (1994). The mathematical model requires the statistical properties of the m-values defined as the ratio of the foundation test capacity and the geotechnical design capacity. The various underlined assumptions of the model are presented and a need for a more robust linear regression model has been identified when the specific assumptions are not met. This is presented in Section 5 under the linear regression model ("alternative method"). An example design problem for a pad and chimney foundation is used in section 5 to demonstrate the applications of simple model as well as the 'alternative model' based on a linear regression analysis.

5.0 CALIBRATION OF FOUNDATION STRENGTH FACTOR (LINEAR REGRESSION MODEL)

5.1 General

As described in Section 4, the foundation designer could initially analyze the foundation test data, based on the assumptions that the intercept is zero and the variance of residuals is non constant. This will lead directly to the $\overline{m}$ - model (Buckley, 1994) and will provide a single foundation strength factor for a given dataset. However, in order to validate these two assumptions the foundation designer should consider using a linear regression model where the foundation strength factor can be obtained directly from the lower limit of the prediction interval.

The prediction interval is very similar to the confidence interval estimate for the mean test capacity value (reference to section 2.3); except that the prediction interval is used to predict a foundation test capacity (new value) for a given single foundation geotechnical design capacity value. The lower limit on the prediction interval in the linear regression model is analogous to the exclusion limit on characteristic capacity described in section 2.4.2. This linear regression model can be applied to a wide variety of foundation test data, where the correlation value may not be high (table 4.1) and the intercept from the linear regression equation may have an effect.

This section of the report correspondingly provides an alternative methodology to calibrate the foundation strength factor of one foundation based on a linear regression model. The regression model developed is for general application; it is also shown that the equation (4.7) is a subset of the generalized linear regression model. In addition, a simple test is provided that can be used to simultaneously accept or reject the "hypothesis" that the intercept is zero or can be neglected and the slope is $\bar{m}$ (equation 4.3), and as such validate the $\bar{m}$ - model. This is a major finding of this study and will validate the use of equation (4.3) across the data range, provided that the hypothesis is acceptable. An example problem (3 point test data), using manual calculations is presented in section 9.0 (APPENDIX) to illustrate the methodology. A design problem based on ESB test data (section 4.0) is presented using the linear regression model ("alternative method").

5.2 Linear Regression Model

5.2.1 General

In the linear regression model, the test capacities (y-values) are plotted against the foundation model predicted geotechnical design capacities (x-values) and a straight line is fitted through the data to ensure that the residual error, ε_i, is minimum. Figure 5.1 depicts a least square fit of a typical test dataset. The residual error, ε_i given in equation (5.1), is defined as the difference between the estimated mean test capacity value, $\hat{y}_i$ (given in equation 5.5), and the actual test capacity value, y_i and is shown in Figure 5.1.

$$\varepsilon_i = y_i - \hat{y}_i \qquad (5.1)$$

This residual error, ε_i, should be minimized to ensure that there is a 'best fit' line to the dataset. The regression model presented is for the case where each test capacity has a corresponding model predicted geotechnical design capacity. This means that each pair of data (x_i, y_i) is one test. If there are n-sample data plotted, it can be shown that the slope, b, and the intercept, a, are given by the following formula:

$$b = \frac{\sum w_i (x_i - \bar{x}) y_i}{\sum w_i (x_i - \bar{x})^2} \tag{5.2}$$

$$a = \bar{y} - b\bar{x} \tag{5.3}$$

where w_i is the appropriate weight applied to each data point (Ang and Tang, 1975). Normally, the variance of residuals in the linear regression model is assumed to be constant and w_i =1.0, unless the residual varies in a non constant manner; this point will be discussed later. The sum is carried out based on a sample size of n.

The slope, b, can also be computed from the following relationship if we know the correlation coefficient, r from Equation 4.1:

$$b = r\frac{s_y}{s_x} \tag{5.4}$$

The estimated mean test capacity value along the regression line corresponding to each x_i, is $\hat{y}_i$, and can be calculated from equation (5.5) as:

$$\hat{y}_i = a + b(x_i) \tag{5.5}$$

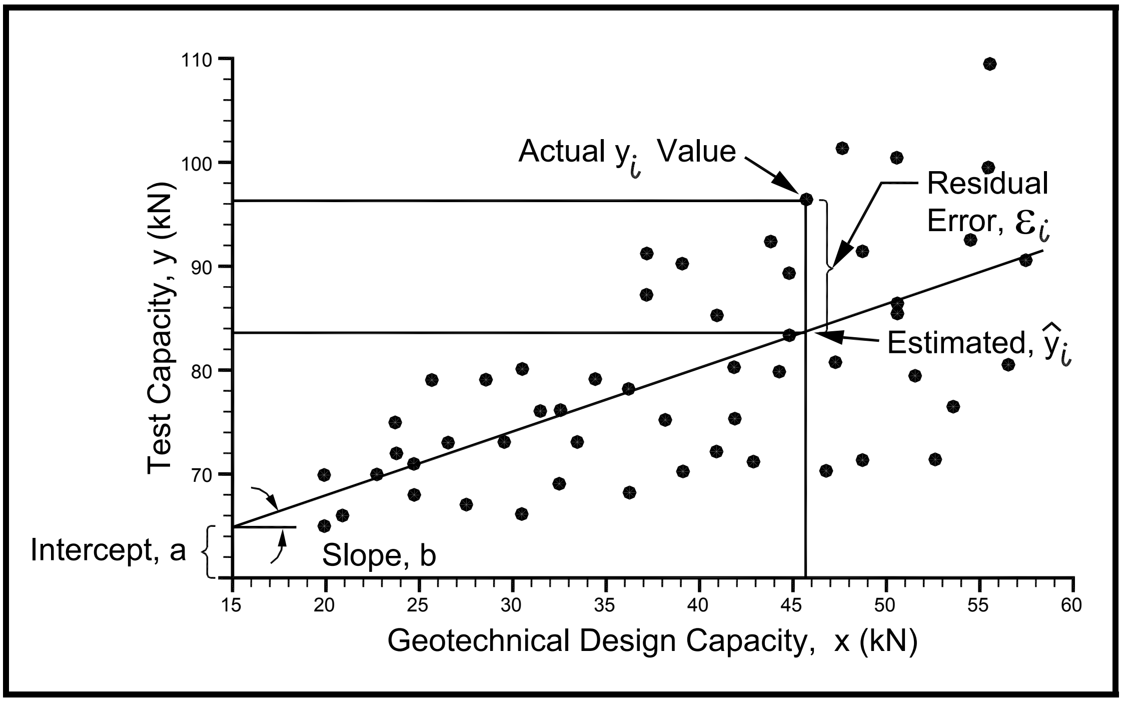


Figure 5.1 - Typical regression plot with intercept and slope

The conditional variance, $s_{xy}{}^2$ about the regression line often called the Standard Error of Estimate (SEE) of y_i and x_i is assumed to be constant within the predictor values (data along the x-axis) and called hereafter, the variance of residuals and is obtained from equation (5.6):

$$s_{xy}{}^2 = \frac{\sum_1^n w_i (y_i - \hat{y}_i)^2}{n-2} \qquad (5.6)$$

where $\hat{y}_i$ is the estimated mean test capacity value of y_i, and is based on Equation (5.5) and w_i=1.0. For non constant variance of residuals where the residuals vary with x_i values, w_i is dependent on the predictor values along the x-axis (Ang and Tang, 1975). Depending on the weight, the unit of s_{xy} could be dimensional or non-dimensional.

The variance of residuals can be also estimated directly from the data without doing a regression analysis and is given by equation (5.7):

$$s_{xy}{}^2 = [\frac{n-1}{n-2}] s_y{}^2 (1 - r^2) \qquad (5.7)$$

For large n,

$$s_{xy}{}^2 \to s_y{}^2 (1 - r^2) \qquad (5.8)$$

All these calculations can be done routinely in a spreadsheet to determine the intercept, slope, variance of residuals and the correlation value.

5.2.2 Confidence interval (general linear regression analysis)

The two-sided confidence interval (CI) on the estimated mean test capacity value, $\hat{y}$ given a geotechnical design capacity value, x_0 in equation (5.5) is given as:

$$\text{CI} = \hat{y} \pm k_{\alpha/2} * s_{xy} [\frac{1}{\sum_1^n w_i} + \frac{(x_0 - \bar{x})^2}{\sum_1^n w_i (x_i - \bar{x})^2}]^{0.5} \qquad (5.9)$$

In this equation, $w_i = 1.0$ because the variance of residuals is constant. For finite sample size, $k_{\alpha/2}$ should be replaced by $t_{\alpha/2,\nu}$ where ν = n-2 degrees of freedom. The equation is very similar to equation (2.6) except that the estimated mean test capacity $\hat{y}$ is a conditional mean given a geotechnical design capacity value x_0. The term s_{xy} is the standard deviation of the residual error of the combined data (square root of equation 5.6), which includes both the actual test capacity and geotechnical design capacity values. Figure 5.2 depicts the confidence interval along the regression line.

For repeated test data at a specific site, $\bar{x} = x_0$ (sections 3.7.1 and 4.1), $r = 0.0$ (equation 4.1), b= 0.0 (equation 5.4), a= $\bar{R}$ (mean value of the test data itself, equation 5.3), $s_{xy} = s_R$ (square root of equation 5.8) and the equation (2.6) is obtained from equation (5.9). Therefore, the confidence level on the mean value of the test capacity data referred in section 2.3 (equation 2.6, for test capacity data only) is a subset of equation (5.9) under these specific conditions.

5.2.3 Prediction interval (general linear regression analysis)

The prediction interval is not only dependent on the variance of residuals s_{xy}^2, but also on the uncertainty in the estimated mean test capacity value, $\hat{y}$ given a geotechnical design capacity value, x_0 (equation 5.5). The lower limit of the prediction interval of the predicted test capacity y_p, based on a single geotechnical design capacity x_0, (same as R_n) is obtained from equation (5.10)

$$y_p = \hat{y} - k_\alpha \, s_{xy} \left[1 + \frac{1}{\sum_1^n w_i} + \frac{(x_0 - \bar{x})^2}{\sum_1^n w_i (x_i - \bar{x})^2}\right]^{0.5} \qquad (5.10)$$

The estimated mean test capacity value $\hat{y}$ is dependent on the regression coefficients such as the intercept a and slope b. These additional factors introduce larger error in the prediction interval estimate when the predictor data x_0 are far from their mean value, $\bar{x}$ i. e., $(x_0 - \bar{x})$ in equation (5.10) is large. The variance of residuals for the prediction must take into account the fact that the quantity being predicted is a random variable (section 2.4). The quality of the prediction will depend on how small is the difference between $\hat{y}$ and y_p and the error $(\hat{y} - y_p)^2$, which is called the mean squared error of prediction.

The equation (5.10) is very similar to equation (2.8) except that the prediction is based on the lower limit of the interval given a geotechnical design capacity value x_0. This is depicted in figure 5.2 where it is shown that for a specified geotechnical design capacity value x_0, the lower limit of the prediction interval point B, is similar to point L, in figure 2.3 (the characteristic capacity R_C). The only difference is that the computation of the lower limit of the prediction interval in figure 5.2 is based on a given conditional geotechnical design capacity value x_0; while in the latter case, it is done on the test data directly (figure 2.3).

For repeated test data at a specific site $\bar{x} = x_0$ (sections 3.7.1 and 4.1), $r = 0.0$ (equation 4.1), b= 0.0 (equation 5.4), a= $\bar{R}$ (mean test capacity value of the test data itself, equation 5.3), $s_{xy} = s_R$ (equation 5.8) and the lower limit of equation

(2.8) is obtained from equation (5.10). Therefore, the prediction interval of a new test capacity referred in section 2.4 (equation 2.8, one sided for test capacity data only) is a subset of equation (5.10) under these specific conditions.

Details of the calculation of intercept, slope, correlation value, variance of residuals, confidence interval and the prediction interval for a three point scattered data (figure 3.11, table 3.6) are given in Section 9 **(APPENDIX)**. The next four subsections discuss various issues that need to be considered in analyzing the data using a linear regression model.

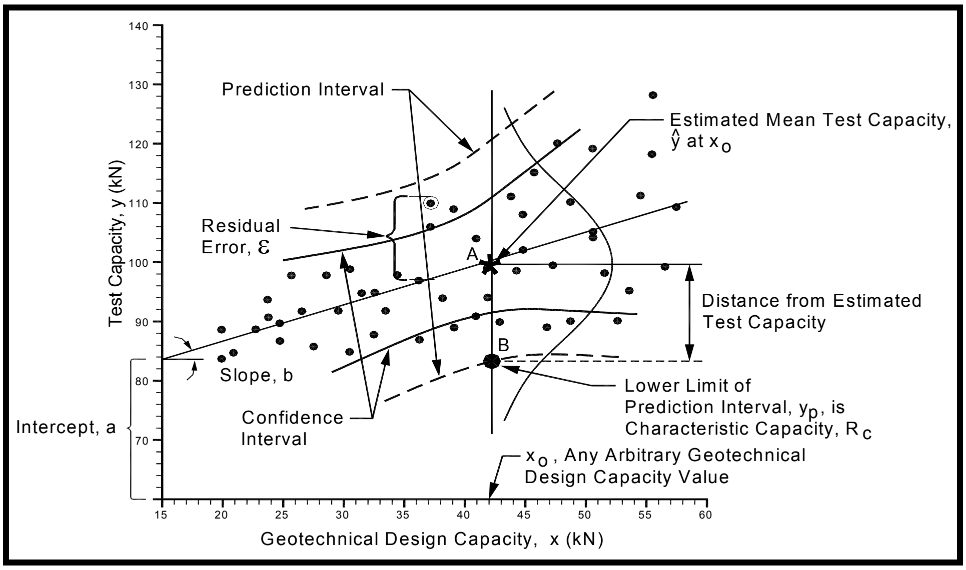


Figure 5.2 - Regression plot with confidence interval on mean test capacity values and prediction interval for new test capacity values

5.2.4 Correlation Issue

Equation (5.8) indicates that the variance of residuals is dependent on the variance of the test data s_y^2 (test capacity data along y-axis) and the correlation value r^2 between the test data and geotechnical design capacity. The correlation value can be obtained from equation (4.1). If a dataset has a high degree of correlation ($r^2 \rightarrow$ 1.0), then every test data point is almost equal to the model predicted geotechnical design capacity (refer to figure 4.1b). In this case, the slope of the line $b \rightarrow 1.0$ and $\bar{y} \cong \bar{x}$ and $s_y \cong s_x$. The intercept will be close to zero (equation 5.3). In this case the user can assume that the line passes through the origin. The correlation value will reflect the realistic characteristic of a design model if the data covers geotechnical design capacity values both in the low (near x =0.0) and high ranges within a sample size.

5.2.5 Linearity Issue

To validate the linearity issue, the user needs to plot the residual errors ε_i (equation 5.1) against the estimated mean test capacity values $\hat{y}_i$ obtained from equation (5.5). A typical graph for the residual error values plotted against the estimated mean test capacity values ($\hat{y}_i$ values) is shown in figure 5.3. The residual errors should produce a band centered along $\varepsilon = 0.0$ line, to demonstrate a linear regression model.

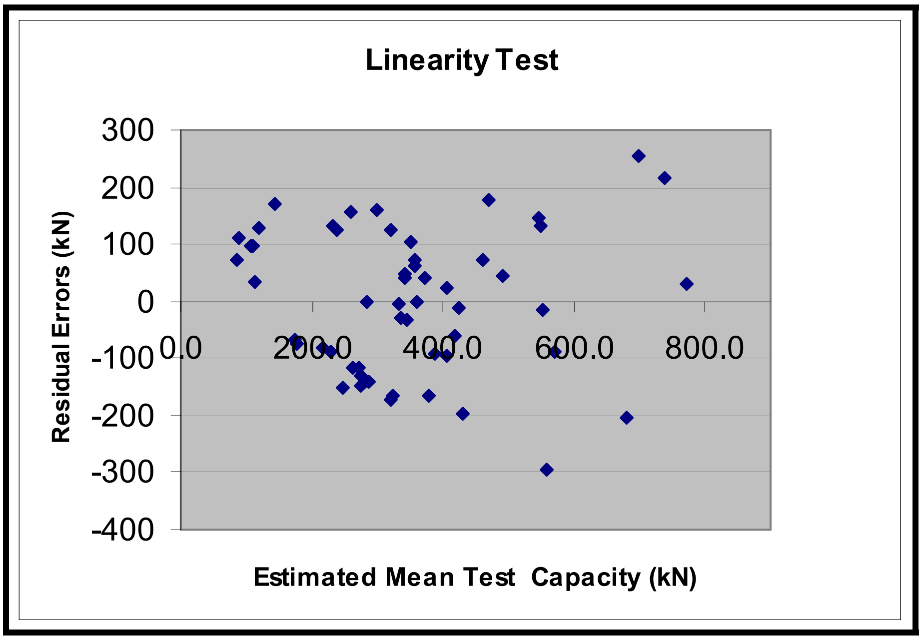


Figure 5.3 - Residual errors banded around $\varepsilon = 0.0$ line

5.2.6 Zero Intercept Issue

As the correlation value tends to decrease ($r^2 \leq 0.8$, table 4.2) with increasing coefficient of variation of the model predicted error ($v_m \geq 0.1$), consideration should be given to include the intercept effect in the prediction interval calculation. For foundation calibrations where the dataset has moderate correlation ($r^2 \leq 0.8$), the intercept has a significant influence in the estimated mean test capacity value $\hat{y}_i$. In this case, by forcing the line to pass through the origin arbitrarily, the foundation designer will make the prediction optimistic (less conservative); while in reality the designer will not be able to validate the predicted foundation test capacity with a very high confidence level.

For example, if a database has a poor correlation ($r \rightarrow 0.0$) and the second assumption (zero intercept) is used arbitrarily, this will make the results acceptable when in reality the test capacity values along the y-axis have no **linear** relationship with corresponding geotechnical design capacity values along x –axis, due to a lack of correlation (Table 4.1). If the foundation design engineer considers the feasibility in reducing the model from $\hat{y}_i = a + b(x_i)$ to $\hat{y}_i = \overline{m}(x_i)$, where the line passes through the origin, then in principle a null hypothesis (H_0) should be tested simultaneously at a specified significance level, α (Benjamin and Cornell,1970). A null hypothesis implies that the proposition

H_0: a=0.0 and b=$\overline{m}$

is believed to be true. The significance level (α), provides a probability measure of rejecting the null hypothesis when in fact it is true. A significance level 0.05 implies that the engineer could reject the hypothesis 5% of the time in the long run when in fact it is true.

A specific test can be used to reject simultaneously a null hypothesis that the intercept, a = a^* (arbitrary value) and the slope b = $\overline{m}$ (H_0: a= a^* and b= $\overline{m}$) using equation (5.11).

$$F^* = [n*(a-a^*)^2 + 2*n*\overline{x}*(a-a^*)(b-\overline{m}) + (\sum x_i^2)(b-\overline{m})^2]/(2*s_{xy}^2) \quad (5.11)$$

If the calculated F^* value is greater than the allowable value $F_{\alpha,2,n-2}$ obtained from F-distribution, with a significance level, (α), and $\nu = n-2$ degrees of freedom then the hypothesis is rejected (Bowker and Lieberman, 1972, Sneddon, 2003). In this case, the intercept should be included in the analysis and restrictions imposed on **how far the data can be extrapolated**. This is particularly important for the database where the correlation is moderate ($0.3 \le r^2 \le$ 0.8, table 4.2) and the data has large coefficients of variation, i. e. all values of v_m, v_x, v_y are considerably greater than 0.1. This situation can also arise when the database does not contain enough points near x=0.0 (low end). It is important that the foundation design engineer collects test data which at least will cover the low as well as high ranges to ensure that the correlation value truly represents the main characteristics of the design model.

The foundation designer should validate that equation (4.7) is satisfied for a^* = 0.0 and b = $\overline{m}$ for the $\overline{m}$ model presented by Buckley (1994). This will be discussed later in the example problem presented in section 5.6.

In equation (5.11), $x_i, (i = 1,...n)$ = model predicted geotechnical design capacities for various test data. Table 5.1 provides some typical values for $F_{\alpha,2,n-2}$ for two significance levels.

Table 5.1 - F -distribution Values (Benjamin & Cornell, 1970)

Sample size. n	$\nu = n-2$	$F_{\alpha,2,n-2}$ **for** α **=10%**	$F_{\alpha,2,n-2}$ **for** α **=5%**
3	1	49.50	199.5
9	7	3.26	4.74
12	10	2.92	4.10

5.2.7 Variance issue (constant versus non constant)

In the linear regression model, it is normally assumed that the residual errors across all geotechnical design capacity values remain fairly constant as depicted in figure 5.4a. However for many datasets, it is possible for the residual errors to vary linearly or non linearly with the geotechnical design capacity values along the x-axis. In this case, it is known as non constant variance of residuals. Figure 5.4(b) presents this case, which is one of the basic assumptions of the $\overline{m}$-model recommended in CIGRE (2002) and presented by Buckley (1994).

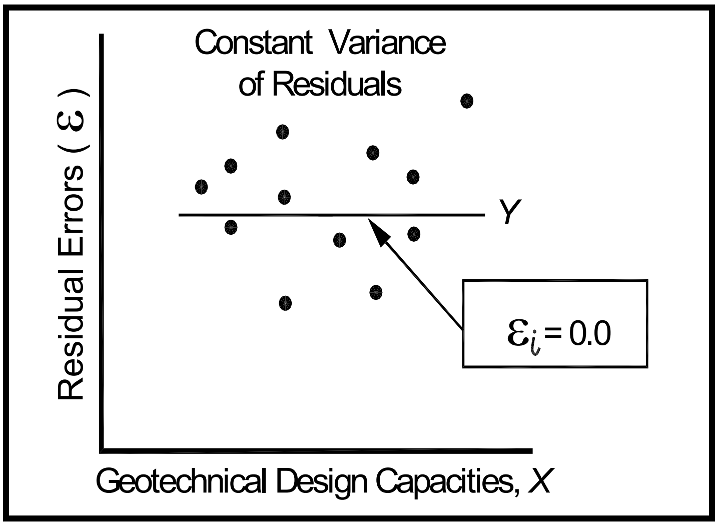


Figure 5.4(a) - Constant variance of residuals

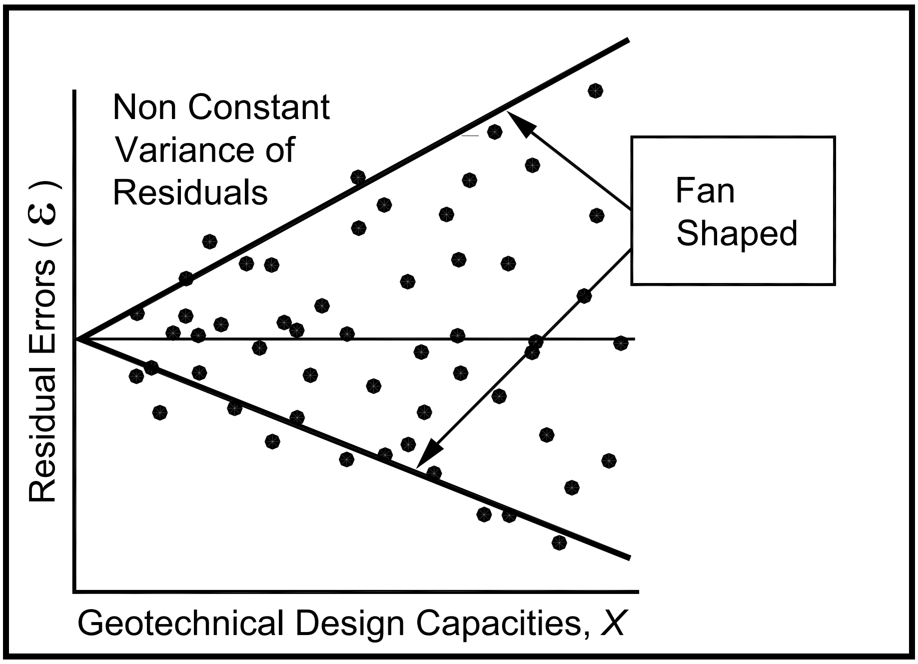


Figure 5.4(b) - Non constant variance of residuals

For non constant variance of residuals, the assumption is that the variance of residuals is proportional to the square of the x-values and an appropriate weight $w_i = \frac{1}{x_i^2}$ is applied to the original dataset and the regression analysis is carried out on the modified data (Ang & Tang, 1975). Figure 5.5 presents a typical residual plot against the geotechnical design capacity values for a large dataset.

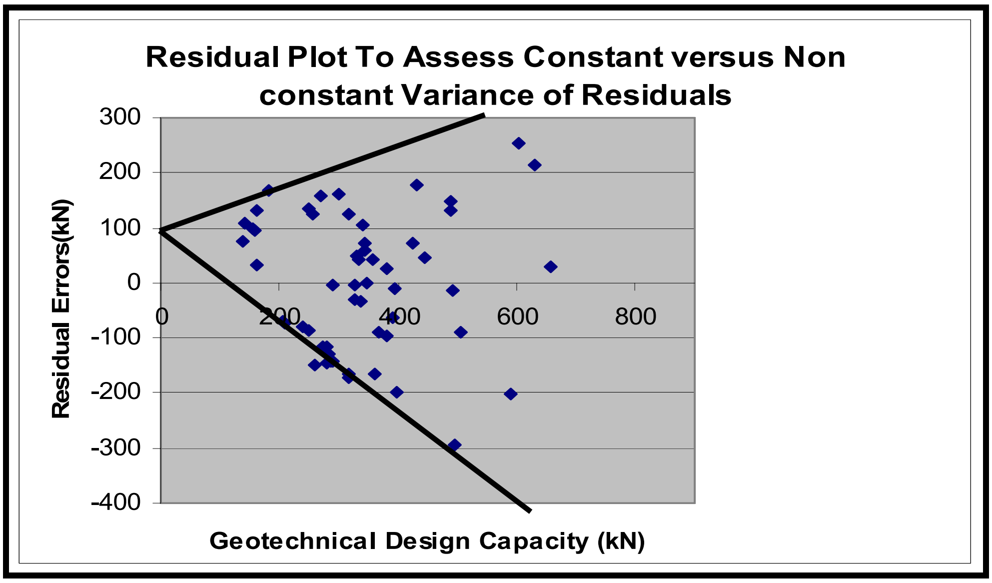


Figure 5.5 - Non constant variance of residuals trend for a larger dataset

5.3 Foundation Strength Factor, ϕ_F

5.3.1 General

The basic linear regression model can be used to provide the confidence interval (CI, equation 5.9) on the estimated mean test capacity $\hat{y}$ (equation 5.5 for $x_i = x_0$), as well, as the lower limit of the prediction interval of a new test capacity y_p (equation 5.10), given a single geotechnical design capacity value x_0 (figure 5.2)

The foundation strength factor ϕ_F is defined as the ratio between the lower limit of the predicted test capacity y_p and the geotechnical design capacity value x_0 used to determine this capacity (figure 5.2, Haldar, 2001, Ang, 2002). If the predicted test capacity with a specified confidence level is y_p and the geotechnical design capacity value is x_0, then the foundation strength factor is:

$$\phi_F = \frac{y_p}{x_0} \tag{5.12}$$

So far we have discussed two scenarios; these are: (1) the line passes through the origin and (2) the line does not pass through the origin and therefore has an intercept. Since there are a further two options (constant versus non constant variance of residuals), under each of these conditions (zero or non zero intercept condition), the foundation designer will end up with four possible cases for analyzing the data. This is described in Table 5.2. Each case in Table 5.2 will provide a lower limit on the predicted test capacity with certain confidence level based on linear regression model outlined in section 5.2.

Table 5.2 - Various Cases in Linear Regression Model

Intercept	Variance of Residuals	
	Constant	Non constant
Non Zero	Case A	Case B
Zero	Case C	Case D

The foundation designer can still obtain the foundation strength factor value using equation (5.12) provided the predicted test capacity is known from any one of the cases in Table 5.2. Equation (5.11) provides a definite test whether to include the intercept in the analysis or not. However, the test for constant variance of residuals versus non constant is not quantitative and is rather subjective because the designer needs to plot the residuals against the geotechnical design capacity values and to derive a trend line similar to figures 5.4a and 5.4b respectively. If it is not clear that the residuals vary either in a constant or non constant manner then the designer will end up with Cases A and B as one group or Cases C and D as the other group respectively. This will provide the upper and lower limits on the foundation strength factor values or in the predicted test capacity values. An example design problem based on ESB data is shown later to indicate how the designer can select a particular option.

The following section summarizes formulae for the four cases that the user may face in analyzing the data. The formulae are presented to predict the lower limit of the foundation test capacity y_p for a given geotechnical design capacity value x_0 for conditions with or without zero intercept and with or without constant variance of residuals. All these formulae also consider the correlation issue and the coefficients of variation of the data. The various parameters such as slope b, variance of residuals $s_{xy}{}^2$, estimated mean test capacity $\hat{y}_i$, confidence interval (two sided) on mean test capacity values CI and the lower limit of the predicted test capacity for a single geotechnical design capacity value x_0 with a given confidence level y_p are presented for each case. Often, the predicted test capacity value will depend on the intercept a, slope b, variance of residuals $s_{xy}{}^2$, the correlation coefficient r and the weight w_i used to estimate the variance of residuals. Since the intercept can be either zero or can have a finite value and the variance of residuals can vary either in a constant or a non constant manner (figures 5.4a and 5.4b), the designer will likely encounter at least two of the four cases as presented in Table 5.2. A flow diagram is also presented later to provide a guidance to the design engineer in the data analysis process.

5.3.2 CASE A - Intercept is non zero and the variance of residuals is constant

This case is applicable to problems where the intercept requires to be included and the residuals plotted against the geotechnical design capacity values produce a band around the $\varepsilon_i = 0$ line (Figure 5.4a, constant variance of residuals). The slope b, intercept a, estimated mean test capacity value $\hat{y}_i$, variance of residuals

$s_{xy}{}^2$ (standard error of estimate), confidence interval (CI) on the estimated mean test capacity value $\hat{y}$ and the lower limit on the prediction interval (PI) of the predicted test capacity y_p given a single geotechnical design capacity x_0 can be obtained from equations (5.2), (5.3), (5.5), (5.6), (5.9) and (5.10) respectively (figure 5.2).

5.3.3 CASE B - Intercept is non zero and the variance of residuals is non constant

This case is applicable to problems where the intercept needs to be included and the residual does follow a non constant variation according to Figure 5.4b. In this case, there is a need to include both intercept and slope in estimating the prediction interval. The slope b, intercept a, estimated mean test capacity value $\hat{y}_i$, variance of residuals $s_{xy}{}^2$, confidence interval (CI) on the estimated mean test capacity value $\hat{y}$ and the lower limit on the prediction interval (PI) of the predicted test capacity y_p given a single geotechnical design capacity x_0 can be obtained from equations (5.2),(5.3), (5.5),(5.6), (5.13) and (5.14) respectively (figure 5.2) with the appropriate weight, $w_i = \dfrac{1}{x_i{}^2}$.

Confidence interval CI = $\hat{y} \pm k_{\alpha/2} * s_{xy} \left[\dfrac{1}{\sum_1^n w_i} + \dfrac{(x_0 - \bar{x})^2}{\sum_1^n w_i \; (x_i - \bar{x})^2} \right]^{0.5}$ (5.13)

Predicted test capacity $y_p = \hat{y} - k_\alpha * s_{xy} \left[\dfrac{x_0{}^2}{1} + \dfrac{1}{\sum_1^n w_i} + \dfrac{(x_0 - \bar{x})^2}{\sum_1^n w_i \; (x_i - \bar{x})^2} \right]^{0.5}$ (5.14)

5.3.4 CASE C - Slope only with constant variance of residuals

This case is applicable to problems where the intercept can be neglected based on equation (5.11) and the residual does not increase with geotechnical design capacity values (Fig. 5.4a). In this case, we only need to include the slope in estimating the prediction interval. The weight $w_i = 1.0$, because the variance of residuals is constant. The slope b, variance of residuals $s_{xy}{}^2$, estimated mean test capacity value $\hat{y}_i$, confidence interval (CI), on estimated mean test capacity value $\hat{y}$ and the lower limit on the prediction interval (PI) of the predicted test capacity y_p, given a single geotechnical design capacity value x_0 with a specified confidence interval can be obtained from equations (5.15), (5.16), (5.17), (5.18) and (5.19) respectively (figure 5.2 with the intercept being zero).

$$b = \frac{\sum w_i (x_i) y_i}{\sum w_i (x_i)^2} \quad (5.15)$$

$$s_{xy}{}^2 = \frac{\sum w_i (y_i - \hat{y}_i)^2}{n-1} \quad (5.16)$$

$\hat{y}_i$ = b x_i where $x_i = x_0$ (5.17)

Confidence interval $\mathbf{CI}$ = $\hat{y} \pm k_{\alpha/2} * s_{xy} [\frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}]^{0.5}$ (5.18)

Predicted test capacity $y_p = \hat{y} - k_\alpha * s_{xy} [1 + \frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}]^{0.5}$ (5.19)

5.3.5 CASE D - Slope only with non constant variance of residuals (Similar to $\overline{m}$ -model)

This case is applicable to problems where the intercept is neglected based on equation (5.11) and the residual does strictly follow a conical (fan shape) variation (Figure 5.4b). In this case there is only a need to include the slope in estimating the prediction interval. This is obtained using the weight $w_i = \frac{1}{x_i{}^2}$. The slope b, variance of residuals $s_{xy}{}^2$, estimated mean test capacity value $\hat{y}_i$, confidence interval (CI) on the estimated mean test capacity value $\hat{y}$ and the lower limit on the prediction interval (PI) of the predicted test capacity y_p, given a single geotechnical design capacity x_0 can be obtained from equations (5.20) to (5.24).respectively (figure 5.2 with the intercept being zero)..

$$\mathbf{b} = \frac{\sum w_i (x_i) y_i}{\sum w_i (x_i)^2} = \frac{\sum (1/x_i)^2 (x_i) y_i}{\sum (1/x_i)^2 (x_i)^2} = \frac{\sum (1/x_i) y_i}{\sum i} = \frac{\sum_1^n y_i / x_i}{n} = \overline{m} \quad (5.20)$$

$$s_{xy}{}^2 = \frac{\sum w_i (y_i - \hat{y}_i)^2}{n-1} = s_m{}^2 \quad (5.21)$$

$\hat{y}_i = \overline{m}\ x_i$ where $x_i = x_0$ (5.22)

Confidence interval $\mathbf{CI}$= $\hat{y} \pm k_{\alpha/2} * s_m\, x_0\, [\frac{1}{n}]^{0.5}$

$$= \overline{m}\ (\mathbf{1} \pm k_{\alpha/2} \frac{v_m}{\sqrt{n}})\, x_0 \quad (5.23)$$

Predicted test capacity $y_p = \hat{y} - k_\alpha * s_m\, x_0\, [1 + \frac{1}{n}]^{0.5}$

$$= \overline{m}\,(1 - k_{\alpha} * v_m [1+\frac{1}{n}]^{0.5})\, x_0 \qquad (5.24)$$

Equation (5.24) is similar to equation (4.3) presented by Buckley (1994) but includes a factor $[1+\frac{1}{n}]^{0.5}$ which was omitted in the earlier study. For small sample size, this may create differences in the foundation strength factor values. For large sample size, the term under the second bracket in equation (5.24) is close to 1.0. This is similar to the difference in the predicted test capacity value and the characteristic capacity presented in section 2.4. The foundation strength factor obtained from equation (5.24) will be **constant** across all model predicted geotechnical design capacity values (Sneddon, 2003). If this is not the case, then the foundation designer may find that the foundation strength factor will be a function of the geotechnical design capacity since the predicted test capacity is also a function of the geotechnical design capacity (Cases A, B and C in table 5.2).

In all the above equations, k_{α} values should be obtained from Table 2.1 for one sided confidence level or use $t_{\alpha,\nu}$ when the sample size is finite (n < 30, Table 2.2). For cases A and B, ν = n- 2 while for cases C and D, ν = n-1.

Equation (5.23) is similar to equation (2.6) where the confidence level is sought on mean test capacity data only. Equation (5.23) provides the confidence interval on the estimated mean test capacity (conditional mean value of $\hat{y}$ given a geotechnical design capacity value x_0 **(or** R_n**),** $\hat{y} \mid x_0$ as long as they are related through a regression line with a slope b= $\overline{m}$ and a coefficient of variation v_m**.** Equation (5.24) similarly provides the lower limit on the prediction for a single geotechnical design capacity value x_0 This also corresponds to equation (2.8), where a minimum strength (characteristic capacity with e% exclusion limit) is obtained from the test data directly**.**

5.4 Flow Diagram

In order to analyze the data in a systematic manner, a flow diagram is presented in Figure 5.6, where all the required validation tests and the corresponding equation numbers are identified.

5.5 Limitation of Case D Model (Equation 5.24)

The foundation designer needs to consider two questions before the equation (5.24) (also applicable for the simple model, equation 4.7), can be applied generally. These are:

- What is the upper limit of v_m beyond which the method is not valid?
- How far do we go along the x-axis to extrapolate the data in providing a prediction interval for the test capacity?

To address the first question, the coefficient of variation cannot be greater than

$$v_m \le [\frac{1}{t_{\alpha,\nu}[1+\frac{1}{n}]^{0.5}}] \qquad (5.25)$$

since $\overline{m}$ > 0.0 (equation 5.24).

This will ensure that the foundation strength factor is positive and provides a reasonable value. Of course it can be argued that even with a very high $\overline{m}$ value, it is always possible to achieve a positive foundation strength factor value when the term inside the bracket (equation 5.24) is very low. However, in such a case the foundation designer should not use the geotechnical design model, because it is underestimating significantly the test data and correlation will be low (table 4.2). Table 5.3 presents some typical values of v_m to ensure that the foundation strength factor does not become negative. For other intermediate value, use equation (5.25).

Table 5.3 - Theoretical upper Limit of v_m for various confidence levels and sample size (Equation 5.25)

Sample Size, n	90% Confidence level		95% Confidence level	
	$t_{\alpha,v}$	$v_m \le \frac{1}{t_{\alpha,v}[1+\frac{1}{n}]^{0.5}}$	$t_{\alpha,v}$	$v_m \le \frac{1}{t_{\alpha,v}[1+\frac{1}{n}]^{0.5}}$
3	1.90	0.45	2.92	0.29
5	1.53	0.59	2.13	0.42
12	1.36	0.70	1.80	0.53
30	1.28	0.76	1.64	0.59

If the test data and the model predicted geotechnical design capacity produce a v_m value greater than the allowable limit shown in Table 5.3, the foundation strength factor ϕ_F could become negative. The foundation design engineer needs to be careful when the data is extrapolated outside the range. Even when the v_m values are close to the values shown in Table 5.3, the foundation strength factor value will be low and probably unrealistic indicating that the error is introduced because of the large coefficient of variation. Similar criteria can also be used for other cases from the above equations or plotting the data graphically. Section 6 will provide some guidelines on interpolation and extrapolation.

5.6 Example Problem (Re-analysis of 9-point ESB Data using Linear Regression Model)

In this section, the 9-point ESB test data (table 4.3) is re-analyzed using the linear regression model where two major assumptions that the intercept is zero and the variance of residuals is non constant are examined. Figure 5.7 presents the regression plot of the data and the correlation value.

Table 5.4 presents the data statistics. The model, in general, appears to be conservative because of large bias, i. e. the geotechnical design model on average underestimates the test capacity value by 30%. The correlation value, r^2 is 0.81 is high (equation 4.1 - Test 1 in flow diagram, table 4.2). A linear regression model (CASE A) is run first following the flow diagram in figure 5.6. The standard deviation of error s_{xy} is 63kN based on constant variance of residuals assumption (CASE A, equation 5.6 with $w_i = 1.0)$). The geotechnical design capacity of data ranged from 150kN to 500 kN.

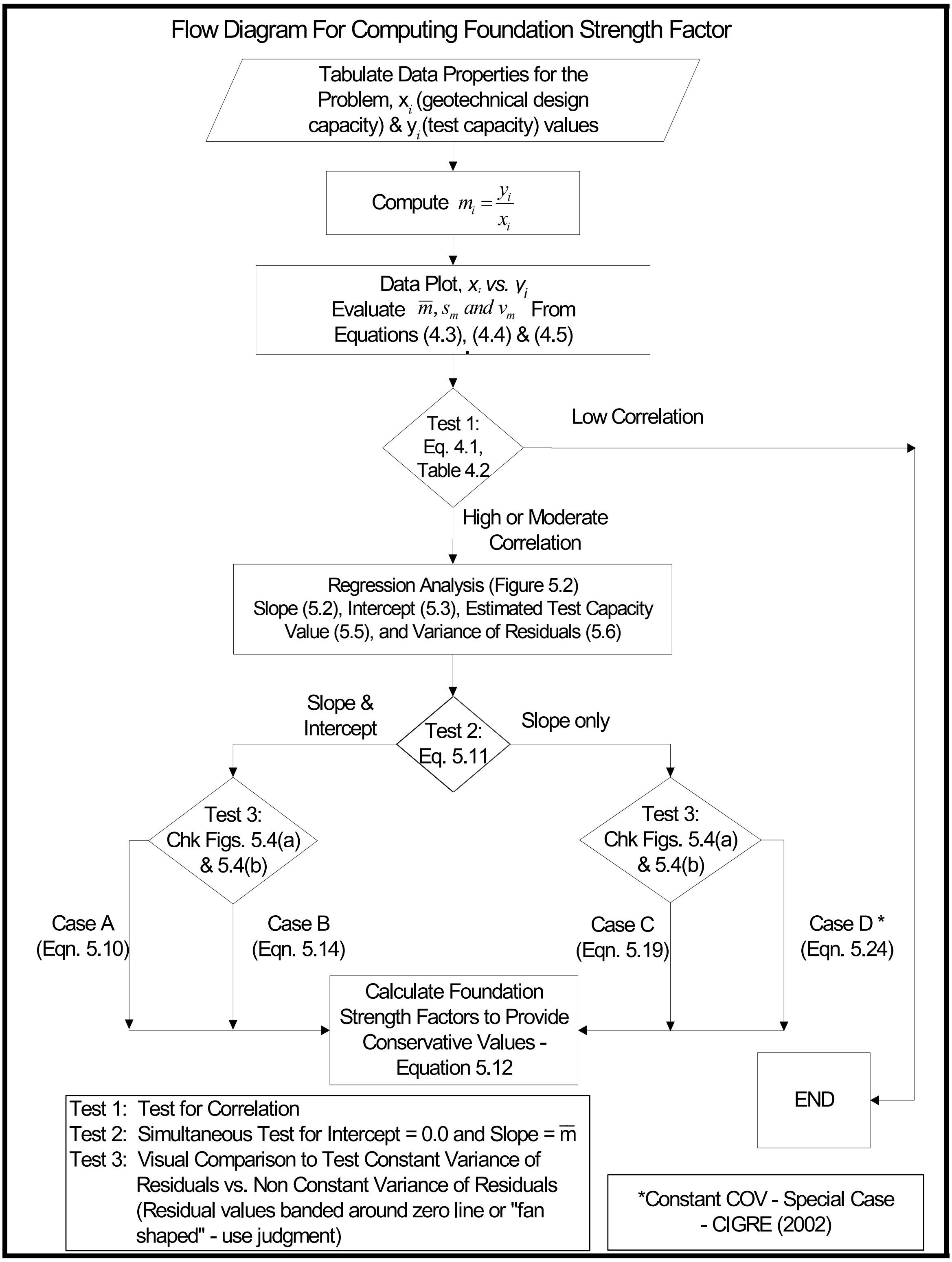


Figure 5.6 - Flow diagram

Table 5.4 presents also the regression parameters for CASE A. The test for intercept indicated (equation 5.11 –Test 2 in flow diagram) that F^*=0.164 which is less than the allowable $F_{\alpha,2,v}$ = 3.26 (Table 5.1). Therefore, the null hypothesis that the intercept is zero and the slope is equal to $\overline{m}$ =1.30 (Table 4.4) can be accepted. Therefore, the designer has to choose between Case C and Case D respectively (figure 5.6).

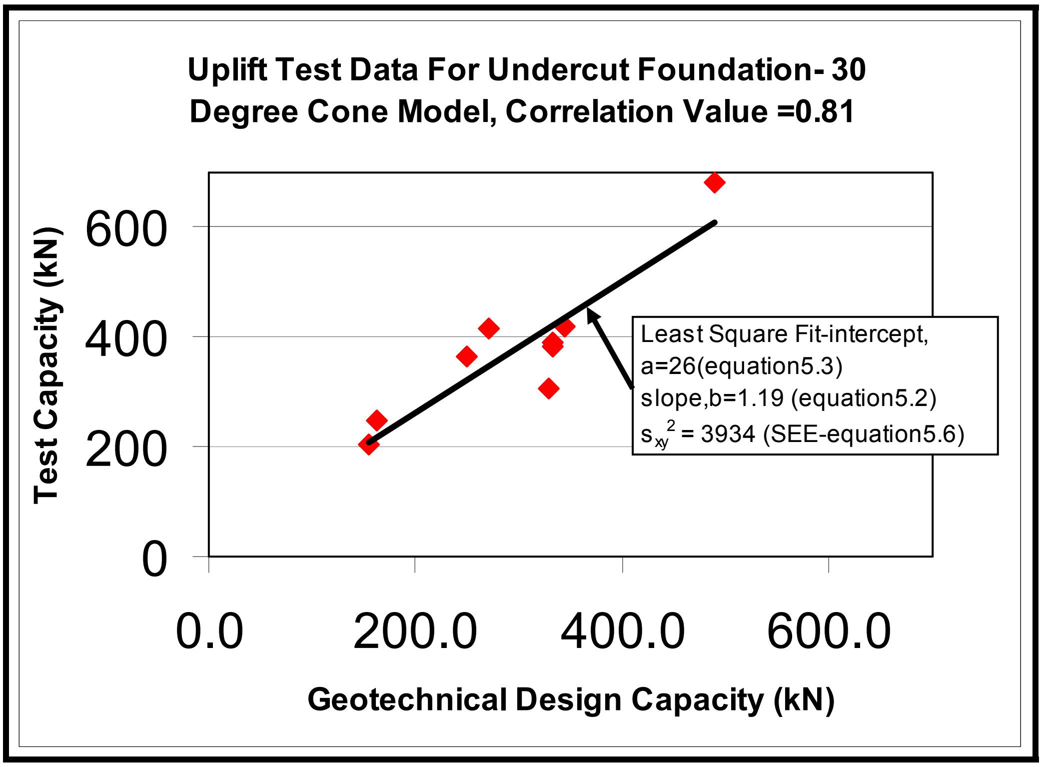


Figure 5.7 - ESB Uplift test data (CIGRE, 2002)

Table 5.4 - Data Statistics for Intercept Check (Equation 5.11)

n	r^2	$\overline{x}$	s_x	$\overline{y}$	s_y	a	b	s_{xy}	$\sum x_i^2$	F^*-value	$\overline{m}$	v_m
9	0.81	296	102	378	135.4	25.8	1.19	63	0.87E+05	0.164 < 3.26	1.3	0.154

Next, the residual values for the nine data points are plotted against the geotechnical design capacity values and Figure 5.8 presents this. Two lines are drawn to see a trend line where the variance of residuals is non constant (similar to figure 5.4 b) and it is not fully definitive that this is the case. Therefore, according to the flow diagram in figure 5.6, the designer should try both Cases C and D for determining the foundation strength factor. Obviously under Case C the foundation strength factor will not be constant and rather will be a function of the geotechnical design capacity; because the predicted test capacity value is also dependent on the geotechnical design capacity value (equation 5.19)

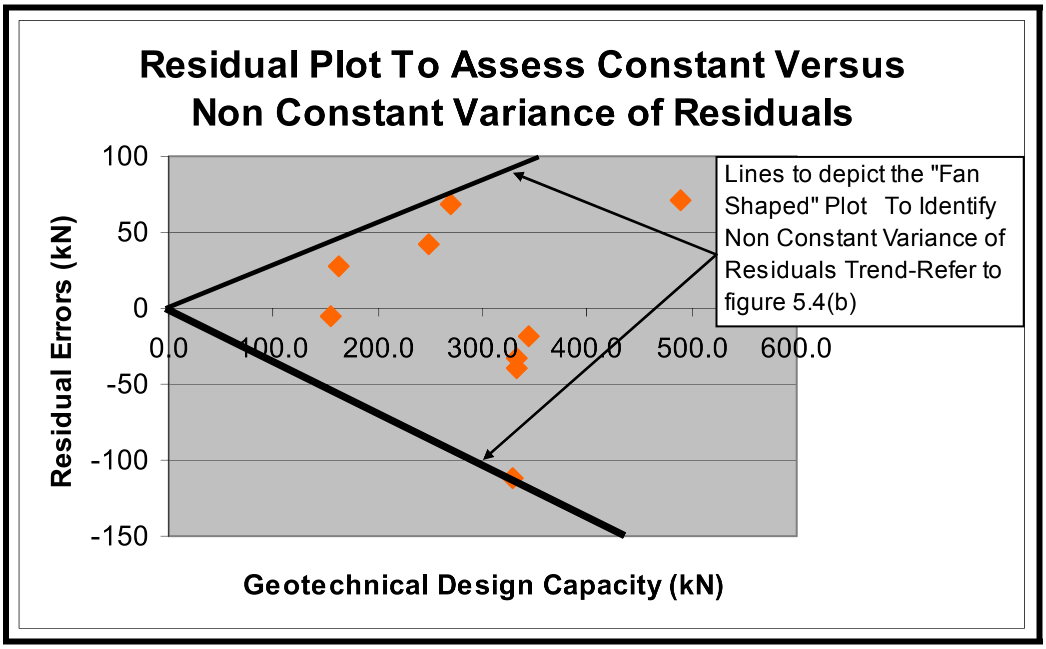


Figure 5.8 - Residual errors versus geotechnical design capacity values

Table 5.5 - (Case C-b=1.27, s_{xy}=59.29, $t_{\alpha,v}$=1.40, $w_i = 1.0$)

Geotechnical Design Capacity Value, x_0	**Estimated Mean Test Capacity Value** $\hat{y} = bx_0$ **(first term in equation 5.19**	**P=$t_{\alpha,v} * s_{xy}$ [1+$\frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}$]$^{0.5}$** **(second term in equation 5.19)**	**$y_p = \hat{y}$ -P (equation 5.19)**	**Foundation Strength Factor=$\frac{y_p}{x_0}$ (equation, 5.12)**
200	254	85	169	0.847
400	509	90	419	1.046
600	763	99	664	1.107
800	1017	109	908	1.135

The lower limit of the prediction interval with (90%) confidence level is presented for both cases, in tables 5.5 and 5.6 respectively. Figures 5.9 and 5.10 present the plots. Figure 5.11 depicts the comparison of foundation strength factor, ϕ_F based on CASE C and CASE D scenarios; it is interesting to note that for CASE D where the foundation strength factor is constant across all geotechnical design capacity values, the interpolation may not be a problem. However, for CASE C scenario, where the residual is interpreted as constant, (figure 5.4a), the foundation strength factor becomes strongly dependent on the geotechnical design capacity value (x-value) chosen as the predictor values and further away from the mean value (297 kN). Therefore, in this situation, the foundation designer may want to consider the lower value of the two foundation strength factors to ensure that the foundation design is conservative (Flow diagram-figure 5.6). Discussions on interpolation versus extrapolation are presented in section 6.0.

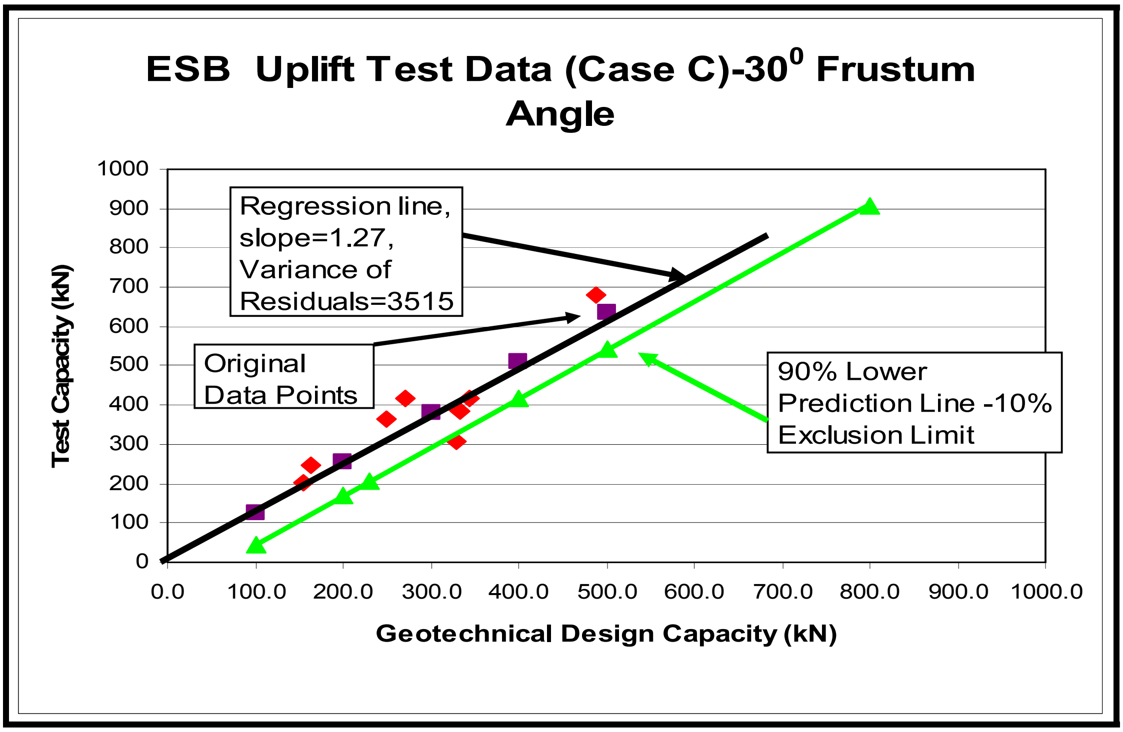


Figure 5.9 - Predicted test capacity with 90% confidence level (Case C, constant variance of residuals)

Table 5.6 (Case D- $\overline{m}$ **=1.30,** $s_{xy} = s_m$ **=0.20,** $t_{\alpha,\nu}$ **=1.40,** $w_i = \frac{1}{x_i^2}$ **)**

Geotechnical Design Capacity Value, x_0	**Estimated Test Capacity Value** $\hat{y} = \overline{m}x_0$ **First term in equation (5.24)**	**P=** $t_{\alpha,\nu} * s_m [1+\frac{1}{n}]^{0.5} x_0$ **Second term in equation (5.24)**	$y_p = \hat{y}$ **-P (equation 5.24)**	**Foundation Strength Factor=** $\frac{y_p}{x_0}$ **(equation, 5.12)**
200	260	59	201	1.00
400	520	119	401	1.00
600	780	178	602	1.00
800	1040	237	803	1.00

If the foundation designer wants to correct for the small sample size, then $t_{\alpha,\nu}$ value should be based on Student t-distribution (reference to Table 2.2) and the foundation strength factor becomes

$\phi_F = \frac{y_p}{x_0} = \frac{R_{e\%}}{R_n} = \overline{m}$ (1- $t_{\alpha,v} * v_m$ [1+ 1/n]$^{0.5}$) = 1.30*(1-1.40*0.146*(1+0.11) ^0.5) = 1.00

This value is shown in Table 5.7. The value is slightly different from the one presented in Table 4.5 because the term $[1+\frac{1}{n}]^{0.5}$ in equation (5.24) was not included in the earlier analysis.

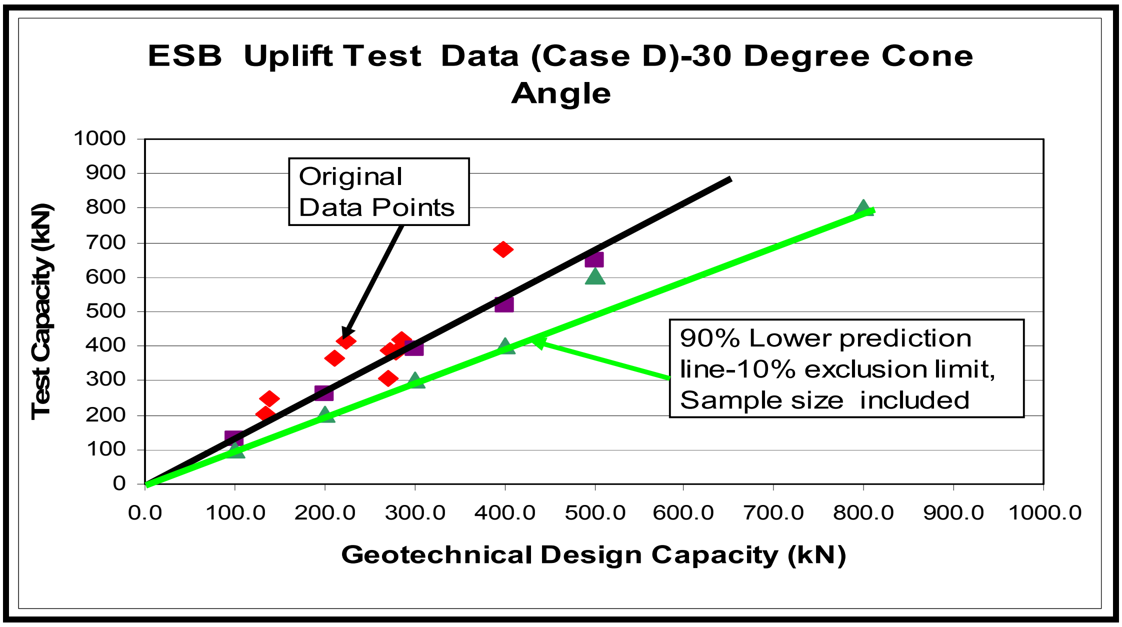


Figure 5.10 - Predicted test capacity with 90% confidence level (CASE D, non constant variance of residuals)

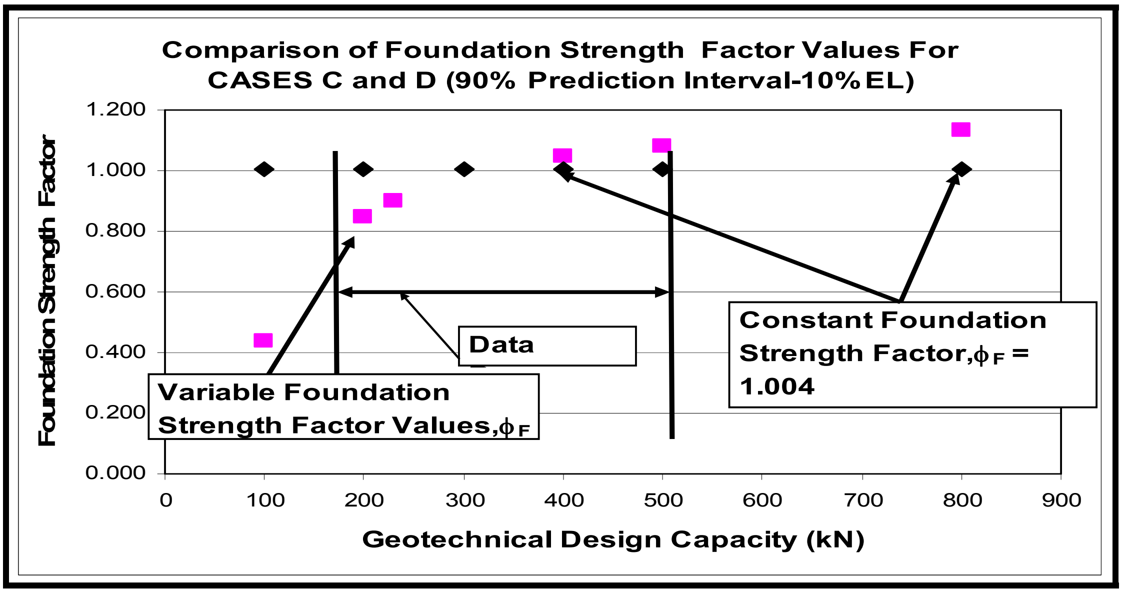


Figure 5.11 - Comparison of foundation strength factor (sample size effect included)

Table 5.7 –Foundation Strength Factor Values (Linear Regression Model)

Confidence Level	CASE D (equation 5.24)	
	90%	**95%**
n	9	9
$\overline{m}$	1.30	1.30
v_m	0.154	0.154
k_α -values	1.28	1.64
ϕ_F for k_α -values (equations 5.24 & 5.12)	1.03	0.97
$t_{\alpha,\nu}$ where ν = n-1 (Table 2.2)	1.40	1.86
ϕ_F for $t_{\alpha,\nu}$ (equations 5.24 & 5.12, replace k_α by $t_{\alpha,\nu}$ in equation 5.24)	1.00	0.93

5.7 Design Problem Using ESB Data (Foundation Depth)

A base case model for an undercut foundation (refer to figure 3.2 and table 5.8) was used to obtain the uplift capacity based on a 30^0 cone model. The unit weights for soil and the concrete were taken as 15.7kN/m^3 and 22.6kN/m^3 respectively. The calculated capacity was 508kN. Next, a sensitivity study was undertaken by changing the depth while keeping the base width constant. Figure 5.12 presents the data and it is apparent that the tripling of the depth has a seven fold increase in the foundation uplift capacity. The equation for computing the depth for a given geotechnical design capacity is also given on the graph. The correlation value is 0.9999 (figure 5.12).

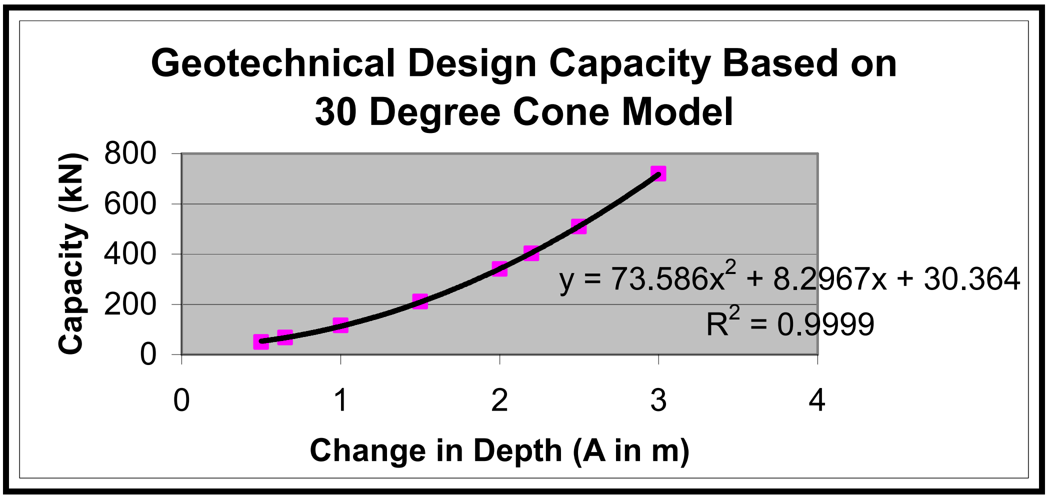


Figure 5.12 - Geotechnical design capacity with respect to change in depth

Table 5.8 - Typical Dimensions (m) of a Pad and Chimney Foundation (Figure 3.2)

Type	A	B	C	D	E	F
Square Based	2.5	2.0	1.9	0.2	0.8	0.1

The design equations (2.18) and (5.12) provide a relationship between the predicted capacity y_p (similar to R_C), and the geotechnical design capacity x_0, (same as R_n)

$$R_C = \phi_F \ R_n$$

For example, if the foundation design load due to extreme wind is Q_T = 500 kN, and the foundation designer wants to provide a 90% confidence level based on nine ESB tests, then the geotechnical design capacity required is (Table 5.5 , ϕ_F =1.00 for 90% confidence, 10% exclusion limit)

$$R_n = \frac{Q_T}{\phi_{F_c}} = \frac{500}{1.00} = 500\text{kN}$$

Once the geotechnical design capacity is known, the required foundation depth can be estimated using figure 5.12. For Q_T = 500 kN, Case C provides a geotechnical design capacity of 465 kN. This can be computed from equation (5.19) by substituting $y_p = Q_T$ and solving for x_0. The foundation designer can also compute the geotechnical design capacity from figure 5.12 for a predicted test capacity, y_p = 500 kN following the lower limit of the prediction interval line (green line). The results show a 7% difference in capacity between Case C and Case D respectively. The corresponding design depth can be estimated either from figure 5.12 or from the equation given on figure 5.12 for these two geotechnical design capacity values. These are: design depths 2.37 m versus 2.47 m (5.0% difference).

However, if Q_T = 200kN, Case C will provide a geotechnical design capacity of 230kN (figure 5.9) while for Case D this would still be 200 kN (figure 5.10, 15% difference in capacity). Figure 5.12 provides the design depth for R_n = 200 kN as 1.46 m while 1.6 m for R_n = 230 kN (9% difference in depth). Based on the analysis, it is recommended that the designer uses the maximum depth for the design to be conservative.

5.8 Summary

This section of the report presented a framework to determine the prediction interval of the foundation test capacity based on a limited number of full-scale test data. The lower limit of the prediction interval is analogous to characteristic capacity with a specified exclusion limit (section 2.4) and can be linked to the foundation strength factor (equation 5.12). The prediction interval is based on the calibration of data on full-scale test capacity against model predicted geotechnical design capacity. The calibration procedure uses a linear regression model. Furthermore, It is shown that the simple model (CIGRE, 2002) is a subset of the linear regression model when certain assumptions are met. The example problem presented in section 4 is reanalyzed using the linear regression model. It is shown that the designer can obtain a comparable foundation strength factor value using the linear regression model if the specific conditions are met.

6.0 CRITERIA FOR INTERPOLATION AND EXTRAPOLATION

6.1 General

The foundation strength factor, ϕ_F , obtained from a linear regression model has a valid range and is normally dependent on the predictor data range during the calibration process (i.e. geotechnical design capacity values along the x-axis x_0 or R_n values). Any interpolation and/or extrapolation should be done properly to ensure that the confidence level on the estimated mean test capacity value, $\hat{y}$ from equation 5.5), and the prediction interval used to determine a new foundation test capacity y_p , for a single geotechnical design capacity value x_0 , are obtained accurately.

6.2 Interpolation

The relationship between the predicted test capacity y_p for a given geotechnical design capacity, x_0 and the predictor (geotechnical design capacity, x_0) used in the calibration model applies strictly when the predictors have similar values that were used in the calibration model. If this is the case, then it is called interpolation.

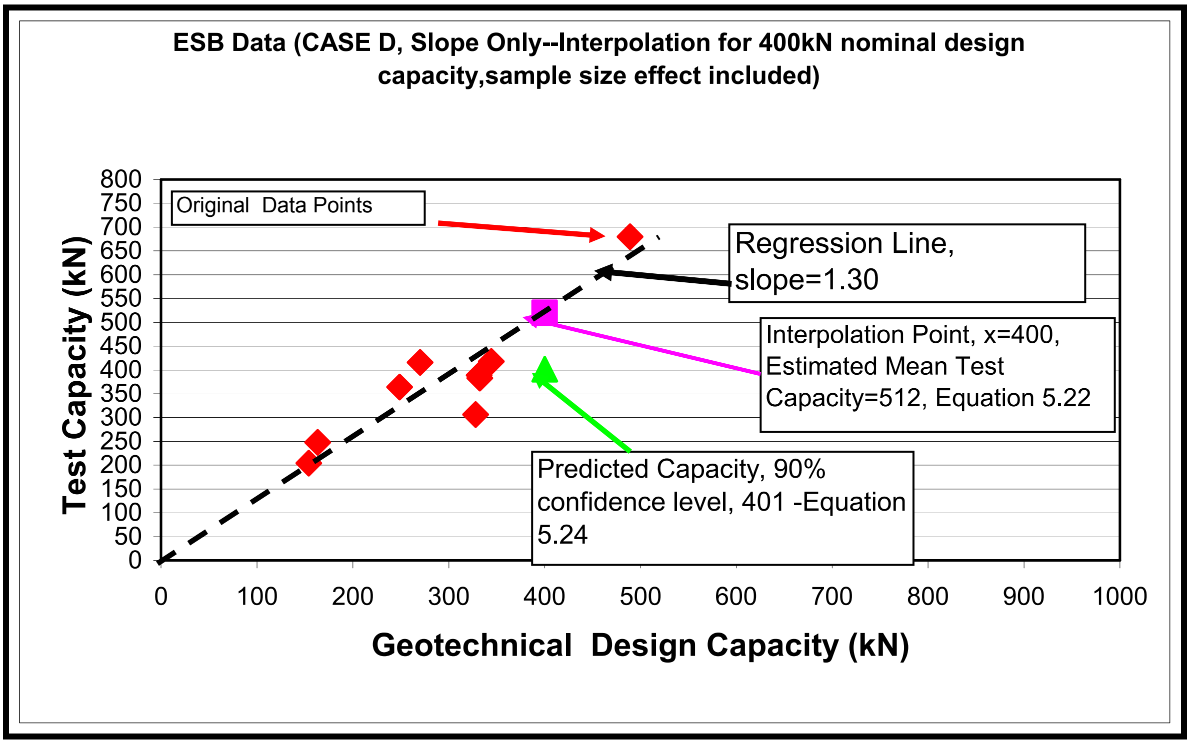


Figure 6.1 - Interpolation

Therefore, it is important to develop the linear regression model based on a wide data range (predictor values, x_0) to ensure that the large error is not introduced during interpolation. In this example (9 points data set), the geotechnical design capacity values along the x-axis (predictor values) range between 150kN and 500kN.

Figure 6.1 presents a case where a geotechnical design capacity value of 400kN is interpolated along the regression line for CASE D (equation 5.22). The estimated mean test capacity is 512kN ($\overline{m}$ =1.30, $\hat{y} = \overline{m}x_0$ = 400 x 1.30=512). On the same graph, the predicted test capacity (90% confidence level - 10% exclusion limit) is also depicted which has a value of 424kN (equation 5.24, $t_{\alpha,\nu} = 1.90$). All these computations are done based on CASE D model where the line passes through the origin and the variance of residuals is assumed to be non constant.

6.3 Extrapolation

The extrapolation process is making a prediction outside the range of values of the predictor used in the sample to generate the model. Therefore, the further the value of the predictor is moved from the mean value, the larger the error in the prediction (reference to section 5.2.2). Besides, there is no guarantee that the linear relationship holds outside the data range (Ang and Tang, 1975).

Figure 6.2 presents a case where the extrapolation is done for a geotechnical design capacity of 800 kN along the regression line (CASE D model). The estimated mean test capacity is 1040 kN (equation 5.22). On the same graph, the predicted test capacity (90% confidence level-10% exclusion limit) is also depicted which has a value of 849 kN (equation 5.24, $t_{\alpha,\nu} = 1.90$) where the slope is only considered (the line passes through the origin) and the variance of residuals is assumed to be non constant.

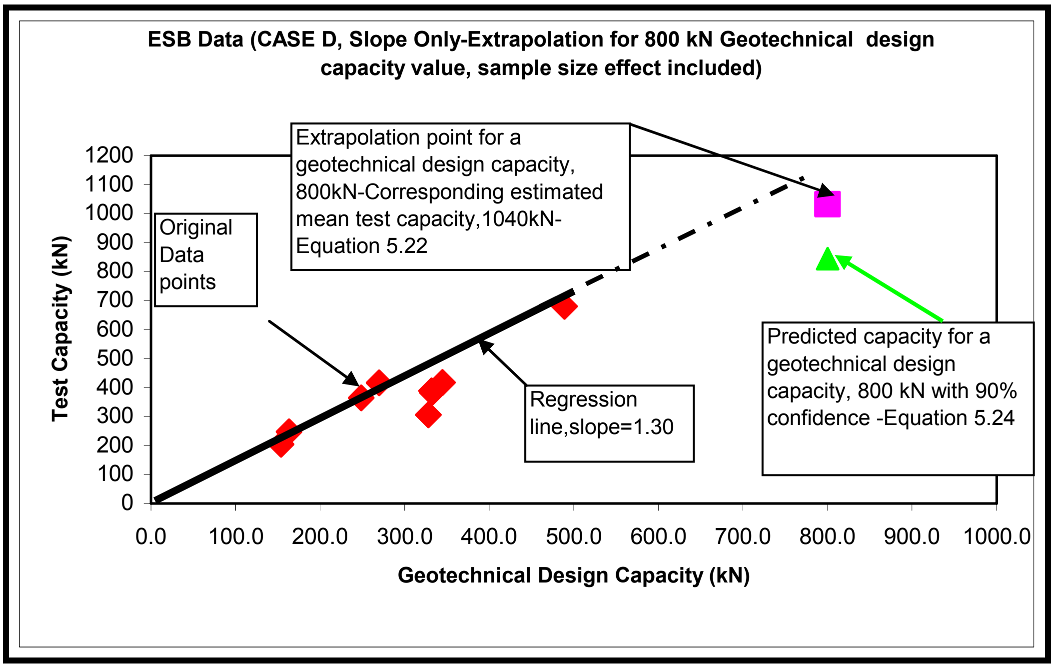


Figure 6.2 – Extrapolation

6.4 Foundation Strength Factor – Interpolation and Extrapolation

Figure 5.11 presents the foundation strength factor plot for various geotechnical design capacity values. The geotechnical design capacity values for ESB data varies between 150kN and 500kN. Between the limits of 300kN to 500kN, both cases (CASE C and CASE D) provide similar foundation strength factor values. However, as the predictor (geotechnical design capacity value) becomes less than 300kN, the foundation strength factor in CASE C starts to decrease compared to the value obtained in CASE D. Therefore, a judgment is needed when one tries to determine the foundation strength factor based on CASE C.

However, if the foundation designer believes that the variance of residuals is fully non constant based on figure 5.8, then a constant foundation strength factor value can be used for the entire data range (150 kN to 500 kN). Of course to extrapolate outside the data range, the assumption still is that the linearity is still valid in this range. As mentioned earlier, this is only valid when the line passes through the origin (no intercept) and the variance of residuals is strictly non constant. If this is not the case, the foundation strength factor can be dependent on the geotechnical design capacity values for certain data range.

6.5 Characteristic capacity given two geotechnical design capacity values

The predicted test capacities, y_p or the characteristic capacities, R_C for two geotechnical design capacity values, R_n are computed as follows:

6.5.1 Interpolation

R_C = $\phi_F R_n$ = 1.046 x 400 = 418kN (Table 5.5, Figure 5.10-Case C)

R_C = $\phi_F R_n$ = 1.00 0 x 400 = 400kN (Table 5.6, Figure 5.11-Case D)

The lower and upper ranges of the design value are 400kN to 418kN

6.5.2 Extrapolation

R_C = $\phi_F R_n$ = 1.135 x 800 = 908kN (Figure 5.10-Case C)

R_C = $\phi_F R_n$ = 1.000 x 800 = 800kN (Figure 5.11-Case D)

The lower and upper ranges of the geotechnical design capacity value are 800kN to 908kN respectively.

The user needs to be careful when extrapolating the data outside the range, particularly when the variance of residuals is constant (Case A and Case C). Also, there are no published guidelines how far one can go to extrapolate the predictor value. Correspondingly, it is recommended here that for constant variance of residuals case, we do not go normally beyond the $x_L - s_x$ or $x_U + s_x$ where x_L and x_U are the lower and upper value of the predictor data range and s_x is the standard deviation of the geotechnical design capacity. The foundation designer therefore needs to use judgment in extrapolating the data outside the range assuming that the linearity relationship holds. As the engineer gets more data from future tests, this data can then be added to the database and the analysis updated. The designer should collect data which covers the low and high ranges, particularly data near the low end (x=0.0); which could allow the intercept effect to be minimum provided the correlation is high. On the other had if the designer collects data only in the high end and the correlation is moderate, then most likely the intercept effect would be considerable and the extra caution should be used in extrapolating the data outside the range (toward the low end). The economical impact of not having data that covers both low and high ranges adequately could be increased cost (increased size of the foundation) depending on the specific case chosen with particular reference to calibration of foundation strength factor.

This section presented the limitation of the data extrapolation and some guidelines how to do this extrapolation. An example problem is shown using the foundation strength factor obtained in Section 5.0 (Tables 5.5 and 5.6).

6.6 Summary

This section presented a discussion on the issues related to interpolation versus extrapolation. Whereby, it is shown that the designer needs to be aware the foundation strength factor value can be dependent on the geotechnical design capacity for constant variance of residuals case, while for a non constant variance of residuals this is always constant. The designer also should be aware that in extrapolating foundation strength factor data outside the data range, the linearity assumption must hold. Furthermore, it is recommended here that the extrapolation should be not done normally beyond the limits of $x_L - s_x$ or $x_U + s_x$ unless the linearity assumption can be validated outside the data range.

7.0 SUMMARY AND CONCLUSIONS

7.1 Summary

This report presents the various steps necessary to determine the lower limit of the prediction interval of the foundation test capacity (similar to characteristic capacity) and to calibrate the foundation strength factor based on a limited number of full-scale foundation test data. The calibration is undertaken within the framework of a linear regression model. Initially, a brief review is presented with respect to confidence interval on the sample mean of the test capacity values and the prediction interval of a new test capacity value given a sample data of finite size. The confidence interval is used in the determination of required sample size for a foundation test programme while the lower limit of the prediction interval is linked to the characteristic capacity with a specified exclusion limit. This is followed by a brief review outlining the deterministic as well as the semi-probabilistic design methodologies such as ASD (Allowable stress design), IEC 60826 (2003) and ASCE (LRFD).

The foundation designer is given guidance on how to undertake a full-scale foundation test programme. Several steps need to be completed before a full-scale foundation test programme can be commenced. These include: (a) selection of the test site(s), (b) selecting the applicable foundation geotechnical design model(s) to determine the foundation's geotechnical design capacity, (c) subsurface investigation to determine the various geotechnical strength parameters for the foundation design model(s) and (d) sample size requirement for the test program. Once the foundation's geotechnical design capacity has been determined, the foundation designer is presented with a number of steps required to carry out a full-scale foundation test programme. This generally includes the choice of test site locations, installation, loading scheme and sequence (load cycles), instrumentation and data collection, data reduction and finally the test results in terms of a typical load versus displacement curve.

Subsequently, a reference is made to IEC 61773 which outlines the available methodologies to determine the foundation's test capacity in terms of damage limit as well as the ultimate limit capacity based on the test result. A review of the information presented shows that the estimated ultimate capacity could vary considerably depending on the method selected. However the foundation designer can also choose a damage capacity (IEC 60826) based on a prescribed displacement (subjective). For a pad and chimney foundation used in the example, load at a 10 mm displacement is defined as the test capacity (Buckley, 1994).

Once the foundation's test capacity and the geotechnical design capacity have been determined for specific site(s), the foundation designer can choose a simple calibration model based on $m = \frac{Test\ \ Capacity}{Geotechnical\ Design\ \ Capacity}$ ratio(s). However, to use this simple model ($\overline{m}$ -model) the foundation designer needs to ensure that the correlation value of the data is high (table 4.2) and the coefficient of variation of the m-values, v_m (model error) is low.

For other situations where this is not the case, the foundation designer may need to carry out a more detailed analysis (full linear regression model) to ensure that the intercept (ordinate at origin) effect can be neglected. This can be undertaken using a simple statistical test. If the intercept effect cannot be neglected, then the linear regression model (an alternative methodology) is also presented on how to include this in the analysis to estimate the prediction interval of the test capacity given a geotechnical design capacity value. Moreover, the effect of constant or non constant variance of residuals is presented and its effect is emphasized in analyzing the data.

Finally a design example is presented based on ESB test data on undercut pad and chimney foundation. A discussion on the limitation of data extrapolation is also presented.

7.2 Conclusions

Some important conclusions are drawn from this report:

- A review of the current **CIGRE** method (CIGRE, 2002) is presented; which has examined the underlying assumptions and their limitations in a systematic manner. The method is primarily based on the linear regression model and it is shown that it is valid under very specific conditions and assumptions, as detailed below.

- The CIGRE Method relies on two very specific conditions which imply that the intercept should be zero and the residual (the difference between the actual test capacity and the estimated mean test capacity) should vary with the geotechnical design capacity values in an increasing manner.

- To apply this CIGRE method to a wide range of reliability based foundation design calibrations with various degrees of correlation, the designer must be aware that unless these specific conditions are met, the prediction interval with a high degree of confidence (90% for IEC 60826 and 95% for ASCE) will produce results that may not be valid.

- A simple statistical test is proposed here to provide guidance to the designer so that the assumption of "zero intercept" is validated properly.

- In addition, it is shown that the CIGRE method does not consider the error due to small sample size in calculating the prediction interval. CIGRE method is similar to Case D (equation 5.24) but does contain an additional term $[1+\frac{1}{n}]^{0.5}$. Therefore the CIGRE method is modified to include the effect of small sample size. The difference in the foundation strength factor value could be 2 % to 6% depending on the sample size, n and v_m values.

- If the correlation value, r^2 is greater than **0.8** and the coefficient of variation, v_m is low, the intercept can be neglected without introducing significant error in the analysis. Under this condition, CIGRE method **(similar to CASE D)** is quite acceptable provided the variance of residuals is non constant.

- However; if the correlation value, r^2 is greater or less than equal to **0.8 (**table 4.2**)** and the coefficient of variation v_m starts to increase (> **0.10)**, the designer needs to carry out a full regression analysis to choose a particular option and/or options from the flow diagram (figure 5.6) to ensure that the analysis result provides not only a predicted capacity which produces a conservative design, but also that the design is optimal (economical).

- A simple example problem with 3-point test data is illustrated in Section 9 ("Appendix") to show the designer how to calculate the various steps manually to check the CIGRE method. The results based on the CIGRE method are not only verified but the limitations are also identified.

- An example problem is illustrated for a typical pad and chimney foundation in cohesionless soils. The comparison is excellent between the results obtained from $\overline{m}$ - model and CASE D.

- Finally, it is shown that the general regression model with the unique test for **"zero intercept"** is quite applicable to the analysis of a wide range of foundation test data with various degree(s) of correlation, coefficient of variations and $\overline{m}$ values. The methodology presented here is quite **robust** and provides a consistent basis for analyzing a wide range of test data. The foundation designer is able to review the details of these calculations in order to understand the intricate relationships of various parameters that contribute to the estimation of the foundation strength factor, ϕ_F values. The foundation designer can make an informed decision to choose a particular option or options in determining the predicted test capacity of the foundation with a specified exclusion limit.

8.0 REFERENCES

Ang, A. H. S. and Tang, W.H. 1975 Probability Concepts in Engineering Planning and Design, Volume 1, John Wiley & Sons

Ang, A. H. S. 2002 Review of the "Current Method" of Calibration and a Proposal for an "Alternate Method" for Calibration of Foundation Strength factor- prepared by Asim Haldar, September 2001, presentation to CIGRE WG07 meeting in Oslo, Norway.

Ang, A. H. S. 2002 Personal communication

Benjamin, J. and Cornell, Allin 1970 Probability, Statistics and Decisions for Civil Engineers, McGraw Hill

Bowker, A. H. and Lieberman, G. J. 1972 Engineering Statistics, Prentice Hall, NY. 641 p

Brink, A.B.A, Patridge, T.C. and Williams, A.A.B. 1982 Soil Survey for Engineering, Clarendom Press, Oxford, 378 p

Buckley, M. 1994 Reliability Based Design of OHL Foundations, CIGRE Publication, SC 22-203, Overhead Lines, Paris Meeting, August 28-September 3,11p.

Buckley, M. 2001 Example of Probabilistic Foundation Design, CIGRE WG meeting, Heidelberg, Germany, 8p

Buckley, M. 2007 Background Information on ESB Tests, e-mail communication, 2p.

CIGRE 2002 The Design of Transmission Line Support Foundations- An Overview, WG 22.07, August, Technical Brochure No. 206

CIGRE 2006 Foundation Installation-An Overview, WG B2.07, May, Technical Brochure No. 306

Davidson, H.L. 1982 Laterally Loaded Drilled Pier Research, Volumes 1 and 2, EPRI EL 2197, RP 1280-1, prepared by GAI Consultant, USA

Digioia et al 1993 A comparison of Various Methods for Predicting the Response of Drilled Shafts Subjected to High Moments, ELECTRA No. 149, 1993

Digioia Jr., **A.M.**, 2000 Reliability Based Design and Assessment of Foundations for Transmission Line Structures, IEEE T & D World Expo, April, 26-28, Cincinnati, Ohio

EN 50341-3-4 2002 Overhead Electrical Lines Exceeding AC 45 kV-Part 3-4: National Normative Aspects (NNA) for Germany

ESB Test Report on Overhead Line Foundations

EPRI EL 2870 1983 Transmission Line Structure Foundations for Uplift-Compression Loadings, Electric Power Research Institute, Palo Alto, California.

IEC 60826 2003 Design Criteria of Overhead Lines

IEC 61773 Testing of Structure Foundations For Overhead Lines.

IEEE 1985 Transmission Structure Foundation Design Guide

Killer, J. 1953 Economical Foundations of Towers for High Voltage Transmission Lines, Third Soil Mechanics Conference, Zurich, Switzerland

Melchers, R. 1999 Structural Reliability Analysis and Prediction, John Wiley

Mors, H. 1964 Methods of Dimensioning for Uplift Foundations of Transmission Line Towers, Paper No 210, CIGRE, Paris, France

Prasad, Yenumula and Haldar, Asim 2001 Full-scale Tests On Grillage Anchor Foundations for High Voltage Lines, Proc. Canadian Geotechnical Conference, Calgary

Rao, S.S. 1992 Reliability Based Design, McGraw Hill Publication

Sneddon, Gary 2003 Review of RBD Reports, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada

Sneddon, Gary 2004 Personal Communications, Department of Mathematics, Memorial University of Newfoundland, St. John's, Canada

9.0 APPENDIX

9.1 Example Problem (refer to Figure 3.13)

This section will provide the user a step-by-step procedure for analyzing the data following the flow diagram in figure 5.6. Each step is shown clearly so that the user can understand the calibration process in determining the foundation strength factor for a specific foundation dataset. The user is still required to use some judgment in selecting a particular option or a combination of options to ensure that the foundation strength factor is not only conservative but also optimum. Details of the calculations are presented here**.** All calculations are based on 90% confidence level (10% exclusion limit). Figure 9.1 presents the three point scattered test data along a transmission line route (section 3.6.2, reproduced figure 3.11). Figure 9.2 presents the flow diagram (reference figure 5.6) which will be followed in computing the foundation strength factor for this dataset. Table 9.1 presents the various parameters that need to be computed to determine the foundation strength factor. A summary of the methodology is presented in two steps.

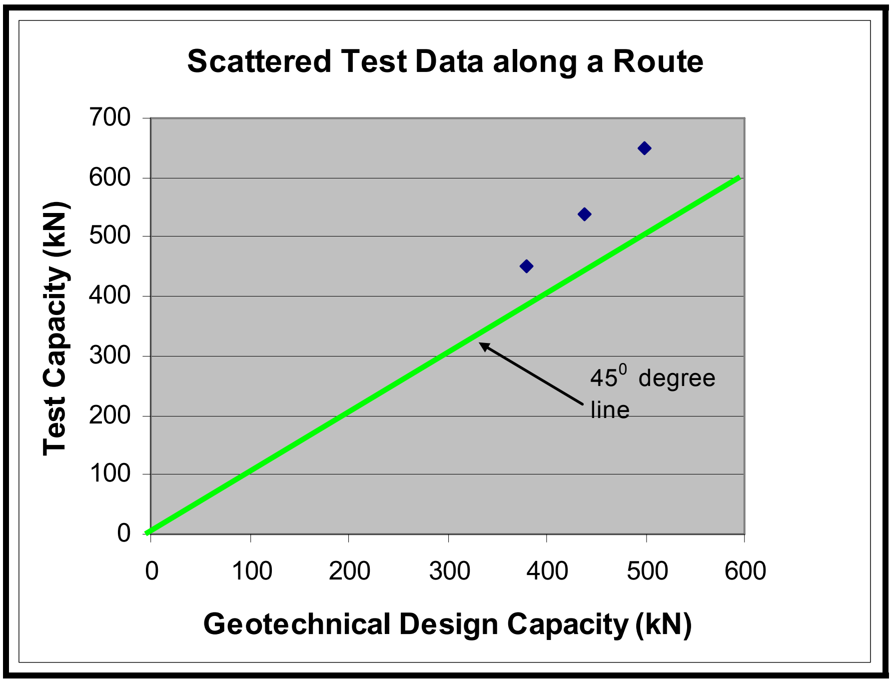


Figure 9.1 - 3-point Scattered test data plot

In the first step, a routine linear regression analysis is carried out by minimizing the residual errors across the all data points. Underlying assumption of the linear regression analysis is that the regression considers an intercept and the variance of residuals is constant. To ensure that the intercept can be neglected, a null hypothesis is tested and a decision is made either to include or exclude the intercept. In the second step, residual error values are plotted to test the assumption of constant variance of residuals. If this is not validated, then an appropriate weight is applied to each data point and the regression analysis is carried out on the transformed data, with or without the intercept. Table 9.2 presents the necessary equation numbers for four cases referred in Table 5.2.

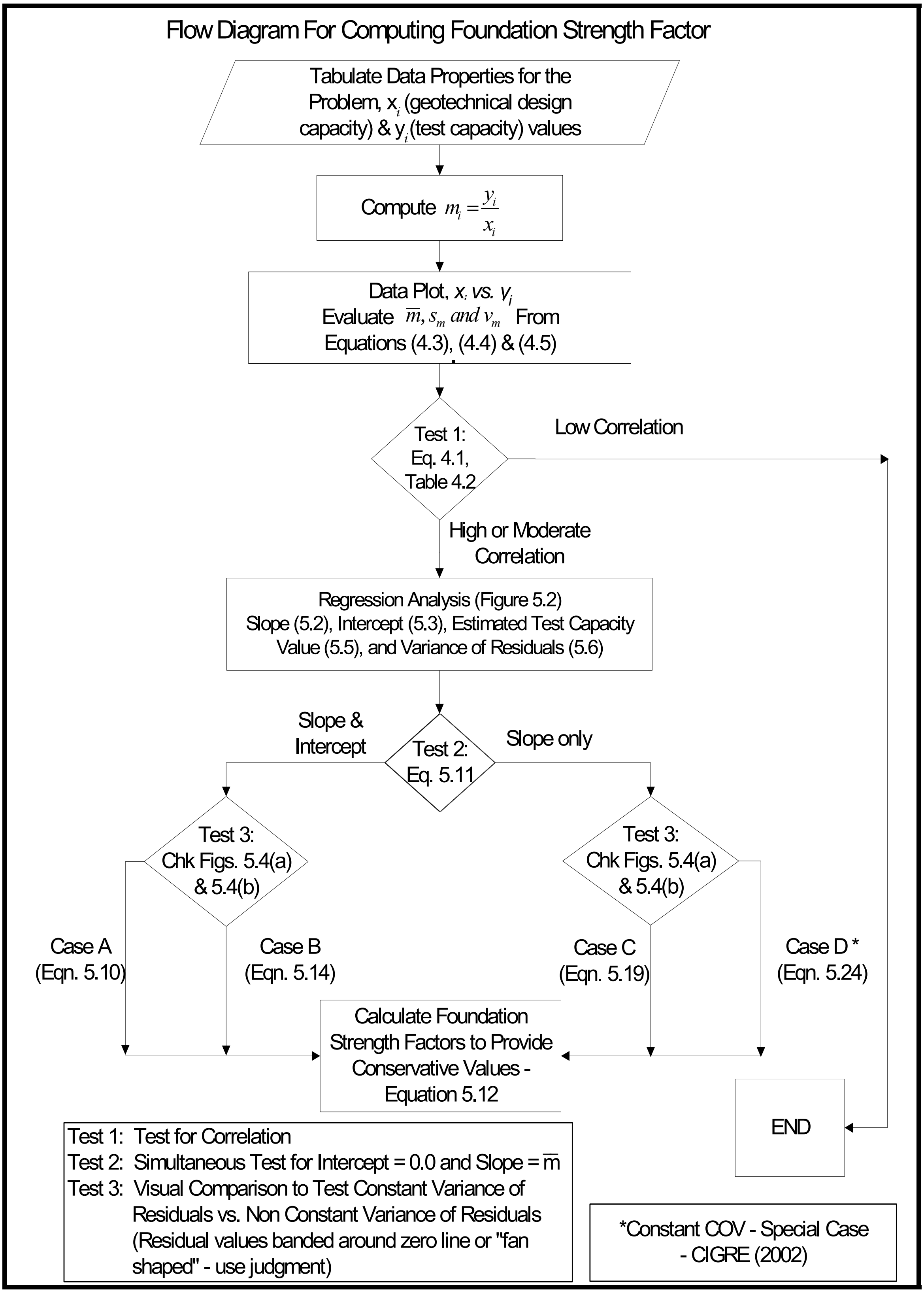


Figure 9.2 - Flow diagram (figure 5.6, reproduced)

9.2 Calibration Methodology (Summary)

Table 9.1: Methodology to Follow the Flow Diagram

Items	Parameters	Sections	Equation Numbers
Data Properties for the Problem	Geotechnical Design Capacities, x_i	§3.2 to §3.4	
	Test capacities, y_i	§3.5 to §3.7	
Calculation of mean value, standard deviation and coefficient of variation x_i and y_i values	$\bar{x}$, s_x & v_x	§4.1	Use equations (2.1) to (2.3) replacing R_i and $\bar{R}$ by x_i (or y_i) and $\bar{x}$ (or $\bar{y}$) respectively.
	$\bar{y}$, s_y & v_y	§4.1	
Compute m_i values;	$m_i = \frac{y_i}{x_i}$	§4.2	4.2
Compute the statistics of m_i values	$\bar{m}$, s_m, & v_m	§4.2	4.3,4.4 and 4.5
Linear regression analysis- ➢ correlation value, r^2	$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1)\, s_x s_y}$	§4.1	4.1
➢ residual error value, ε_i	$\varepsilon_i = y_i - \hat{y}_i$	§5.2.1	5.1 and 5.5
➢ variance of residuals, $s_{xy}{}^2$:	$s_{xy}{}^2 = \frac{\sum_1^n w_i (y_i - \hat{y}_i)^2}{n-2}$	§5.2.1	5.6

Test 1 on correlation value, r^2 according to Table 4.2

Test 2 on intercept, "a" according to Equation (5.11) and Table 5.1

Test 3 on variance of the residuals of errors, ε_i according to Figures 5.4 a & b

Table 9.2: Reference Equation Numbers for Various Cases (Table 5.2)

Case	ν	w_i	b	a	$\hat{y}^{(1)}$	s_{xy}	y_p (analogous to R_C) for a single geotechnical design capacity, x_0 (or R_n)	ϕ_F
A	n-2	1.0	(5.2)	(5.3)	(5.5)	(5.6)	(5.10)	(5.12)
B	n-2	$\frac{1}{x_i^2}$	(5.2)	(5.3)	(5.5)	(5.6)	(5.14)	(5.12)
C	n-1	1.0	(5.15)	0.0	(5.17)	(5.16)	(5.19)	(5.12)
D	n-1	$\frac{1}{x_i^2}$	(5.20)	0.0	(5.22)	(5.21)	(5.24)	(5.12)

(1) $\hat{y}$ *is* $\hat{y}_i$ *at* $x_i = x_0$

9.3 Table 9.3- Scattered test data (Table 3.6 reproduced)

Table 9.3- Scattered test data

Foundation Capacity (kN)	**Site A**	**Site B**	**Site C**
Geotechnical design capacity	379	439	498
Test Capacity (at 10mm displacement)	450	540	680
$m = \frac{Test\ Capacity}{Geotechnical\ Design\ Capacity}$	1.19	1.23	1.37

9.4 Compute m_i values – (use equation 4.2)

$\overline{m} = (\sum_{1}^{n} m_i)/n$ = $[\frac{450}{379}+\frac{540}{439}+\frac{680}{498}]$/3 = [1.19 +1.23+1.37]/3 = 1.261

Next we calculate s_m and v_m as using equations (4.3) and (4.4)

$$s_m{}^2 = \frac{1}{n-1}\sum_{1}^{n}(m_i - \overline{m})^2 = \frac{1}{3-1}[(1.19-1.261)^2 + (1.23-1.261)^2 + (1.37-1.261)^2]$$

=0.009

$$v_m = \frac{s_m}{\vec{m}} = \frac{0.093}{1.261} = 0.0737$$

9.5 Correlation check (Test No. 1- equation 4.1 and replace y by R)

To do the following calculations use equations (2.1-2.3);

$$\overline{R} = (\sum_{1}^{n} R_i)/n = \text{(450+540+680)/3} = 556.7 \text{ kN}$$

$$s_R{}^2 = \frac{1}{n-1}\sum_{1}^{n}(R_i - \overline{R})^2 = \frac{1}{3-1}[(450-556)^2 + (540-556)^2 + (680-556)^2]$$

$$= (115.9)^2 \text{ kN}$$

$$v_R = \frac{s_R}{\overline{R}} = \frac{115.9}{556.7} = 0.208$$

Similarly the mean, standard deviation and the coefficient of variation of x-Values (Geotechnical design capacity) are $\overline{x}$ = 438.7 kN, s_x =59.5 kN and v_x = 0.136. The correlation coefficient r , value is calculated directly from equation (4.1).

$$r = \frac{1/(n-1)[\sum_{1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{s_x s_y}$$

$$= \frac{1/2[(379-438.7)(450-556.7)+(439-438.7)(540-556.7)+(498-438.7)(680-556.7)]}{59.5x115.9}$$

$$= 0.992$$

Therefore, the correlation value r^2 = 0.983 is high and the linear association between the test capacity y and the geotechnical design capacity x is strong (table 4.2). Therefore, the dataset passed the **Test 1** according to the Flow diagram (Figure 9.2).

9.6 Regression Analysis (variance of residuals is constant)

Table 9.4 - Section 5.2.1 (Case A, $\overline{x}$ = 438.7 kN, $w_i = 1.0$ *)

Test number	Geotechnical design capacity, x_i	Damage Limit Capacity, y_i	$w_i(x_i - \overline{x})y_i$	$w_i(x_i - \overline{x})^2$
1	379	450	-26850	3560
2	439	540	180	0.11
3	498	680	40347	3520
Sum = $\sum$			13676	7080

* For basic regression analysis, the variance of residuals is always constant unless it is proven otherwise. Therefore, $w_i = 1.0$

The slope b, is calculated from equation (5.2) b = $\frac{\sum w_i (x_i - \bar{x}) y_i}{\sum w_i (x_i - \vec{x})^2}$ = $\frac{13676}{7080}$ =1.93

The intercept is calculated from equation (5.3)

a = $\bar{y} - b\bar{x}$

a = 556.7- 1.9316x 438= -290kN

The estimated mean test capacity $\hat{y}_i$ values are obtained based on regression analysis from equation (5.5)

$\hat{y}_i = a + b(x_i)$ = -290 + 1.9316 x_i

Table 9.5 – Variance of residuals calculation ($w_i = 1.0$)

Test Number	**Geotechnical design capacity** x_i	**Damage Limit Capacity** y_i	$\hat{y}_i = a + b(x_i)$	**Residual** = $(\hat{y}_i - y_i)$	**Square of Residuals** = $(\hat{y}_i - y_i)^2$	x_i^2
1	379	450	441.4	8.58	73.65	143641
2	439	540	557.3	-17.31	299.65	192721
3	498	680	671.3	8.72	76.17	249004
Sum = $\sum$				1.7e-13	449.49	584366

$$s_{xy}^2 = \frac{\sum_{1}^{n} w_i (y_i - \hat{y}_i)^2}{n-2} = \frac{[(450-441)^2 + (540-557)^2 + (680-671)^2]}{(3-2)}$$ = 449.49;

Therefore, s_{xy} **= 21.1;**

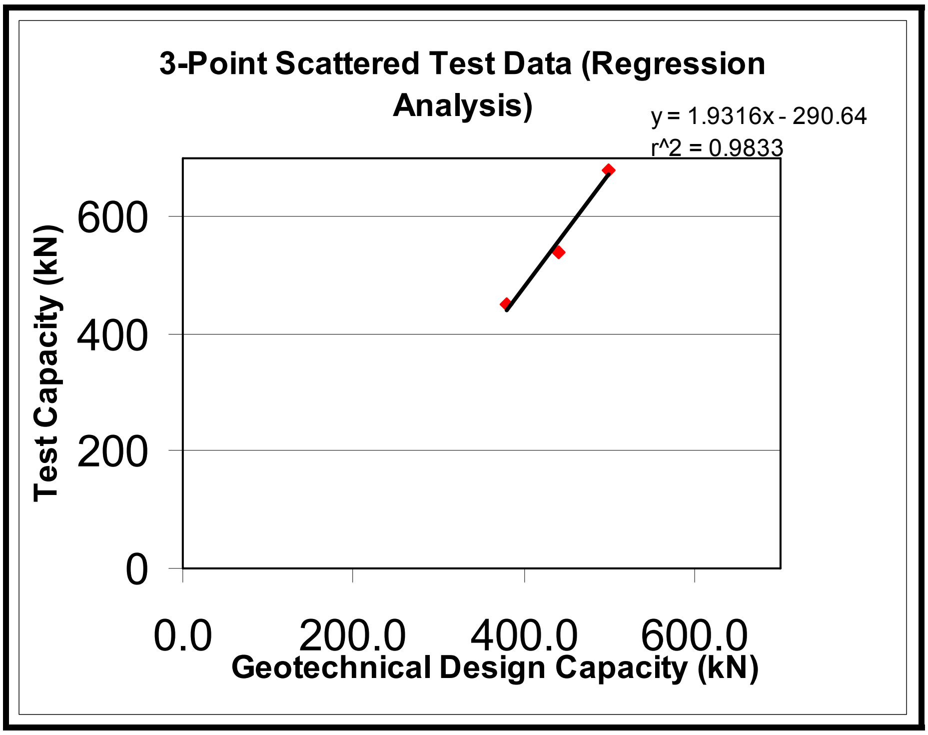


Figure 9.3 - Linear regression plot

9.7 "Zero Intercept" Check – [Test No. 2 - equation 5.11]

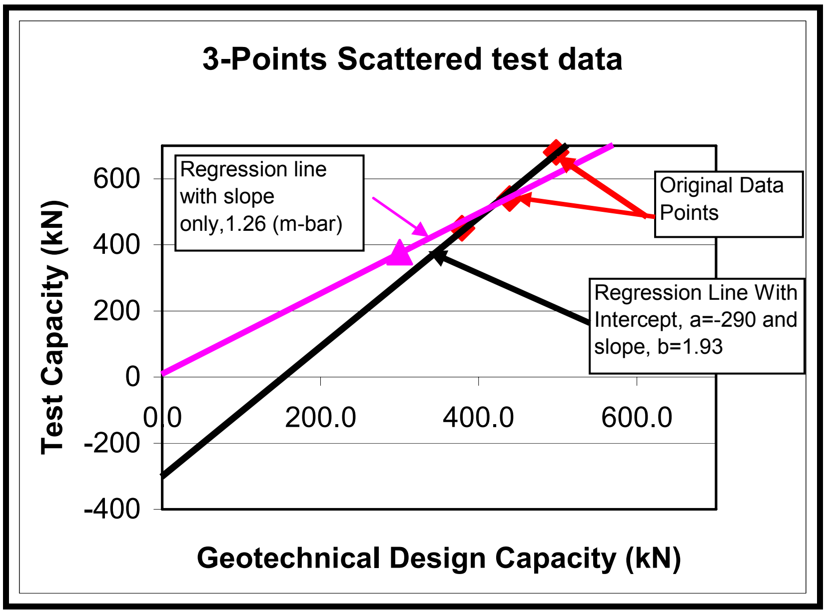


Figure 9.4 - Linear regression plot (with intercept and without intercept)

$$F^{*}=[n*(a-0.0)^{2}+2*n*\bar{x}*(a-0.0)(b-\bar{m})+(\sum x_i{}^{2})(b-\bar{m})^{2}]/(2*s_{xy}{}^{2})$$

where

n= 3 (sample size),
a = -290kN (intercept),
b = 1.93 (slope),
$\overline{m}$ =1.26 (slope based on "zero intercept")
$\sum x_i^2$ = 584366
s_{xy} **=** 21.1kN (standard deviation of the error in the linear regression analysis)
$\overline{x}$ = 438kN (mean value of 3 geotechnical design capacities)
$F_{\alpha,2,n-2}$ = $F_{0.10,2,1}$ =49.50 (Table 5.1)

The three terms in equation 5.11 are calculated below.

Table 9.6 - Terms in equation (5-11)

First Term- **n (a-0.0)^2**	**Second Term-** **2n** $\overline{x}$ **(a-0.0)(b-** $\overline{m}$ **)**	**Third Term-** $(\sum x_i^2)(b-\overline{m})^2$	**F*=** $\sum Sum$
3(-290.64)^2= 253415	2*3*438(-290-0.0)(1.93-1.26) = -483114	584366*(1.93-1.26)^2 = 233078	3378

$$F^* = \frac{\sum Sum}{2*s_{xy}^2} = \frac{3378}{2*21^2} = 3.78 < F_{\alpha,2,n-2} = 49.50 \text{ (Table 5.1)}$$

Therefore, we accept the Hypothesis that the intercept is zero and the slope is equal to $\overline{m}$ = 1.261 ($\overline{m}$ **- model, Section 4**); All results are summarized in Table 9.5.

Table 9.7- Data statistics for 3 – point scattered test data problem

n	R^2	$\overline{x}$	s_x	$\overline{y}$	s_y	**a**	**b**	s_{xy}	$\sum x_i^2$	**F-value**	$\overline{m}$	v_m
3	0.983	438	59	556	116	-290	1.93	21	584366	**3.78 < 49.50**	1.26	0.0737

Note:

Similar calculations can be done for ESB data and Table 5.4 can also be produced following the above steps.

9.8 Variance of residuals-Test (Test No. 3- Figures 5.4a and 5.4b)

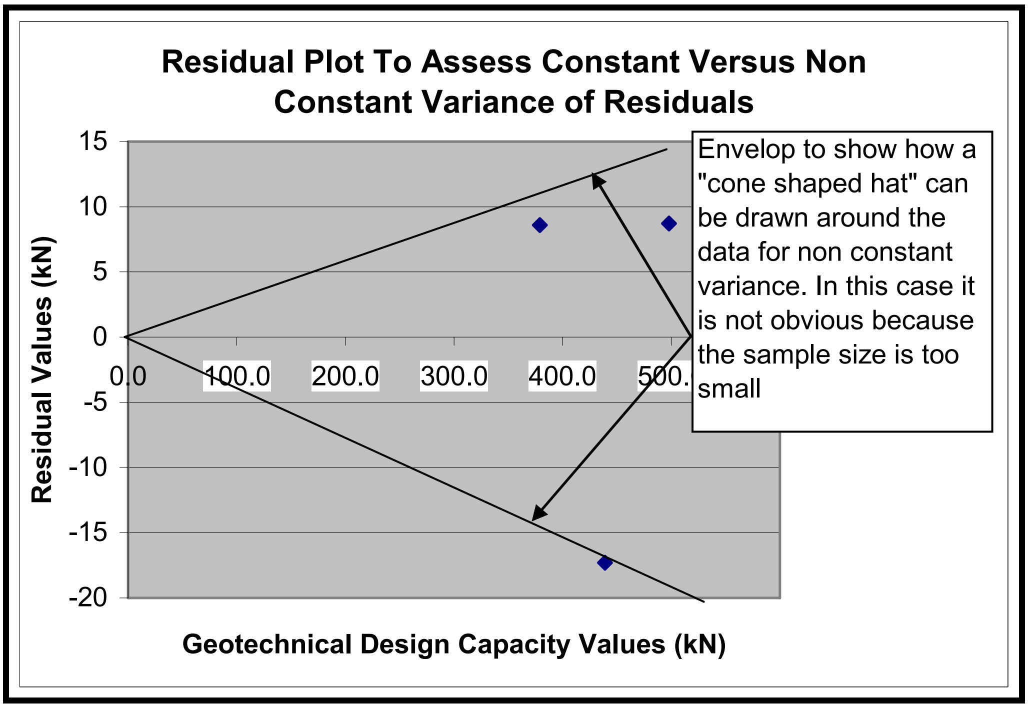


Figure 9.5 - Residual values versus Geotechnical design capacity

In this example we assume the variance of residuals is not fully non-constant because of limited data point. In actual foundation design problem, the user will have more data points and therefore, a more rational decision can be made with respect to residual variation. For details refer to Section 5.0 of the report.

9.9 Predicted Test Capacity (sample size effect included)

CASE C (w_i **= 1.0) - equations (5.15) – (5.19)**

Table 9.8 - Variance of Residuals-Calculations

Test Number	Geotechnical design capacity, x_i (kN)	Test Capacity, y_i (kN)	$w_i(x_i)y_i$	$w_i(x_i)^2$	$\hat{y}_i = bx_i$	$(\hat{y}_i - y_i)^2$
1	379	450	170550	143641	484	1155
2	439	540	230760	192721	560	425
3	498	680	338640	248004	638	1940
Sum			746350	584366		3520

Following section 5.3.4 and table 9.2, the slope, b is calculated as

$$b = \frac{\sum w_i(x_i)y_i}{\sum w_i(x_i)^2} = \frac{746350}{584366} = 1.277$$

The standard error of estimate (SEE) is obtained from equation (5.17)

$$s_{xy}{}^2 = \frac{\sum w_i(y_i - \hat{y}_i)^2}{n-1} = \frac{3520}{2} = 1760$$

Therefore, s_{xy} = 41.9

The predicted test capacity is computed from equation (5.19) for $k_\alpha = t_{\alpha,\nu}$ (small sample size n=3)

$$y_p = \hat{y} - t_{\alpha,\nu} * s_{xy} [1 + \frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}]^{0.5}$$

For a sample size n=3, $t_{\alpha,\nu}$ = 1.90 and is obtained from Table 2.2 (90% value for one sided interval). Therefore, the second term in the above equation for x_0 =200 kN becomes

$$P = t_{\alpha,\nu} * s_{xy} [\frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}]^{0.5} = 1.90 * 42 * [1 + \frac{200^2}{584366}]^{0.5} = 82.39$$

Table 9.9 presents the data for four geotechnical design capacity values.

Table 9.9 - Predicted Test Capacity

Geotechnical design capacity, x_0	**Estimated Mean Test Capacity Value** $\hat{y} = bx_0$ **(first term in equation 5.19)**	**P=** $t_{\alpha,\nu} * s_{xy} [1 + \frac{(x_0)^2}{\sum_1^n (x_i)^2 w_i}]^{0.5}$ **(second term in equation 5.19)**	$y_p = \hat{y}$ **-P (equation 5.19)**
200	255	82	173
400	511	90	421
600	766	101	665
800	1022	115	907

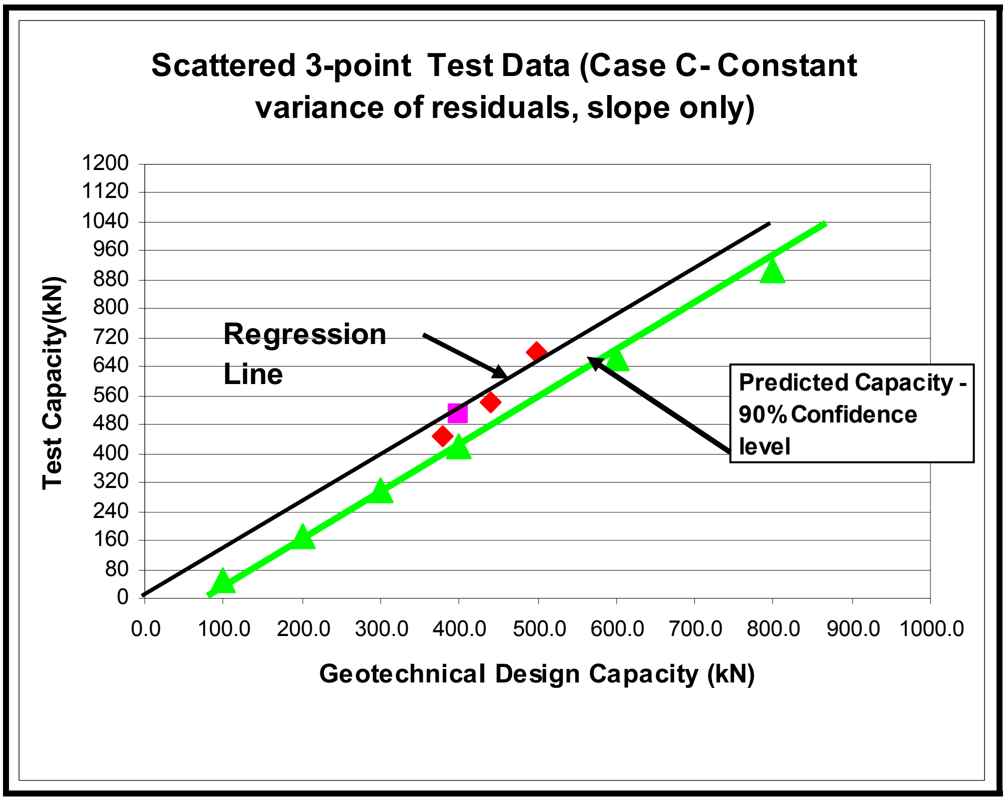


Figure 9.6 - Limit of Prediction Interval (case C)

CASE D ($w_i = \dfrac{1}{x_i{}^2}$) – (equations 5.20 -5.24)

Table 9.10 – Variance of Residuals Calculations

Test Number	Geotechnical design capacity, x_i (kN)	Test Capacity, y_i (kN)	$w_i(x_i)y_i$	$w_i(x_i)^2$	$\hat{y}_i = bx_i$	$w_i\,(\hat{y}_i - y_i)^2$
1	379	450	1.19	1.0	478	0.0054
2	439	540	1.23	1.0	554	0.0010
3	498	680	1.37	1.0	628	0.0109
Sum			3.78	3.0		0.0173

The slope is calculated from Equation (5.20)

$$\mathbf{b} = \frac{\sum w_i(x_i)y_i}{\sum w_i(x_i)^2} = \overline{m} = \frac{3.78}{3} = 1.261;$$

The variance of residuals (standard error of estimate) is obtained from equation (5.21)

$s_{xy}{}^2 = \frac{\sum w_i (y_i - \hat{y}_i)^2}{n-1} = \frac{0.0173}{2}$ =0.00865

s_{xy} = 0.093; in this case $s_{xy} = s_m$ (equation 5.21)

The "hat" on y-values represents the estimated mean test capacity and is obtained from equation (5.21) as $\hat{y} = \overline{m} x_0$ where $\overline{m}$ is the slope. Therefore the coefficient of variation v_m = $\frac{s_m}{\overline{m}} = \frac{0.093}{1.26}$ =0.0737. For a geotechnical design capacity value x_0 = 200kN, the estimated mean test capacity value is obtained from equation (5.22) in which $\overline{m}$ =1.26 and the estimated mean test capacity value is 252 kN. Equation (5.24) provides

$$y_p = \overline{m}\ x_0 - t_{\alpha,\nu}\ s_m\ x_0\ [1+\frac{1}{n}]^{0.5}$$

Therefore, for x_0= 400kN, y_p = {1.26 * 400- 1.90 * 0.0735* $[1+\frac{1}{3}]^{0.5}$ * 400} = 422.78kN

Table 9.11- Predicted Test Capacity

Geotechnical design capacity, x_0	**Estimated Mean Test Capacity Value** $\hat{y} = \overline{m} x_0$ **First term in equation (5.24)**	**P=$t_{\alpha,\nu} * s_m [1+\frac{1}{n}]^{0.5} x_0$** **Second term in equation (5.24)**	**$y_p = \hat{y}$ -P (equation 5.24)**
200	252	41	211
400	504	82	422
600	757	122	635
800	1009	163	846

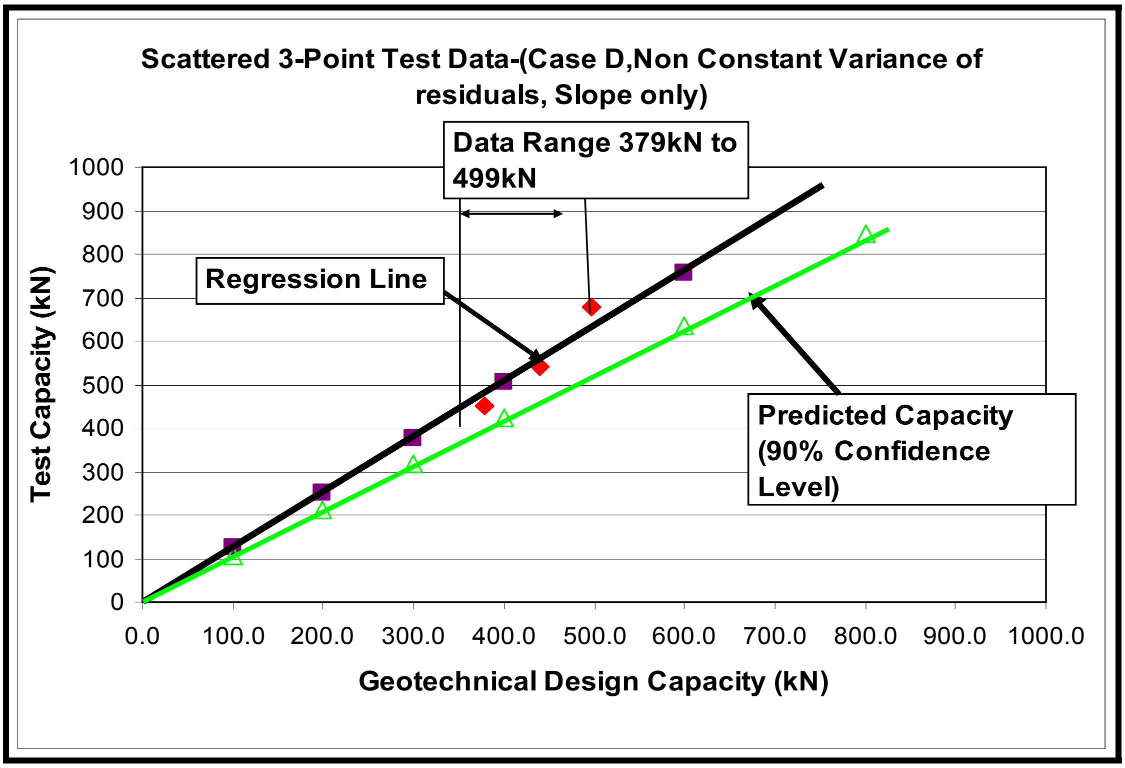


Figure 9.7 – Lower Limit of Prediction Interval (Case D)

9.10 Comparisons of foundation strength factor values (Case C and Case D)

Sample size effect included

The foundation strength factor is obtained from equation (5.12)

$$\phi_F = \frac{y_p}{x_0} ..$$

For a geotechnical design capacity value x_0 = 400 kN we obtained y_p =423 kN for CASE D; Therefore, $\phi_F = \frac{422.78}{400}$ =1.057

Table 9.12 - Comparison of foundation strength factor (Case C & Case D)

Geotechnical Design Capacity Value, x_0	CASE C		CASE D	
	Predicted Test Capacity	Foundation Strength Factor	Predicted Test Capacity	Foundation Strength Factor
200	173	0.865	211	1.057
400	421	1.052	422	1.057
600	665	1.108	635	1.057
800	906	1.133	846	1.057

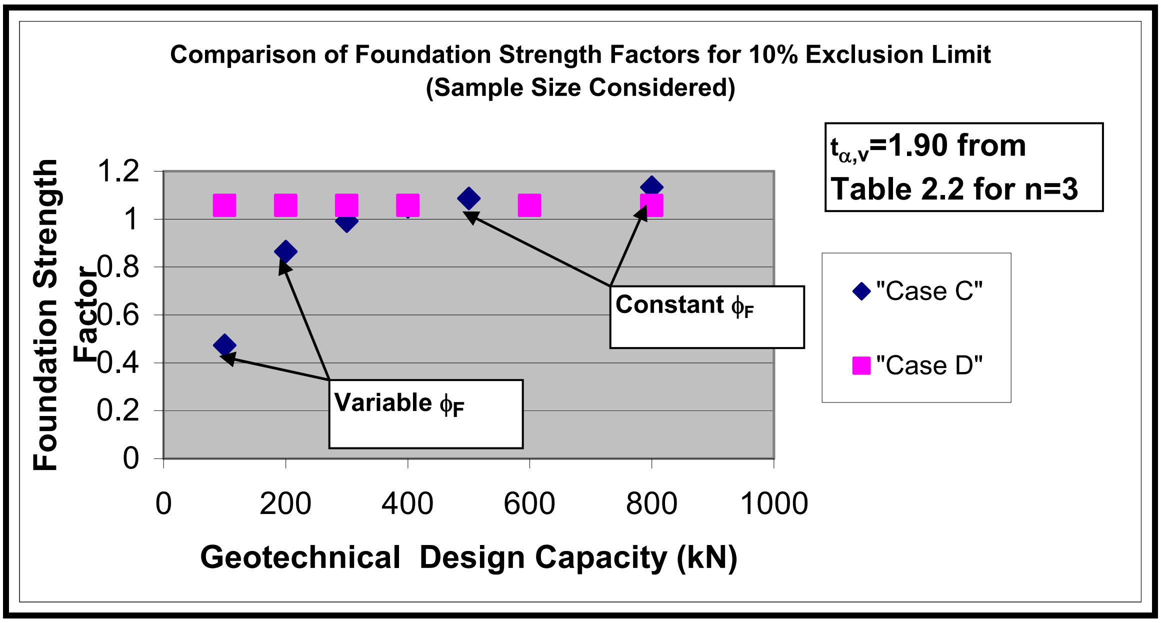


Figure 9.8 - Comparison of foundation strength factor

9.11 CIGRE (2002)

$\phi_F = \overline{m}\,[1- t_{\alpha,v} * v_m\,]$ = 1.261 (1-1.90 x 0.0737) = **1.084 (CIGRE, 2002);**

$\phi_F = \overline{m}\,[1- t_{\alpha,v} * v_m\,[1+1/n]^{0.5}]$ = 1.261 (1-1.90 x 0.0737[1.33]$^{0.5}$) = **1.057; (CASE D)**

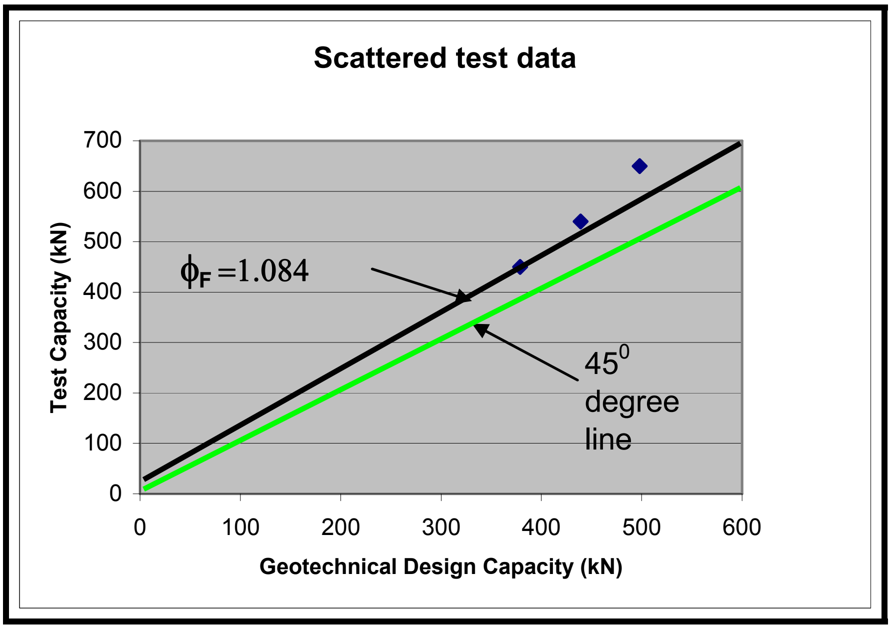


Figure 9.9 – Foundation Strength factor (Buckley, 1994 & CIGRE, 2002)